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# ***Energy***

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## ***Conversion and Utilization***

***Second Edition***

**JERROLD H. KRENZ**

**University of Colorado**

***Allyn and Bacon, Inc.***

***Boston      London      Sydney      Toronto***



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# Preface

Since the publication of the first edition of *Energy: Conversion and Utilization*, the perception of energy-related problems has changed markedly. While there has been a concerted research and development effort in the area of energy supplies based upon alternatives to fossil fuels, there also has been a greater realization that fossil fuels will continue to supply a substantial portion of the energy needs of industrial societies for the next few decades. Concurrently, it has been "discovered" that the potential for reducing energy usage is far greater than was generally accepted during the early 1970s. The new revised and greatly expanded edition addresses these important developments.

Chapter 1, in addition to providing an updated quantitative perspective on energy usage, highlights the importance of a systems perspective. It is, generally, the accomplishment of various tasks, such as keeping warm or cool, that is of importance whereas the supplying of a given quantity of energy is of secondary importance. It is shown that policies based upon constant growth rates are unrealistic since this results in unbounded exponential increases. A mathematical discussion of exponential growth is included.

An extensive treatment of fossil fuel reserves (Chapter 2) has been added. Not only are estimates of various fuel reserves included, but the methodology for dealing with a depletable resource is also developed. Although estimates of reserves are expected to change, this methodology provides a perspective by which the reasonableness of new estimates may be judged. The refining of petroleum fuels is covered as well as methods by which they may be synthesized from coal. Chapter 3, *Terrestrial Limitations*, has been revised to reflect an increased knowledge of the earth-atmosphere system. Growing concentrations of carbon dioxide, an unavoidable

consequence of the combustion of fossil fuels, and acid rain continue to be a source of concern.

Chapter 4 has an elementary-level introduction to thermodynamics based upon the behavior of an ideal gas. The concepts of a cyclic heat engine and the Carnot efficiency are developed. A discussion of available work and second-law efficiencies has been added to this chapter. These topics are generally ignored in the popular thermodynamics texts, but they must be understood if energy is to be used in an efficient manner. The utility of these concepts is emphasized in Chapter 5, *Electrical Energy from Fossil Fuels*. The advantage of cogeneration, that is, supplying process steam or thermal energy for a district heating system, along with electrical energy, is stressed. Again, it is through a systems approach to supplying jointly thermal energy and electricity that primary energy needs may be reduced.

While the desirability of fission-produced energy (Nuclear Energy, Chapter 6) continues to be debated, estimates of the energy that will be supplied by this technology have been declining. In addition to a general discussion of nuclear physics and of reactors, a discussion of radioactivity and a quantitative calculation of fission waste products is presented. Although the quantity of wastes is not debated, the safety with which they can be isolated from the environment is questioned. (It is interesting to note that a discussion of waste products is often omitted in texts on nuclear reactors.) This new edition provides a detailed description of the sequence of events of the Three Mile Island-2 reactor accident. Both the likelihood of a catastrophic reactor failure and the effects of low-level ionizing radiation remain a part of the debate on fission-produced energy.

Chapter 7, *Fusion*, is probably the most difficult one in the book. An appreciation of the problems associated with achieving a controlled plasma reaction requires a quantitative treatment of fusion physics. This is necessary for an understanding of the experimental results obtained and the developments that are yet needed to achieve an energy-producing reaction. Developments that have occurred since the publication of the first edition are included.

The analysis of solar energy systems (Chapter 8) is based upon principles of basic physics. The chapter has been expanded to include developments that have occurred over the past several years. A cost-benefit analysis, suitable not only for solar energy systems but also for other capital-intensive, energy-related activities, has been also added to the chapter. Several examples illustrative of economic cost-benefit analysis are included. In addition, the treatments of both photovoltaics and photosynthetic reactions for producing biomass have been expanded. The data of Chapter 9 (*Water, Wind, and Geothermal Power*) has been updated and recent developments are included in this new edition.

Chapter 10, dealing with energy and the economy, has been expanded in an effort to make this material more accessible to technically oriented students. Input-output techniques provide a convenient approach to obtaining a moderately accurate estimate of the total energy required for producing a particular economic good or service. The interrelatedness between economic activity and energy usage is also emphasized.

A quantitative treatment of the technical limitations and potentials of energy-

related processes requires a background in physics and calculus. The first edition of the text has been successfully used in classes of senior-level engineering, physics, architecture, and mathematics students. It is hoped that the new edition will also be of value to graduates in its coverage of the alternative energy conversion processes that are now receiving considerable attention. Estimates of the potential of various energy conversion processes are included even though vast quantitative differences often exist among various "experts." For cases in which large uncertainties exist, the different assumptions used in arriving at the estimates are pointed out.

A complete and rigorous in-depth treatment of all limitations associated with energy conversion and usage is obviously beyond the capability of a single text. Very likely it is even beyond that of a four-year curriculum, since ideally what is needed for it is not only a good background in mathematics, science, and engineering but also in economics, geology, biology, ecology, as well as climatology. However, the understanding of the fundamental limitations associated with many basic energy conversion processes requires only a minimal background in physics and mathematics. Technically educated students seldom realize the potential of their general background. One of the objectives of this text is to demonstrate that one need not be an expert in a particular field to be able to appreciate fundamental limitations. Hopefully, students will develop an interest in specialized areas and will consequently enter specialized courses, at the same time keeping in mind the broad general picture.

The author has attempted to be thorough in his treatment of the subjects covered, but completeness was obviously impossible. Many topics discussed in single sections are frequently the subject of book-length treatments. Sufficient references are included for the reader who wishes to pursue a specific topic. The aim of the text is to develop an energy overview and hence interrelate the various aspects of energy conversion and usage that are all too often regarded as unrelated concepts. For example, the treatment of thermodynamics is obviously incomplete. (The introductory sections may be omitted if students have had an introductory thermodynamics course.) Overly sophisticated introductory treatments of thermodynamics, however, can leave students confused and despondent. Also, more rigorous treatments often ignore the very thermodynamic processes used for large-scale energy conversion. The concept of available work, discussed in Chapter 4, which is the key to understanding energy conservation, is frequently ignored in thermodynamics texts. While attempting to be comprehensive, an encyclopedic cataloging of numerous details has been avoided.

A multitude of units of measure exist for specifying energy and power. Even though technologists and scientists frequently express a preference for the metric system, the corresponding unit of energy, the joule, is almost totally neglected. The mks unit of power, the watt ( $\text{J/s}$ ), is frequently used and a physical feel tends to exist for it. Even though energy is a topic of almost daily discussion and news, a physical feel for energy units is lacking. While one may know the quantity of gasoline required for a given automobile trip, it is unlikely that one would know the required energy expressed in joules (or, for that matter, in any other units). Energy consumption rates, that is, the quantity of energy consumed in a given interval (often for a calendar year), are important. Energy consumption rates, however, have the dimension of

power. If the energy consumed is divided by the corresponding time interval, an average power is obtained. Units of power expressed in watts are therefore appropriate for specifying consumption rates.

Hybrid energy quantities based on nonfundamental units of time are presently used; witness the watt-hour and kilowatt-hour (a watt-second is a joule). Similarly, an average power for a year, the time interval often used to specify consumption rates, results in an energy quantity which, if expressed in watt-years, is numerically equal to the average power. While mks units are emphasized, units commonly used for particular fields are added when appropriate. A multilingual understanding of physical units will no doubt be required for many years.

Important quantities, such as the rate of energy consumed by the United States, for example, change with time. A quantitative perspective, however, necessitates the use of such quantities, even though they tend to date the material of the text. The base year for most of the data utilized was 1979 for the United States and 1978 for the world. In the United States, 1979 corresponded to a peak in energy usage, and consumption from 1980 to 1982 declined, with consumption in 1982 being about the same as in 1972, the base year for the first edition of the text. If consumption again increases, the data for 1979 will tend to be typical of that for future years.

In addition to providing a needed coverage of energy, it is hoped that this text will serve as a stimulus to develop similar comprehensive courses. Hopefully these courses will not only provide a dimension needed in the technical curricula, but also can have a major effect in revitalizing the present educational process.

Special appreciation is due to my colleagues at the University of Colorado, John Cooper, Jan F. Kreider, Jerome B. Martin, and Ronald E. West, and also to Charles D. Beach, then at the solar energy laboratory at Colorado State University, for reading portions of this manuscript and providing many useful comments. I am especially indebted to Frank Kreith at the Solar Energy Research Institute in Golden, Colorado, for his continued assistance in completing both editions of the book. While deeply appreciative of the help provided, the author accepts full responsibility for errors that have gone undetected as well as for any misconceptions that may exist.

I would also like to acknowledge the support of my wife, Maria, without whose patience and encouragement the original manuscript and its revision would not have reached fruition. Since Maria both edited and typed the manuscript, the completed work truly reflects a joint effort.

# CHAPTER 1

## *Introduction*

### 1. THE ENERGY CRISIS

As early as 1969 New York City had experienced an electrical energy shortage. On August 5, 1969, *The New York Times* carried a front page story with the following title [1]:

***Con Ed Power Cut 20% by Mishaps; Long Crisis Ahead  
Generator Repair May Take up to Month—Consumers Help Save  
Electricity***

While this particular crisis was precipitated by an untimely series of breakdowns, it provided a warning signal for events to come. Since the 1969 crisis, Consolidated Edison Company has moved from a position of advocating increased use of electricity to urging customers to conserve electricity [2].

#### ***Con Ed Says It Will Confine Ads to Urging Power Conservation***

The Chairman of the Consolidated Edison Company, Charles F. Luce, said yesterday that the utility has disbanded its sales promotion department and would use advertising only to urge customers to conserve electricity. Mr. Luce said he believed it was the first such action by any utility in the nation.

Electrical energy consumption within the United States had, prior to the 1970s, been increasing at an average annual rate of 7% since 1900. At this rate, the consumption doubles every ten years. Therefore, generating capacity must also double every ten years. A continuation of this growth rate implies a quadrupling



in 20 years and an eightfold increase in 30 years. Over an interval of 30 years, seven additional generating units would need to be added for each initial unit.

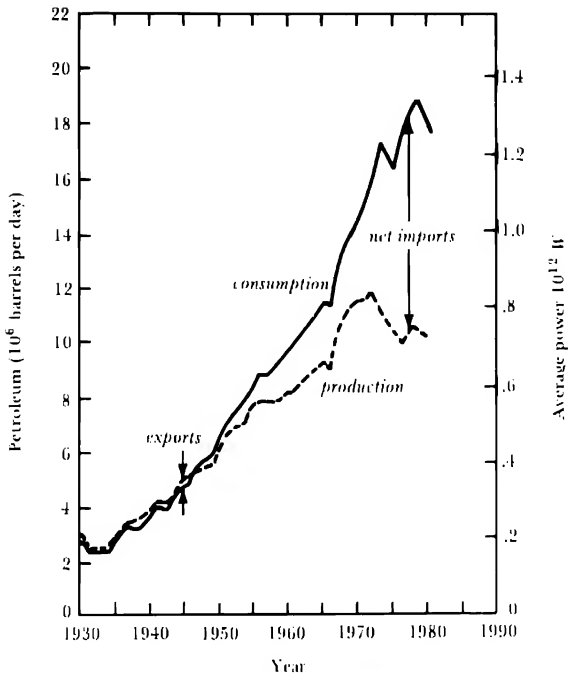
The energy crisis is not only associated with electrical energy. The total consumption of energy had, prior to 1974, been increased at a 4 to 5% annual rate within the United States as well as the world. Approximately 31% of this energy is used for the generation of electricity. The other 69% is used for transportation (electric propulsion is negligible in the United States), industrial processes, and heating. Although the known reserves of coal are sufficient for at least the next millennium at the present rate of consumption, this is not true for petroleum and natural gas. The known world reserves of natural gas and petroleum will last, based upon present consumption rates, many decades. If, however, consumption of these premium hydrocarbon fuels continues to increase, the world's reserves will be depleted in the early part of the twenty-first century.

While scientists have for many years been concerned with the rapid increase in energy, the oil embargo during the latter part of 1973 and the beginning of 1974 served to focus the public attention on energy issues. The curtailment of exports by the Arab petroleum producing countries resulted in serious economic disruptions throughout the world. Petroleum consumption during this period in the United States was 14% less than anticipated prior to the embargo [3]. The reporting of this and other petroleum shortfalls has tended to be misleading. The impression often conveyed is that petroleum supplies fall short of a quantity that has traditionally been used. This is not the case. Petroleum usage, even following the embargo and the numerous subsequent OPEC price increases, has been increasing. The aforementioned 14% shortfall reduced consumption, temporarily, to a level corresponding to that of 1970, a year not noted for shortages.

Petroleum consumption in the United States, as indicated in Figure 1.1, had been increased rapidly [4-6]. Domestic production, even including the crude oil now obtained from Alaska (the initial pipeline capacity was  $1.2 \times 10^6$  bbl/day), has been insufficient to meet the growing demand. Hence imports have steadily increased. Imported crude oil and petroleum products in 1979 supplied nearly half of the nation's demand for petroleum. Although the imports attract the most attention, because of their cost in foreign exchange and the uncertainty of supplies resulting from the political instability of many of the exporting nations, the United States is still one of the largest petroleum producers in the world. Prior to 1966, U.S. production exceeded that of the entire Middle East. It was not until 1975 that the Soviet Union's production surpassed that of the United States, moving the USSR into the lead [7, 8].

Domestic production of petroleum and natural gas, the premium hydrocarbon fuels that provide almost three-quarters of the nation's energy inputs, has been declining. If presently accepted estimates of yet-to-be discovered hydrocarbon fuels prove valid, half of the nation's original endowment has already been depleted. The remaining reserves are therefore insufficient to justify a significant increase in production.

The increasing scarcity of fuels is not the only factor contributing to the crisis. Associated with the combustion of fossil fuels is the unavoidable release of pollutants.



**FIGURE 1.1 U.S. Petroleum Consumption and Production (References 4-6).**

Coal and petroleum contain sulfur impurities which form sulfur oxides during combustion. At the high temperatures typical for the internal combustion engine, nitric oxides are produced. Incomplete combustion produces carbon monoxide and hydrocarbons. Even if these undesirable and biologically hazardous by-products are eliminated, the generation of carbon dioxide is unavoidable. The atmosphere contains carbon dioxide (an essential component of the life processes), but the effect of an increase in the atmospheric content is uncertain. Finally, essentially all energy generated creates heat, due both to the inefficiencies of various conversion processes and to its eventual utilization. The present rate of energy consumption in heavily populated metropolitan areas is sufficient to substantially increase the ambient temperature.

Before shortages of fuels were experienced, excessive levels of atmospheric pollutants were recognized as a national problem. As a consequence, the Clean Air Act (PL 91 604) was enacted in 1970 and the Environmental Protection Agency

(EPA), also established in 1970, was given the responsibility of implementing the provisions of the act. Ambient air quality standards were established that provided a quantitative measure for assessing air quality. To reduce pollutant levels which for most urban areas were deemed excessive, a set of permissible pollutant emission levels was established that would gradually become effective. This legislative action, in effect, recognized the societal costs of air pollution and by means of emission restrictions transferred the costs directly to energy users. Automobile owners, electric power plant operators, and other energy consumers had to bear the cost of reducing pollutant emissions.

Many have recommended that alternative supplies of energy be developed. Only in the area of atomic energy, however, has there been a sustained research effort commensurate with the magnitude of the crisis. Shortly after World War II, nuclear energy was hailed as the promise for the future. Electricity produced by nuclear energy was to be so cheap that metering would be unnecessary. Although the first nuclear chain reaction was produced in 1942, 30 years later reactors were only beginning to emerge from the experimental stage. In 1979, 3.5% of the energy used in the United States depended upon the splitting of the atom. Proponents of nuclear energy envision that a substantial fraction of the energy generation will be nuclear in the future. Opponents seriously question both the prediction and the wisdom of nuclear-derived energy.

Nuclear fusion, the process of putting light atoms together (as opposed to fission, the process of breaking apart heavy atoms), is seen by many as an ultimate energy solution. Radiation problems are less severe and nonradioactive waste products are generated. Unfortunately, a controlled fusion reaction (the hydrogen bomb is an uncontrolled fusion reaction) had eluded scientists. Very few doubt that a controlled fusion process is possible, but several more years of development will be necessary. Unless an unforeseen major breakthrough occurs, even its most optimistic proponents do not expect nuclear fusion to modify the energy picture for several decades.

As a result of the 1973-1974 oil embargo and subsequent events, technologies that rely either directly or indirectly on solar energy received by the earth have received considerable attention. Even though these technologies are frequently viewed as new, solar collectors for producing both heat energy and mechanical work were utilized at the beginning of the century. Interest in these technologies rapidly waned since plentiful supplies of fossil fuels were discovered during the first half of the century. There was a renewed interest in solar-derived energy during the 1950s but this interest too was short-lived. Not only were petroleum supplies abundant, but gluts of petroleum in which crude oil was sold as cheap as 10¢ a barrel at the wellhead were not uncommon.

Solar-derived energy, other than that of fossil fuels which are the result of photosynthetic reactions that occurred several hundred million years ago, will undoubtedly be the future energy source for the world. But a direct replacement of the energy provided by fossil fuels may not be in order. Solar-derived energy, if present indications prove valid, will be expensive. To collect the "free" solar energy, very large capital investments are required. For example, one square foot of a flat-plate collector system which may cost \$25 will yield on a daily basis the amount of thermal

energy provided by a mere thimblefull (1 oz) of petroleum when used in a conventional furnace. While lower priced solar collection devices are envisioned (as a result of being mass-produced), the price of energy derived from solar technologies will probably remain considerably higher than the price of fossil fuels of the past. This implies that it will be economic to use energy more effectively and efficiently.

For the U.S. energy system, approximately half of the fossil fuels consumed (over 90% of the energy inputs) is used to power heat engines of electric power plants and internal combustion engines employed by the transportation sector. The other half is used for producing heat, very high temperatures for industrial activities such as reducing ores, moderate temperature process steam, and fairly low temperatures to supply comfort heat and hot water needs. Heat engines, owing primarily to the nineteenth-century thermodynamic developments, are considerably more efficient and reliable than their early predecessors. Their conversion efficiencies have been markedly improved by reducing avoidable irreversibilities; that is, entropy increases have been minimized. Heat engines, however, are the only portion of the nation's energy system that may be characterized as relying upon highly sophisticated thermodynamic principles. While these principles apply equally to all aspects of energy usage, they have been extensively utilized only in the design of heat engines, and they have tended to be neglected in the heat-providing portion (furnaces) of the energy system or in the usage of electrical and mechanical energy derived from heat engines. Hence the potential for using energy more effectively is very great.

As evidenced by a three-article series carried by *The New York Times*, it was recognized as early as 1971 that a national energy dilemma existed [9-11].

***Nation's Energy Crisis: It Won't Go Away Soon***

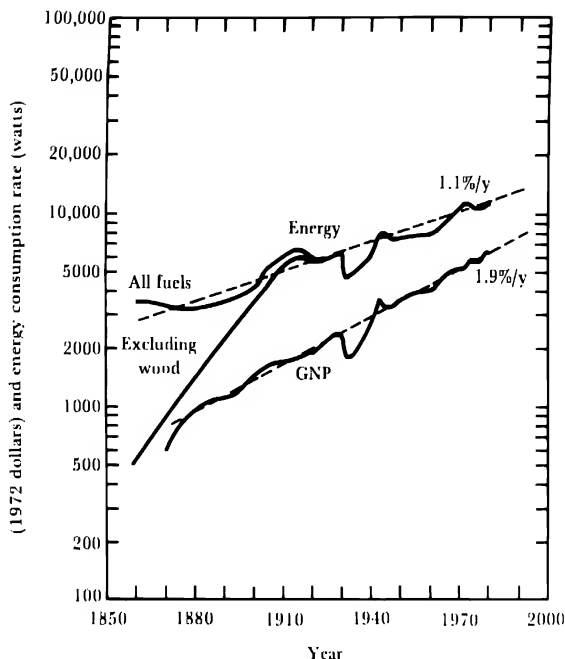
***Nation's Energy Crisis: Nuclear Future Looms***

***Nation's Energy Crisis: Is Unbridled Growth Indispensable to the Good Life?***

Despite subsequent presidential calls for energy independence and a moral equivalent of war, little visible change occurred in the nation's energy policy during the decade of the 1970s. While temporary shortages of fuels and electricity occurred and energy prices increased substantially, there was little change in the manner in which energy was used. However, during this decade, not only politicians but also physical, social, and political scientists began to grapple with the problems of energy production and utilization. As a result, there is a considerably better understanding of the philosophical issues involved. An intellectual base from which rational policy decisions could be derived is beginning to emerge [12].

## **2. ENERGY CONSUMPTION**

While world population has been increased at an annual rate of approximately 2%, energy consumption prior to the 1973-1974 embargo had been increased at a higher rate of 5%. High energy consumption has traditionally been associated with a high

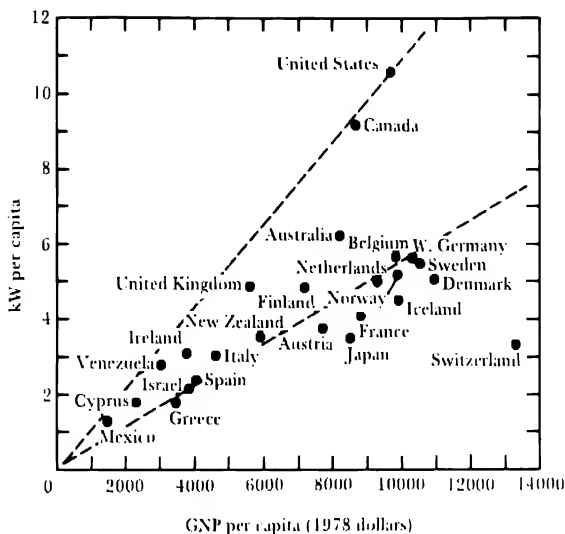


**FIGURE 1.2** *U.S. per Capita Energy Consumption Rate and Gross National Product (Reprinted by Permission from Energy, 2, 2 (June 1977), p. 116).*

quality of life, which is often associated with a high gross national product. In the United States both the rate of which energy is utilized and economic activity have been increasing [4, 5, 6, 13] (Figure 1.2).

Although the long-term average per capita growth in energy usage has been approximately  $1.1\%/y$ , the growth over the decade preceding the 1973-1974 oil embargo was slightly greater than  $3\%/y$ . Though the large increase in both energy consumption and economic activity that occurred during this period was unusual if not unique—projections for future demand based upon the growth rates of this period have been common. From 1973 to 1979, however, per capita energy usage changed very little.

Energy usage and economic activity tend to be similarly related for other nations [8, 14]. The data in Figure 1.3 for 1978 provides a comparison in the manner in which energy is used by developed nations. The United States and Canada, on a per capita basis, use considerably more energy than do equally affluent European nations. This comparison has received considerable attention [15-21], since it raises

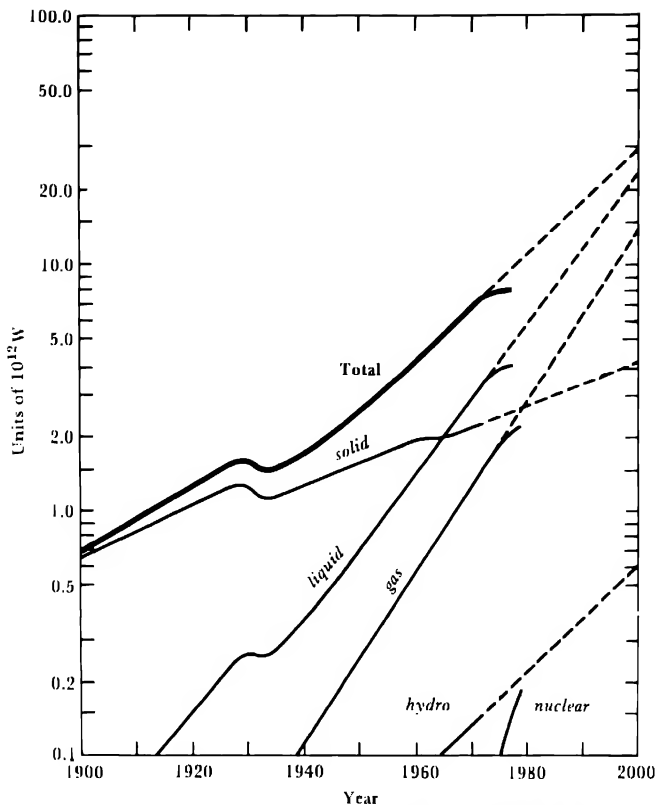


**FIGURE 1.3 International Comparison of per Capita Rate of Energy Consumption and Gross National Product, 1978—Market Economies (References 8 and 14).**

the distinct possibility that the level of economic activity and energy usage need not be correlated; that is, economic activity could increase while the rate of energy consumption remained unchanged.

Figure 1.4 is a graph of the world rate of energy consumption for the twentieth century [7, 8, 22]. To emphasize the exponential nature of the consumption curves (prior to the 1973-1974 oil embargo), a semilogarithmic set of scales has been used. Exponential functions result from constant yearly growth rates and appear as straight lines on such a plot.\* The projections to the year 2000 are simply straight-line extrapolations based upon a continuation of the pre-1973 growth rates, a 5%/y growth for total consumption, 1%/y for solids, in excess of 7%/y for liquids and natural gas, and 2%/y for hydropower. Although these or similar projections have frequently been used for predicting future energy requirements, neither the long-term consumption patterns nor those following the 1973-1974 oil embargo justify their use. The growth rates declined significantly for the years immediately following the embargo, and if OPEC restricts its oil exports (petroleum products account for about 45% of the world's energy usage), low growth rates (or even no growth) may prevail. While the growth in energy consumption may be limited, technical innovations may reduce energy needs and energy may be used more effectively in the future.

\* If  $y = Ae^{at}$ ,  $\log y = \log A + at \log e$ . Therefore, if  $\log y$  is plotted as a function of  $t$ , a straight line with a slope of  $a \log e$  results.



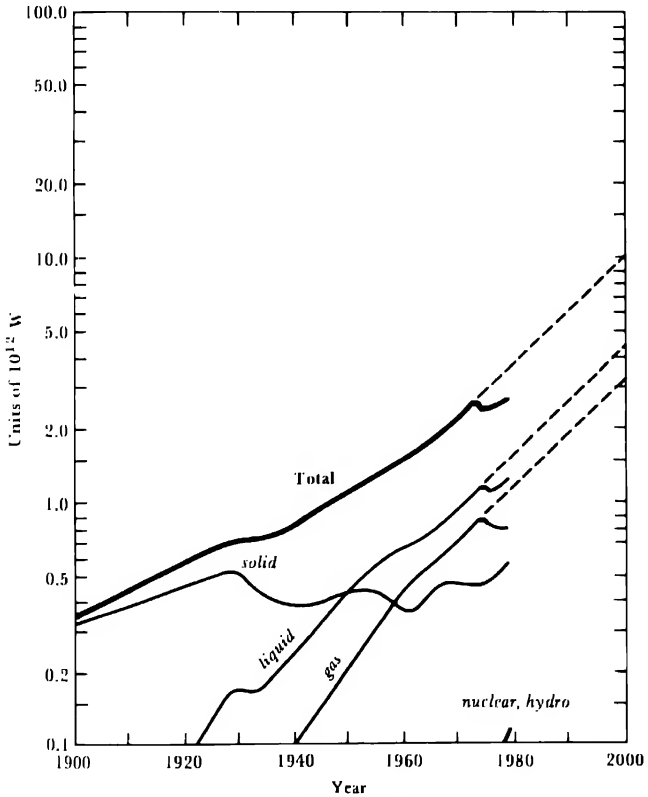
**FIGURE 1.4 World Energy Consumption Rate (References 7, 8, 22).**

Solids include both coal and wood. Wood was important in the nineteenth century, but the quantity used now is very small. Liquid (petroleum) and gas consumption has shown the greatest growth rate. Hydropower remains an order of magnitude less than fossil fuels. All significant amounts of hydropower, however, are used to generate electricity. In this respect, it is more effective than fossil fuels used for the same purpose. Thermal generation of electrical energy is presently no more than 40% efficient, whereas efficiencies for hydroelectric generation can approach 100%.

The 1978 world rate of energy consumption was approximately  $8.1 \times 10^{12}$  watts. Units of  $10^{12}$  watts or  $1000 \times 10^9$  watts are beyond normal everyday experience. Large-scale fossils and nuclear fuel electric power plants are presently being

designed and constructed for electrical outputs of  $10^9$  watts. For a conversion efficiency of 40%, the heat value of the fuel would be  $2.5 \times 10^9$  watts. Therefore, the present world consumption rate corresponds to the consumption rate of more than 3000 of these plants.

Figure 1.5 is a plot of the energy consumption rate for the United States [4-6]. The average growth rate for the total U.S. energy consumption prior to 1973 was 4.2%, a value slightly less than that for the world. However, for the following six-year period, 1973 to 1979, overall energy usage in the United States increased by only 5.9%, approximately the percentage of population increase. Following the peak of

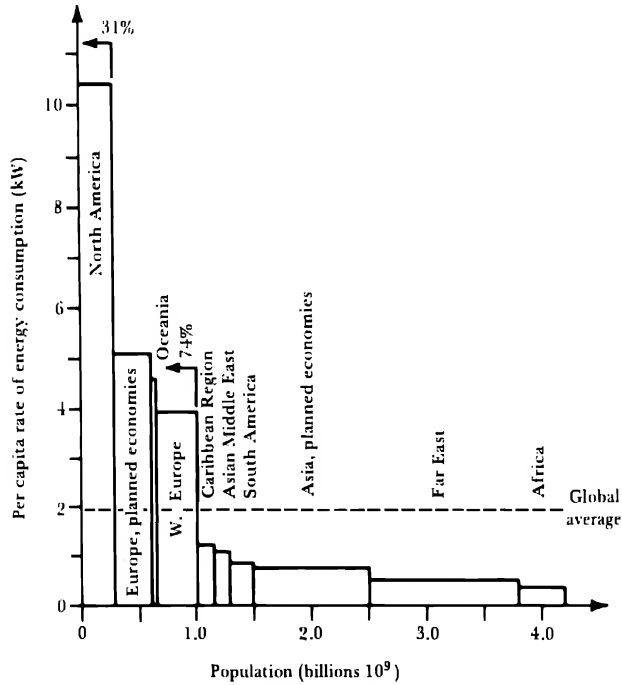


**FIGURE 1.5** *United States Energy Consumption Rate (References 4-6).*



1979, energy consumption has declined; that of 1982 was 10% less than that of 1979. Not only are the once generally accepted projections that the national energy usage in 2000 will be two to three times that of 1972 unlikely to be fulfilled, but consumption could even become less than that of the 1970s. With increased energy prices, energy usage may continue to decline, since it will be more economic to rely on smaller energy inputs but to use them considerably more efficiently in 2000 than at present.

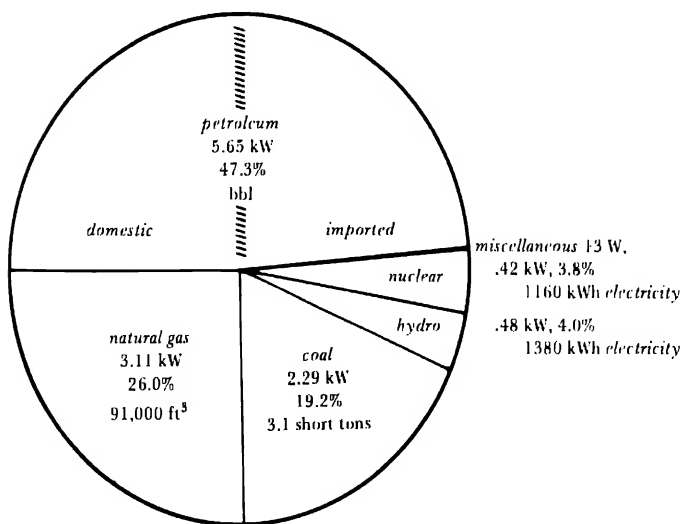
It is obvious from Figures 1.3, 1.4, and 1.5 that large differences exist in the per capita rate at which energy is utilized throughout the world. Figure 1.6 illustrates these differences [8]. Since the horizontal scale is population and the vertical scale is the per capita energy consumption rate, the area of a bar representing a particular global region is proportional to the overall energy usage rate for that region. North America (the United States and Canada; according to U.N. data, Mexico is included in the Caribbean region), with only 6% of the world's population, accounts for 31% of the energy used. Europe (including all of Russia), Oceania, and North America,



**FIGURE 1.6 Per Capita Rate of Energy Consumption by Global Regions (Reference 8).**

the more affluent regions of the world, account for 74% of the energy usage even though they contain only 24% of the world's population. While the average global per capita energy usage rate is approximately 2 kW (less than one-fifth that in the United States), 69% of the world's population subsists on energy inputs of 1 kW or less.

Frequently lacking in energy discussions is a realization of the enormous quantities of energy utilized by developed societies. If U.S. energy inputs were to be derived entirely from coal (Figure 1.7), over one ton of coal per capita would be required each month. If derived entirely from petroleum, approximately five barrels (each 42 gallons) would be needed each month. It is the immense quantities of energy that are used that makes significant changes in the energy system difficult to achieve. Extremely large-scale (and usually costly) new energy-producing endeavors are necessary before a statistically significant (1% of total consumption) quantity of energy is produced. Therefore, not only must new technologies (solar, for example) be developed, but a very widescale utilization of the technology must also be achieved before its effect may be considered significant.



**FIGURE 1.7 U.S. per Capita Energy Consumption Rate by Source, 1979 (Fuel Quantities Are Based on Average Energy Values) (Reference 4).**

### 3. UNITS OF ENERGY AND POWER

Many types of measuring units are commonly used to express the rate of energy consumption. A yearly basis is usually used for the quantities involved, that is, the number of tons (metric or English) of coal, barrels of oil, or cubic feet of gas used. These quantities are often converted to a common energy term so that the energy consumed in a one-year period is obtained. The resulting yearly rate averages out seasonal variations.

Power is the rate at which energy is consumed or generated. It is usually thought of in instantaneous terms. Mathematically, it is the derivative with respect to time of the quantity of energy.

$$P = \frac{dE}{dt}$$

Despite the multitude of units commonly used, an attempt to relate all quantities to mks units has been made in this book. The basic unit of energy in the mks system is the joule and the unit of power is the watt. Dimensionally, a watt is one joule per second. The common unit of electrical energy is the watt-hour or kilowatt-hour, that is, 3600 or 3,600,000 joules, respectively. The graphs in the previous section expressed the energy consumption rate averaged over a one-year period. That is, the total number of seconds in a year. This is simply the instantaneous power averaged over a one-year period.

$$E_{\text{year}} = \int_0^T P_{\text{inst}} dt$$
$$P_{\text{av}} = \frac{1}{T} E_{\text{year}} = \frac{1}{T} \int_0^T P_{\text{inst}} dt$$

The quantity  $P_{\text{av}}$  is thus the unit that was used, while the period  $T$  corresponds to one year, that is,  $3.15 \times 10^7$  seconds.

In addition to the watt-hour or kilowatt-hour, the watt-year is also used. One watt-year is simply a power of one watt for a period of one year, or  $3.15 \times 10^7$  joules. Calories are also used since energy is usually in the form of heat. A calorie is defined as the quantity of heat necessary to raise the temperature of one gram of water at 15°C one Celsius degree. A kilogram calorie, which is equal to 1000 calories, is also frequently used. Experimentally, one calorie has been found to be equivalent to 4.184 joules (Joule's constant).

The 1978 world energy consumption rate of  $8.1 \times 10^{12}$  watts may readily be converted to both joules and calories per year.

$$(8.1 \times 10^{12} \text{ joules/second}) (3.15 \times 10^7 \text{ seconds/year}) = 2.55 \times 10^{20} \text{ joules/year}$$

$$\frac{(2.55 \times 10^{20} \text{ joules/year})}{4.184 \text{ joules/calorie}} = 6.1 \times 10^{19} \text{ calories/year}$$

The world therefore consumed  $6.1 \times 10^{16}$  kilogram calories of energy during 1978. The energy consumption rate for the U.S.,  $2.6 \times 10^{12}$  watts (1979) is equal to a yearly consumption of almost  $2 \times 10^{16}$  kilogram calories.

Metabolism is usually expressed in kilogram calories. The diet of a person living in the U.S. (except for those on diets) contains roughly 3100 kilogram calories per day. This rate can be divided by the number of seconds in a day and then be converted into watts, as follows:

$$\frac{3100 \text{ kilogram calories/day}}{8.64 \times 10^4 \text{ seconds/day}} = .036 \text{ kilogram calories/second}$$

$$= 36 \text{ calories/second}$$

$$(36 \text{ calories/second}) (4.184 \text{ joules/calorie}) = 150 \text{ joules/second}$$

$$= 150 \text{ watts}$$

The U.S. nonmetabolic per capita rate of almost 12 kilowatts is 80 times its metabolism rate. The world nonmetabolic per capita rate of 2 kilowatts is more than 13 times the U.S. metabolism rate.

Highly developed societies depend primarily upon nonhuman sources of energy. Although the work output of humans varies greatly, it is not unreasonable to estimate that a work output of 50% of the metabolism rate over eight hours a day can be achieved. A work output of 75 watts for eight hours averaged over a 24-hour day yields an average output of 25 watts ( $\frac{1}{3}$  of the 8-hour rate). In the U.S., human-derived energy, using this estimate, accounts for only approximately .2% of the total energy consumed. If only the work force is considered (children and the aged excluded), less than .1% of the productive energy is derived from humans.

Horsepower is also a commonly used measure of power. One horsepower is equivalent to 746 watts and supposedly represents the work output of a typical horse. The daily average work output of a horse working an eight-hour day is thus approximately 250 watts, or ten times that of a human.

The British Thermal Unit, Btu, is also a widely used energy measure. It is defined in a manner similar to the calorie, namely, the amount of heat necessary to increase the temperature of one pound of water at 39.1°F one Fahrenheit degree. One Btu is equivalent to 252 calories. The world energy consumption rate is thus  $2.4 \times 10^{17}$  Btu per year, while the rate of the U.S. is  $7.9 \times 10^{16}$  Btu per year.

Another widely used practice is to relate energy to the quantity of coal necessary to provide the same heat energy. Historically, this was introduced when coal was the major energy source. While there are variations in the heat values of different coals, an average value of 28 million Btu per metric ton (1000 kg) is commonly used. The world consumption rate is thus  $8.7 \times 10^9$  tons of coal equivalent, while that of the U.S. is  $2.8 \times 10^9$  tons. On a per capita basis, the world average is 2.1 tons, while the U.S. averages 12.9 tons.

As a result of the growing importance of petroleum as a fuel, energy consumption is also frequently expressed in terms of petroleum equivalent. The average energy content of a barrel (42 gallons) of petroleum is  $5.8 \times 10^6$  Btu; thus the world energy

**TABLE 1.1 Fossil Fuels: Approximate Energy Content***Coal*

Metric ton (1000 kg or 2205 lb)

28 million Btu

29.5 billion J

1 ton/y; 940 W

*Petroleum*

Barrel (42 gal, approximately 300 lb)

5.8 million Btu

6.1 billion J

1 bbl/day; 70,000 W (70 kW)

1 bbl/y; 194 W

1 gal gasoline

125,000 Btu

130 million J

3 gal/h, 110,000 W (110 kW)

*Natural gas*1000 ft<sup>3</sup>

1.0 million Btu

1.05 billion J

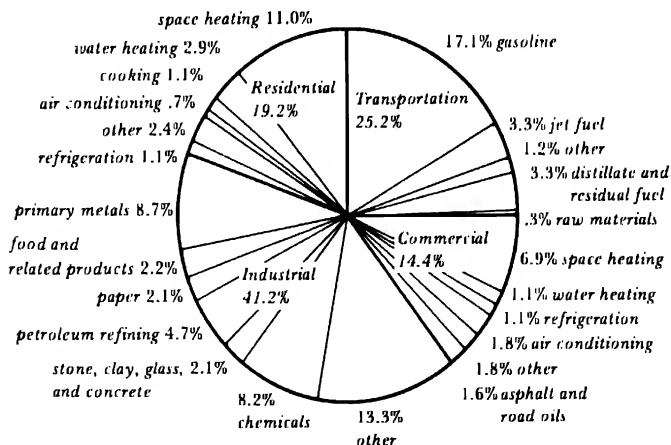
1000 ft<sup>3</sup>/day; 12.2 kW**TABLE 1.2 Energy Data for the World and the United States**

<i>Units</i>	<i>Total consumption rate</i>	<i>Per capita consumption rate</i>
<i>World 1978</i>		
Watts	$8.1 \times 10^{12}$ W	1.9 kW
Joules	$2.6 \times 10^{20}$ J/y	$6.1 \times 10^{10}$ J/y
Calories	$6.1 \times 10^{19}$ cal/y	$1.4 \times 10^{10}$ cal/y
British thermal units	$2.4 \times 10^{17}$ Btu/y	$5.7 \times 10^7$ Btu/y
Metric tons of coal equivalent	$8.7 \times 10^9$ ton/y	2.1 ton/y
Barrels of oil equivalent	$4.2 \times 10^{10}$ bbl/y	9.9 bbl/y
Barrels of oil equivalent (per day)	$1.1 \times 10^8$ bbl/day	.027 bbl/day (1.1 gal/day)
<i>United States 1979</i>		
Watts	$2.6 \times 10^{12}$ W	11.9 kW
Joules	$8.3 \times 10^{19}$ J/y	$3.8 \times 10^{11}$ J/y
Calories	$2.0 \times 10^{19}$ cal/y	$9.0 \times 10^{10}$ cal/y
British thermal units	$7.9 \times 10^{16}$ Btu/y	$3.6 \times 10^8$ Btu/y
Metric tons of coal equivalent	$2.8 \times 10^9$ ton/y	12.7 ton/y
Barrels of oil equivalent	$1.4 \times 10^{10}$ bbl/y	61 bbl/y
Barrels of oil equivalent (per day)	$3.7 \times 10^7$ bbl/day	.17 bbl/day (7.1 gal/day)

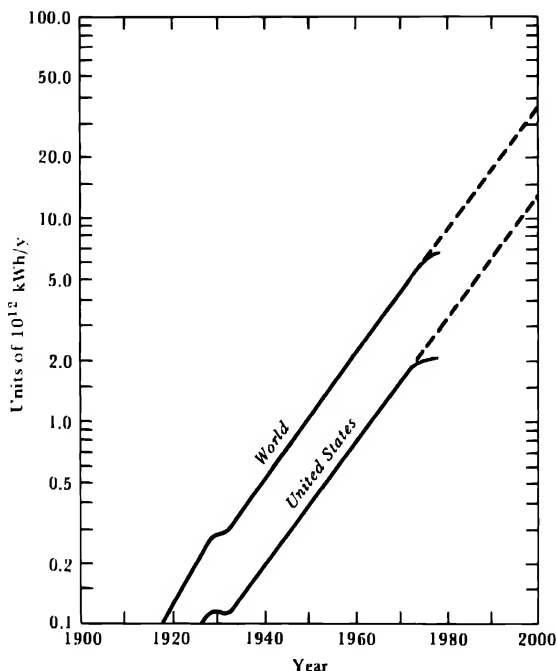
consumption rate is  $4.2 \times 10^{10}$  bbl (equivalent)/year. Often daily rates are used for petroleum, the world rate being  $1.1 \times 10^8$  bbl (equivalent)/day. For the United States, the rates are  $1.4 \times 10^{10}$  bbl (equivalent)/year and  $3.7 \times 10^7$  bbl (equivalent/day). Summarized in Table 1.1 are energy contents of the principal fossil fuels. Table 1.2 summarizes the data for world and U.S. energy consumption rates.

Major uses of energy in the U.S. economy are shown in Figure 1.8 [24]. Residential usage accounts for approximately one-fifth the energy used while the industrial sector accounts for over one-third.

All sectors of society utilize electrical energy. Furthermore, electrical usage is rapidly increasing as shown in Figure 1.9. Prior to the 1973-1974 oil embargo, the yearly growth rate both for the world and for the United States has been nearly a constant 7%. The 1979 U.S. yearly consumption of  $2.2 \times 10^{12}$  kilowatt-hours (utility generated) corresponds (divided by 8760 hours per year) to an average power of  $2.5 \times 10^{11}$  watts. For an average conversion efficiency of one-third, this corresponds to slightly more than one-quarter of the total U.S. energy consumption. All energy sources (solids, liquids, natural gas, hydro, and now nuclear energy sources) contribute to the energy used for electrical generation.



**FIGURE 1.8 Major Uses of Energy in the United States. This Data Is from a Stanford Research Institute Report for Energy Uses in 1968. This Is One of the Most Detailed of All Energy End-Use Studies. While Overall National Energy Usage Has Increased, It Appears That the End-Usage Distribution Has Not Changed Greatly. (Reference 24.)**



**FIGURE 1.9** *Electrical Energy Consumption Rate (References 4-6, 13).*

#### 4. GROWTH RATES

Energy consumption rates from 1964 to 1972 have nearly constant growth rates. A constant percentage growth rate implies that the increase in a quantity is proportional to that quantity. If  $y$  is a function of time, and the growth rate is  $a$ , the following is true for increments.

$$\Delta y = ay \Delta t$$

If time is expressed in years, then  $a$  is the growth rate per year. For example, if the yearly growth rate is 5%, then  $a$  is equal to .05. In the case of interest compounded annually,  $t$  is unity, and therefore the interest earned is  $\Delta y$ .

$$\Delta y = ay$$

The interest earned each year depends upon the value of the account for that year ( $y$  dollars).

The doubling time, it will be noted, is independent of the value of the function,  $y$ , or the time. In the interval of  $t_D$ , the quantity doubles regardless of its initial size. The relation between yearly growth rates and the doubling time in years is given in Table 1.4.

**TABLE 1.4 Doubling Times for Selected Yearly Growth Rates**

Yearly growth rate	$a$ per year	$t_D$ years
1%	.01	69.3
2%	.02	34.7
3%	.03	23.1
4%	.04	17.3
5%	.05	13.9
7%	.07	9.9
10%	.10	6.93

For a yearly growth rate of 5% (such as occurred for world energy consumption), the doubling time is 13.9 years. If this growth rate is maintained, energy consumption would approximately double in a 14-year period. During a second 14-year period, consumption would double again, resulting in a consumption rate four times as great as the initial value. If this growth rate is maintained for three doubling periods (42 years), consumption would be eight times as great. Population trends also exhibit exponential growth. The present world growth rate of 2% per year will double the population in approximately 35 years. That is, if the growth rate remains constant, the world population will be twice that of 1965 in the year 2000.

Many economic processes are based upon growth. A 5% yearly growth rate for the GNP is considered healthy, while a one or two percent growth rate is associated with a lagging economy. A constant 5% yearly growth rate yields a GNP that will be eight times its present value in three doubling periods (42 years). Since the United States' GNP was a trillion dollars ( $\$10^{12}$ ) in 1971, it will have a value of eight trillion dollars ( $\$8 \times 10^{12}$ ) in the year 2013 if the average growth rate is 5%/y. Even relatively small growth rates (such as 5% per year) can result in exceedingly large changes in a rather short period.

In considering exponential growth, the area under the consumption rate curve is also of interest. The rate at which energy is consumed is power. For an increment in time  $\Delta t$ ,  $P \Delta t$  is the energy consumed in that increment. Within an interval between  $t_1$  and  $t_2$ , the area is simply the integral between these limits.

$$\int_{t_1}^{t_2} P \, dt$$

This is the energy consumed in the period  $t_2 - t_1$ . If  $P$  is in watts and the time in seconds, the result is in joules. If, however, years are used for the time scale, the result is in watt-years.



$$\Delta t = 1/m$$

$$\Delta y = (a/m)y$$

If the result at the end of  $t$  years is desired,  $m \times t$  computations will be necessary. Starting with an initial value of  $y_0$ , the following is obtained at the end of the first period.

$$\Delta y = (a/m)y_0$$

$$y = y_0 + \Delta y = (1 + a/m)y_0$$

After  $m \times t$  such computations, the following is obtained.

$$y = (1 + a/m)^{mt} y_0$$

As  $m$  becomes very large, that is, as the interest periods become very small, the above approaches an exponential. To show this, let  $a/m$  equal  $x$ .

$$x = a/m$$

As  $m$  becomes infinite,  $x$  approaches zero.

$$m = a/x$$

$$y = (1 + x)^{at/x} y_0$$

$$\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$$

Therefore, in the limit, the following is obtained for  $y$ .

$$y = y_0 e^{at}$$

Energy consumption as well as most other quantities are rarely as well ordered as the computation of monetary interest. Consequently, an exponential formulation is likely to be as accurate as any other formulation. The growth rate,  $a$ , for most quantities will represent an average rate.

A quantity often of interest when dealing with exponential growth is the doubling time, that is, the time necessary for the quantity to double. Let  $y_1$  be the value at  $t_1$  and  $y_2$  the value at a later time  $t_2$ .

$$y_1 = y_0 e^{at_1}$$

$$y_2 = y_0 e^{at_2}$$

The quantity  $y_0$  may be eliminated by taking the ratio of  $y_2$  to  $y_1$ .

$$y_2/y_1 = e^{a(t_2 - t_1)}$$

When  $y_2/y_1$  equals 2, the time difference  $t_2 - t_1$  is the doubling time  $t_D$ .

$$2 = e^{at_D}$$

$$at_D = \ln 2 = .693$$

$$t_D = .693/a$$

The doubling time, it will be noted, is independent of the value of the function,  $y$ , or the time. In the interval of  $t_D$ , the quantity doubles regardless of its initial size. The relation between yearly growth rates and the doubling time in years is given in Table 1.4.

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3%	.03	23.1
4%	.04	17.3
5%	.05	13.9
7%	.07	9.9
10%	.10	6.93

For a yearly growth rate of 5% (such as occurred for world energy consumption), the doubling time is 13.9 years. If this growth rate is maintained, energy consumption would approximately double in a 14-year period. During a second 14-year period, consumption would double again, resulting in a consumption rate four times as great as the initial value. If this growth rate is maintained for three doubling periods (42 years), consumption would be eight times as great. Population trends also exhibit exponential growth. The present world growth rate of 2% per year will double the population in approximately 35 years. That is, if the growth rate remains constant, the world population will be twice that of 1965 in the year 2000.

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$$\int_{t_1}^{t_2} P \, dt$$

This is the energy consumed in the period  $t_2 - t_1$ . If  $P$  is in watts and the time in seconds, the result is in joules. If, however, years are used for the time scale, the result is in watt-years.

**TABLE 1.5 Exponential Index for a Fuel Reserve**

Static index: 100 years = $E_T/P_0$			
		$E_T/P_0 = 100 \text{ y}$	$E_T/P_0 = 200 \text{ y}$
Exponential growth %/y	$a$	$t_f$ years	$t_f$ years
0	0	100	200
1	0.01	69.3	109.9
2	0.02	54.9	80.5
3	0.03	46.2	64.9
4	0.04	40.2	54.9
5	0.05	35.8	48.0
7	0.07	29.7	38.7
10	0.1	24.0	30.4

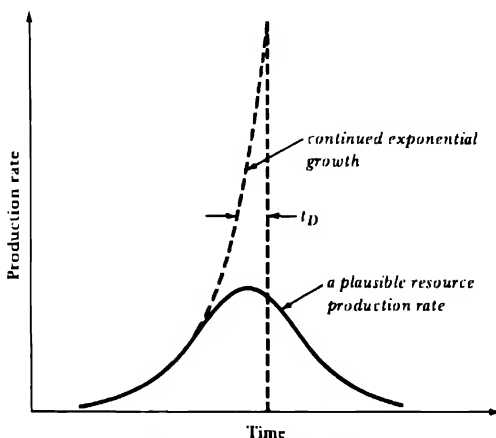
The index,  $t_f$ , becomes infinite when the argument of the natural logarithm goes to zero (assuming  $a$  is negative).

$$aE_T/P_0 = -1$$

$$a = -P_0/E_T$$

Therefore, if the static index for a reserve is 100 years, the reserve would last forever if usage decreased by a yearly rate of 1%. By adjusting to an exponentially declining rate of usage ( $-P_0/E_T$ ), a society can assure that a given finite reserve of a fuel (or other resource) will be available, in a decreasing quantity, in perpetuity.

Even though exponential growth in consumption of fuels has been common

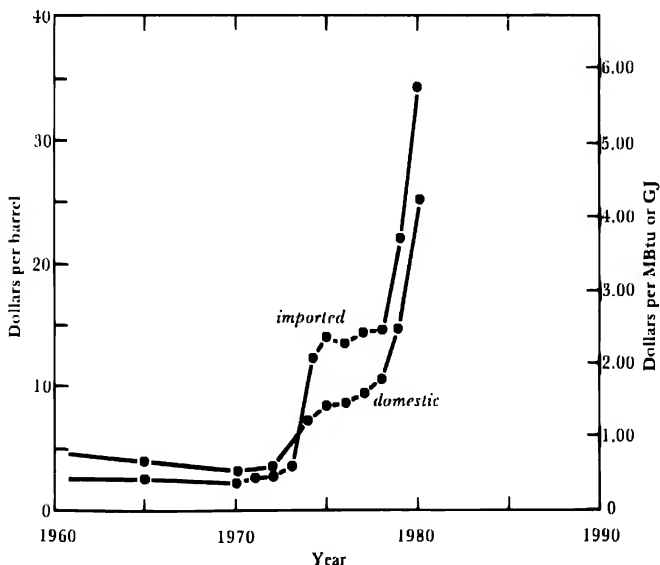


**FIGURE 1.11 A Symmetrical Production Curve of an Exhaustible Resource.**

in the past, it is not likely that such growth will continue until a resource is totally depleted. Initially, the most accessible reserves are exploited. Since those that are the least accessible (and hence cost the most to extract) are used last, production would be expected to decline before a resource were totally depleted. Indicated in Figure 1.11 is a symmetrical production curve in which an exponential growth and decline were assumed, the two curves being arbitrarily connected by a smooth peak. The production rate (tons of coal produced per year, for example) integrated over a time interval yields the quantity produced during that interval (tons of coal). Therefore, the area under an assumed production curve is the total extractable reserve. For a symmetrical production function, the peak occurs when a reserve is half exhausted, not at the depletion of the reserve. As indicated in Figure 1.11, the peak in production occurs approximately one doubling interval earlier than the depletion time predicted by the exponential curve.

## 5. ENERGY UTILIZATION

Many significant changes related to energy utilization occurred during the decade of the 1970s. Not only were there shortages but energy prices also increased by

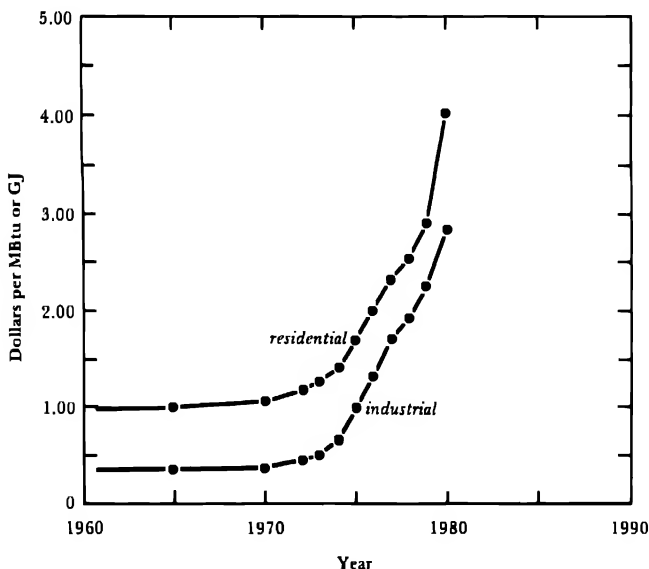


**FIGURE 1.12** *Average Imported and Domestic Crude Oil Prices (References 4, 25, 26).*

unprecedented amounts. Following the 1973–1974 oil embargo by the Arab oil exporting nations, the nations forming the OPEC cartel quadrupled oil prices. By the end of the decade, the cost of imported crude oil to U.S. refiners was approximately 13 times that at the beginning of the decade (Figure 1.12). During the 1960s imported oil remained about \$2.50 a barrel whereas at the beginning of the 1980s prices of \$34.00 per barrel and even more prevailed [4, 25, 26].

The price of domestically produced oil also increased significantly, though not as much as that of imported oil. Higher domestic prices reflect the increased scarcity of reserves and the added difficulty of producing oil in geographically remote and hostile areas such as the Alaskan North Slope. With deregulation, domestic petroleum and natural gas prices are expected to reach world market prices if demand persists. In essence, OPEC pricing policies have greatly enhanced the monetary value of the nation's reserves of hydrocarbon fuels.

The prices set by the oil exporting nations are in one sense arbitrary in that the prices reflect the exporting nations' perceived value of their oil reserves. In another sense, the higher prices are not arbitrary since importing nations have been willing to pay them. The United States, for example, imported an average of  $8.5 \times 10^6$  bbl/day in 1979, compared to  $3.4 \times 10^6$  bbl/day in 1970. Despite a 1200% price increase (13 times the original), imports increased by 150%. But since

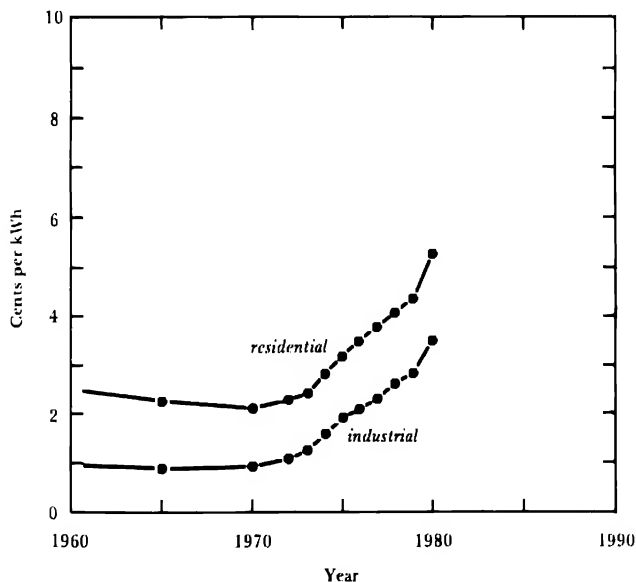


**FIGURE 1.13** Average Natural Gas Utility Prices (Reference 25).

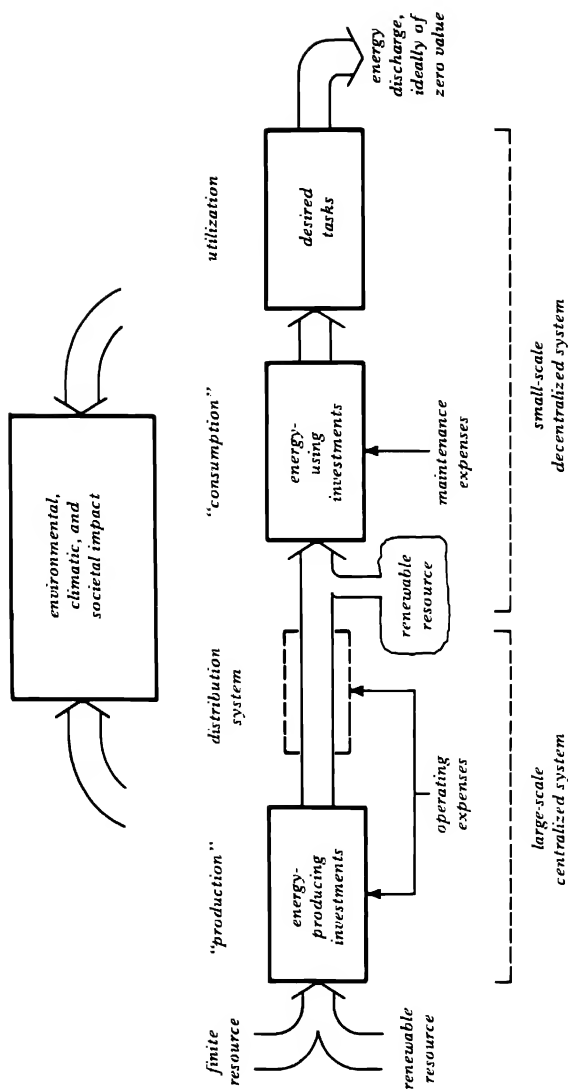
then imports have been declining; in 1982 they were 41% less than in 1979. This decline may be directly attributed to reduced consumption since domestic production rates changed little over this period.

One measure of the value of an energy resource when production costs are small compared to the market price is the cost of providing a substitute. This, in the United States, could be the cost of extracting kerogen from oil shale and refining it into petroleum products or the cost of synthesizing liquid fuels from coal. While several years would be required to establish a synthetic fuels industry with an output sufficient to reduce imports, it appears that as the price of imported oil has not reached a level that justifies such an industry. If the anticipated prices for synthesized fuels were significantly less than the price of imports, a profitable industry would emerge.

The prices of both natural gas and electrical energy, as indicated in Figures 1.13 and 1.14, have also increased significantly since 1970, after a stable period during the 1960 decade [25]. The energy value of a fuel, based upon its heat of combustion, may be specified in million British thermal units, MBtu ( $10^6$  Btu) or in billion joules, GJ ( $10^9$  J). Fuel prices are usually expressed per MBtu or GJ (1 MBtu = 1.054 GJ), the two prices being nearly (within 5.4%) equal. For example, oil costing \$30/bbl ( $5.8 \times 10^6$  Btu/bbl) implies a fuel cost of \$5.17/MBtu or \$4.91/GJ, that is, \$5/MBtu or \$5/GJ in round numbers. Electrical energy could be expressed in terms of MBtu



**FIGURE 1.14** Average Electric Utility Prices (Reference 25).



**FIGURE 1.15 An Energy System.**

or GJ to provide a comparison with the fuels used by a specific consumer or consuming group. Such comparisons, however, may not always be appropriate since electrical energy is, from a thermodynamic perspective, equivalent to work, whereas a fuel, when used for combustion, provides heat. This distinction between heat and work is the basis of thermodynamic theory (Chapter 4). Since approximately three units of fuel are usually expended to produce one unit of electrical energy, electricity might be considered to be three times more valuable than the fuel from which it is produced.

The cost of providing a quantity of energy depends upon several factors, but these factors can often be grouped into two categories: operating costs (such as the labor cost of mining coal) and investment costs (such as the capital expenditure necessary to develop a coal mine or to construct an electric power plant). In addition, there are other costs usually more difficult to quantify. The depletion of a finite reserve of a fuel, for example oil, may result in an added cost to future generations. Also, since all energy related activities modify the environment, usually in an undesirable manner (pollution), some could result in undesirable climatic changes (carbon dioxide from the combustion of fossil fuels), and many of the larger energy projects have significant societal impacts (the construction of the Alaskan pipeline). Ideally, the benefit achieved through the use of the energy to accomplish a desired task should be weighed against all costs. It is the impact of the complete energy system (Figure 1.15) that is important in assessing alternative systems. The desired task has been included in the energy system of Figure 1.15 since it is generally not the energy itself that is of ultimate value to a consumer but that which is accomplished through its use.\*

According to the first law of thermodynamics, energy is always conserved. Therefore, energy merely flows through (its flow rate being power) the system of Figure 1.15. The component of the system commonly referred to as production is really an energy *conversion* process, such as the extraction of an energy resource like oil from deep wells and its refining to desired products. The term energy consumption is also technically incorrect. Energy per se is not consumed but rather its thermodynamic value (available work is one such measure of value) is lessened. In keeping a building warm, for example, heat must be provided by a heating system to compensate for the heat leaking from the building (and hence warming the outdoors). Energy is "consumed" in keeping the building warm only in the sense that the heat escaping through the structure could not again be used-- its temperature is too low. Except for the energy that might escape from parts of the system, for example, flue losses or electrical transmission line losses in which electrical energy is converted to heat, the system's components merely alter the form of the energy flowing through it. In the process of accomplishing a desired task (or tasks), the thermodynamic as well as economic value of the energy is, ideally, reduced to zero.

The U.S. energy system might be characterized as having, primarily, large-scale, centralized energy producing and distributing facilities which supply both large- and

---

\* For example, in a home, the task might be the maintenance of a desired interior temperature. The quantity of fuel utilized depends not only upon the efficiency of the device used to convert it to heat but also upon the thermal integrity of the building.



small-scale users. An example of a small-scale, decentralized conversion system is an individualized solar space- and water-heating unit with a roof-mounted collector. Also, some tasks are accomplished directly such as the heat provided by a south-facing window (a passive solar collector) and windows that provide natural lighting. While these inputs do not appear on energy accounts, added inputs of energy would be required were these inputs not utilized.

A large-scale energy supplying system, such as an electric utility, provides simultaneously the energy required for a multitude of tasks throughout the residential, commercial, and industrial sectors. Each individual task, however, demands a share (often very small) of the energy supplied by the utility. One might, therefore, associate with each energy-using task a small share of the supplying system. An electric refrigerator, for example, could be rated in terms of its average electrical power. (Even though the refrigerator's demand is intermittent, the supplying utility must consider an average load.) An average electrical generating capacity equal to that required to power the refrigerator is thus necessary. For each new refrigerator, or any other electricity-using component added to the system, a generating capacity sufficient for the demand must be added to the system.

Similarly, a portion of the natural gas supplying system might be directly associated with supplying the heating needs of an individual residence, each newly constructed residence necessitating an incremental expansion of the supplying system. For petroleum products, each vehicle added to the existing stock of automobiles (as opposed to a vehicle replacing one that is scrapped) requires an expansion of the supply system (new drilling and increased transportation, refining, and distribution).

In the past, additional energy needs were generally satisfied by expanding the supply system. An alternative to an expansion of the supply system is to increase the efficiency of energy usage. To the extent that energy is "saved" by increasing the efficiency or effectiveness with which it is used, the "saved" energy is available for fulfilling a new need without expanding the supply system.

## 6. SOURCE MATERIALS

The number of publications by both private organizations and governmental departments dealing with energy is enormous. Even a listing of only the more significant publications is impossible and, furthermore, many of the materials would soon be dated. Fortunately, many periodically published materials are readily available in most libraries and can be consulted for recent information.

Statistical data on energy production and consumption in the United States is available on a monthly basis:

*Monthly Energy Review*, published by the Energy Information Administration of the U.S. Department of Energy

Other statistical data, often of interest, is also available:

*Statistical Abstract of the United States*, published yearly by the U.S. Bureau of the Census (U.S. Department of Commerce)

Historical data which includes energy tabulations for the past century have also been compiled:

*Historical Statistics of the United States, Colonial Times to 1970*, also published by the U.S. Bureau of the Census

Economic and business data is available on a monthly basis:

*Survey of Current Business*, published by the Bureau of Economic Analysis (U.S. Department of Commerce)

The United Nations also publishes numerous sets of data that are of value for doing research on world energy production and usage problems:

*U.N. Statistical Yearbook*, published yearly

*Monthly Bulletin of Statistics*

*World Energy Supplies 1950-74* (and various updating supplements)

These are all published by the U.N. Department of Economic and Social Affairs.

Several general publications, dealing with a wide range of energy issues, are frequently helpful:

*Annual Review of Energy*, published yearly by Annual Reviews

*Energy*, published most months by Pergamon Press

*Energy Policy*, published quarterly by IPC Science and Technology Press Limited

Since its inception in 1977, the Department of Energy publishes an annual report which contains not only statistical data but also policy recommendations:

*Annual Report to Congress*, Department of Energy

Special publications by professional organizations and trade groups cover various specialized aspects of energy. These should obviously be consulted if the topic of interest falls into a readily distinguishable category (solar energy, for example). There are a few publications which are not exclusively oriented toward energy issues but which have tended to treat topical energy issues that the author has found of value:

*Science*, published weekly by the American Association for the Advancement of Science

*The Bulletin of the Atomic Scientists*, published monthly (except July and August) by the Educational Foundation for Nuclear Science

*Technology Review*, published monthly by Massachusetts Institute of Technology

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The doubling time, it will be noted, is independent of the value of the function,  $y$ , or the time. In the interval of  $t_D$ , the quantity doubles regardless of its initial size. The relation between yearly growth rates and the doubling time in years is given in Table 1.4.

**TABLE 1.4 Doubling Times for Selected Yearly Growth Rates**

Yearly growth rate	$a$ per year	$t_D$ years
$1\%$	.01	69.3
$2\%$	.02	34.7
$3\%$	.03	23.1
$4\%$	.04	17.3
$5\%$	.05	13.9
$7\%$	.07	9.9
$10\%$	.10	6.93

For a yearly growth rate of  $5\%$  (such as occurred for world energy consumption), the doubling time is 13.9 years. If this growth rate is maintained, energy consumption would approximately double in a 14-year period. During a second 14-year period, consumption would double again, resulting in a consumption rate four times as great as the initial value. If this growth rate is maintained for three doubling periods (42 years), consumption would be eight times as great. Population trends also exhibit exponential growth. The present world growth rate of  $2\%$  per year will double the population in approximately 35 years. That is, if the growth rate remains constant, the world population will be twice that of 1965 in the year 2000.

Many economic processes are based upon growth. A  $5\%$  yearly growth rate for the GNP is considered healthy, while a one or two percent growth rate is associated with a lagging economy. A constant  $5\%$  yearly growth rate yields a GNP that will be eight times its present value in three doubling periods (42 years). Since the United States' GNP was a trillion dollars ( $\$10^{12}$ ) in 1971, it will have a value of eight trillion dollars ( $\$8 \times 10^{12}$ ) in the year 2013 if the average growth rate is  $5\%/y$ . Even relatively small growth rates (such as  $5\%$  per year) can result in exceedingly large changes in a rather short period.

In considering exponential growth, the area under the consumption rate curve is also of interest. The rate at which energy is consumed is power. For an increment in time  $\Delta t$ ,  $P \Delta t$  is the energy consumed in that increment. Within an interval between  $t_1$  and  $t_2$ , the area is simply the integral between these limits.

$$\int_{t_1}^{t_2} P \, dt$$

This is the energy consumed in the period  $t_2 - t_1$ . If  $P$  is in watts and the time in seconds, the result is in joules. If, however, years are used for the time scale, the result is in watt-years.

1930, approximately 13 billion barrels were produced and consumption was 12.1 billion barrels (.9 billion were exported).

- (a) What was the total production and consumption of petroleum prior to 1970?
  - (b) Assume that the production and consumption rates can be approximated with exponentials of the form  $P_0 e^{at}$ . What are the appropriate values of  $a$  if the exponentials are to give the correct rates for 1970 and to result in the same total production/consumption prior to 1970? The exponentials may be assumed to be valid from  $t = -\infty$  to 1970.
  - (c) What are the production and consumption rates, expressed in bbl/day, for 1930 and 1950 predicted by the exponential approximation?
  - (d) What rates do the exponential approximations yield for 1985 and 2000?
  - (e) What is the total production and consumption of petroleum for the 1970 to 2000 period predicted by the exponential functions?
4. While the global average per capita rate of energy consumption was approximately 1.9 kW in 1978, the variation between the rates of different regions was very large (Figure 1.6). A quantitative measure of this effect is the variance of the per capita rate of energy consumption. The square of the variance is equal to the average of the square of the deviation from the average for each population group. As for determining average values, the contribution of each population group is weighted according to its population.
- (a) What is the per capita variance in the rate of energy usage for the world's inhabitants?
  - (b) Suppose the rate of per capita energy usage is increased to 1.9 kW for those regions that are below this level and the per capita rates for the other regions remain unchanged. What is the new global average rate of per capita energy consumption? What is its variance for this condition?
5. The yearly growth rate of a particular quantity is 5%. Assume the initial value of the quantity is 1.0000.
- (a) Determine its value at the conclusion of 14 years if the growth is compounded once a year.
  - (b) Repeat part (a) for semiannual compounding.
  - (c) Repeat part (a) for continuous compounding, that is, exponential growth.
  - (d) Using the result of part (c) as the reference quantity, determine the percentage difference in the results of parts (a) and (b).
6. The cost of a fuel for a particular need is presently  $R$  dollars per year. Assume that its cost increases by a constant percentage each year, that is, it increases exponentially with an exponential coefficient of  $a$ .
- (a) Obtain an expression for the total cost of the fuel over the next  $T$  years. What is the ratio of this expenditure to that which would occur if its cost remained constant at  $R$  dollars per year?
  - (b) Consider a 20-year period. What is the effect of a 3% yearly rate of increase? What are the effects of 5%/y, 10%/y, and 20%/y rates of increase?
7. A particular automobile has a mass of 1.5 metric tons.
- (a) What is the kinetic energy of the automobile when it is traveling at a speed

of 100 km/h? Express the result in Joules, British thermal units, calories, and kilograms of coal equivalent and in barrels of oil equivalent.

- (b) What is the potential energy gained by an automobile for an elevation increase of 1.0 km? Express the result in the same units as in part (a).
  - (c) The automobile uses gasoline for a fuel and its engine/drive system is 10% efficient. What is the quantity of gasoline required to achieve the kinetic energy of part (a) and the potential energy of part (b)?
8. An individual, 80 kg mass, climbs to the top of a peak 1.5 km above the starting point in a time of 6 hours.
- (a) What is the potential energy, expressed in Joules and British thermal units, gained by the individual? What is the average power, expressed in watts and horsepower, expended in achieving the change in potential energy?
  - (b) What food input, expressed in kilogram calories, would be necessary to compensate for the work expended in doing the climb? Assume a metabolic efficiency of 10%.
  - (c) Assume that food has an energy value, per unit mass, that is one-half that of coal. Determine the quantity of food (in kilograms) that would be required.
9. A single-family residence has walls with an average thermal resistance value ( $R$ ) of 8. This implies that the heat loss is .125 Btu/h ft<sup>2</sup> (1/ $R$ ) for each Fahrenheit degree temperature difference. The outside temperature is 20°F, the indoor temperature is 70°F, and the wall area is 1000 ft<sup>2</sup>.
- (a) What is the heat loss for the walls expressed in British thermal units per hour? What would be the total daily heating requirement, expressed in British thermal units, if the temperature difference remains unchanged?
  - (b) What is the heat loss expressed in watts? What is, expressed in joules, the daily heating requirement?
  - (c) Suppose the heat is provided by a natural-gas furnace in which 75% of the fuel is converted to useful heat. What is, expressed in cubic feet, the daily fuel requirement?
  - (d) Suppose heating oil (140,000 Btu/gal) is used with a similar furnace. What would be this daily fuel requirement?
  - (e) Electric resistance heaters convert all electrical energy to useful heat. What would be the daily electrical energy requirement expressed in kilowatt-hours?
  - (f) Assume the walls account for one-third of the residence's heat loss. The other losses are due to heat leakages through the floor and ceiling and due to air infiltration. What would be the daily cost of heating the residence for the temperature difference specified?
- |             |                         |
|-------------|-------------------------|
| Natural gas | 50¢/100 ft <sup>3</sup> |
| Heating oil | \$1.50/gal              |
| Electricity | 7¢/kWh                  |
10. The inlet temperature for a residential water heater is 10°C and its outlet temperature is 55°C. Suppose the daily hot water usage is 200 l. A natural gas or oil-fueled heater transfers about 50% of the energy of the fuel to the water while an electric heater transfers essentially all the electrical energy to the water.

- (a) Determine the daily quantities of fuel or electricity needed to provide the hot water. Using the energy costs of Problem 9, determine the daily and yearly cost of providing the hot water.
- (b) Suppose a source of "waste" heat, say that exhausted by a refrigerator, is available to preheat the water entering the heater. What fraction of the heating would a source with an average power of 100 W provide?
11. The energy intensiveness of a particular transportation mode is usually expressed in terms of the energy required per passenger per unit of distance. Determine, for the following modes, the energy intensiveness expressed in British thermal units per mile and in joules per kilometer. Fuel oil may be assumed to have an energy value of 140,000 Btu/gal and gasoline 125,000 Btu/gal.
- (a) Automobile: 20 miles/gal, two occupants
  - (b) Diesel train: 400 gal fuel/h, 50 mph, 300 passengers
  - (c) 747 jet: 3000 gal fuel/h, 550 mph, 300 passengers
  - (d) Concorde: 10,000 gal fuel/h, 1200 mph, 90 passengers
  - (e) Ocean liner: 10,000 gal fuel/h, 30 mph, 1000 passengers
12. Assume the following estimates for remaining U.S. reserves of fossil fuels are valid:
- |             |                                      |
|-------------|--------------------------------------|
| Coal        | $350 \times 10^9$ metric tons        |
| Petroleum   | $125 \times 10^9$ barrels            |
| Natural gas | $6.5 \times 10^{14}$ ft <sup>3</sup> |
- (a) What is the total energy reserve expressed in joules, British thermal units, watt-years? What fraction of the total energy is due to each source?
  - (b) Using the 1979 national energy consumption rate of  $2.6 \times 10^{12}$  W, determine the static index for the total reserve. What is the index for a 5%/y growth? Determine the percentage decline in energy usage that results in the reserve's lasting in perpetuity.
  - (c) Using the consumption rates for each fuel, determine the static index for each fuel. What is the index, in each case, for a 5%/y growth in usage?
13. Residential and commercial space heating represents a substantial portion of U.S. energy consumption (Figure 1.8). Assume an overall consumption rate of  $2.6 \times 10^{12}$  W and a fractional allocation corresponding to the data of 1968.
- (a) What is the energy consumption rate for space heating?
  - (b) Assuming a U.S. population of 220 million, determine the per capita consumption rate for heating.
  - (c) Determine the coal equivalent per capita rate for heating in metric tons per year.
  - (d) Determine the oil equivalent (barrels and gallons per year) and gas equivalent (cubic feet per year) per capita for heating.
14. What is the per capita energy consumption for transportation (1968 allocation and a total consumption rate of  $2.6 \times 10^{12}$  W)? What is this in barrels and gallons of oil per year?
15. The consumption of electrical energy is given in Figure 1.9.
- (a) What were, for the world and the United States, the average growth rates for the 1940 to 1970 period?

- (b) What was, for 1978, the rate of electrical energy usage, expressed in watts, for the world and the United States?
  - (c) On the average, the conversion of fuels to electrical energy is about 30% efficient. Determine the rate fuels are used, expressed in watts, to generate electricity for the world and the United States. What is the approximate fraction of overall energy usage that is attributable to electricity generation?
  - (d) What are the average per capita rates of electrical energy usage for the world and the United States? Express the results in watts and kilowatt-hours per year.
16. In the United States, residences account for approximately 30% of the electrical energy consumption. Consider a residence occupied by four persons and use the 1978 U.S. per capita result of Problem 15.
- (a) What was, based on the average per capita rate of electricity consumption, the yearly electrical energy usage, expressed in kilowatt-hours, by this residence?
  - (b) What was the average monthly usage and the cost of the electricity based on the rates of Figure 1.14?
  - (c) What was the average electrical power used by the residence?
  - (d) Assume a generating facility supplying the electricity has an average load factor of .6 and that the generating investment cost is \$1000/kW of peak power. What was the investment required for supplying the residence with electricity?
17. Garbage is a potential source of energy. The average quantity of garbage produced each day per person in the United States is approximately 2 kg. Assuming the energy content of garbage is approximately half that of coal, determine the per capita power, in watts, that could be obtained through its combustion. Using a per capita consumption rate of 11.9 kW, determine the fraction of this quantity that could be supplied by garbage.
18. A source of petroleum in the western United States is oil shale. Suppose that the retorting of oil shale produces petroleum at the rate of one million barrels per day. Determine the power equivalent (in watts) of this source of energy. What fraction of the total U.S. energy consumption rate does this source represent?
19. In producing petroleum from oil shale, approximately 1.5 metric tons of shale need to be processed to produce one barrel of oil. (One barrel of oil has a mass of approximately 140 kg.) Determine the mass and volume of spent oil shale necessary to produce one million barrels of petroleum (the density of loosely dumped spent shale is approximately 1.2 metric tons per cubic meter). What would be the volume produced in one year for a production rate of one million barrels per day? Assuming that the spent shale is uniformly distributed with a height of 100 m, determine the land area necessary to dispose of one year's waste.



## CHAPTER 2

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# *Fossil Fuels*

### 1. INTRODUCTION

The combustion of fossil fuels is the main source of thermal energy for the world. Fossil fuels are the rather fortuitous result of photosynthesis that occurred over the past several hundred million years. Coal, crude oil, and natural gas were formed when living materials died, decayed, and carbon dioxide and water squeezed from them while they were trapped by geological formations. They remained entrapped in a manner that prevented their oxidation. When these fuels are oxidized (burned), a portion of the solar energy absorbed by the once living materials is recovered.

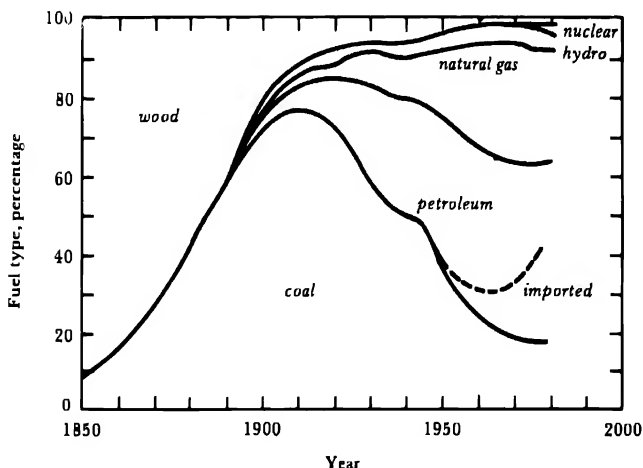
Fossil fuels are formed at an exceedingly slow rate. Compared to the present extraction rates of the fuels, their formation rates are totally negligible. Hence, fossil fuels may be treated as a finite energy source which will eventually be depleted, the time of their depletion being dependent only upon extraction rates.

Based on energy content, coal forms the largest of the fossil fuel reserves, accounting for approximately 90% of the world's fossil fuel reserves. It is principally the carbon of the coal when oxidized that results in a release of thermal energy. The hydrocarbon fuels, crude oil and natural gas, consist of organic molecules of carbon and hydrogen, both elements releasing energy when oxidation occurs. Crude oil is obtained primarily from deep (and often expensive to drill) wells and consists of a mixture of many organic compounds of hydrogen and carbon. Natural gas is often, though not always, found in or adjacent to oil fields. At recovery, it consists of a gaseous fuel, methane ( $\text{CH}_4$ ), and of hydrocarbon liquids. The fuel is known as *dry natural gas* after the liquids are removed. The liquids obtained from natural gas and

crude oil are refined into numerous petroleum products: the smallest molecules are gaseous ethane,  $C_2H_6$ , and propane,  $C_3H_8$ , and the larger molecules consist of a variety of liquids ranging from gasolines (the lightest) to asphalts (the heaviest). World reserves of crude oil and natural gas which may be recovered by conventional techniques are estimated to be about 5% each of the total fossil fuel energy endowment of the earth.

Energy usage in the United States since 1850, according to fuel type, is indicated in Figure 2.1 [1, 2]. Two fuel transitions have occurred [3]. The first, a transition from fuel wood to coal, occurred during the second half of the nineteenth century. While coal comprised less than 10% of energy inputs for the nation in 1850, it reached a peak of over 75% the first decade of the twentieth century. The second, a transition from coal to petroleum and natural gas, occurred in the twentieth century. Even though the first oil well was drilled in Titusville, Pennsylvania, in 1859, hydrocarbon fuels accounted for only 5% of the nation's energy input at the turn of the century. However, by 1940 the hydrocarbon share was 40% and at present it is approximately 70%.

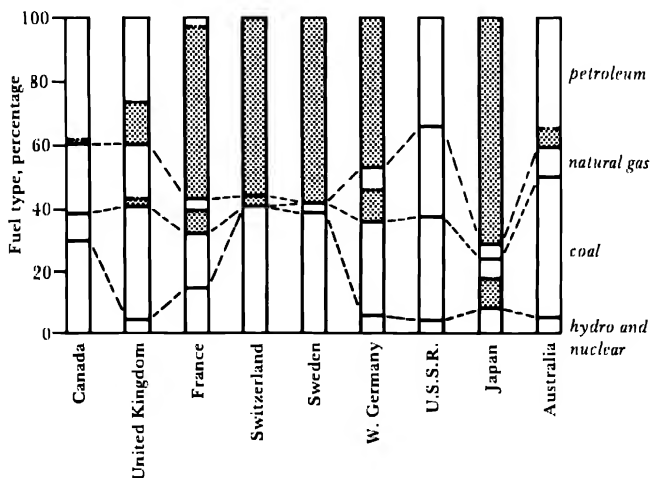
The energy transitions of Figure 2.1 are changes in the fractional composition of total energy usage. Neither is a transition in the sense that one energy form was substituted for another. In each case, the new energy source augmented rather than



**FIGURE 2.1 U.S. Energy Usage by Type, 1850 to Present. Hydro-power and Nuclear Power Have Been Rated on a Fuel Equivalent Basis, That Is, in Terms of the Energy Value of a Fossil Fuel That Would Have Otherwise Been Needed (References 1 and 2).**

replaced the earlier source. The use of fuel wood in the United States peaked at about  $10^{11}$  W in 1870 ( $2.9 \times 10^{15}$  Btu/y, approximately 4% of the present overall energy consumption rate). At the turn of the century, fuel wood consumption declined by only about one-third. Present use of fuel wood, although no longer included in national energy statistics, is still about one-half of that of 1900 (about  $3.2 \times 10^{10}$  W, 1.3% of overall energy usage). During the wood to coal transition, growing inputs of coal provided for new energy demands while the use of fuel wood declined only slightly. The coal to hydrocarbon transition was similar in that growing inputs of hydrocarbon fuels supplied new energy demands. While coal production has fluctuated, the consumption rates of 1910 and 1970 were nearly equal ( $4.2 \times 10^{11}$  W). Hydrocarbon fuels displaced coal only in the sense that had they not been available the consumption of coal would very likely have increased significantly.

Another energy transition, not indicated in Figure 2.1, also occurred during the twentieth century: the expanded use of electrical energy. Fossil fuels as well as hydropower and now nuclear power are transformed into a secondary energy form, electricity, which may be readily distributed. While at the beginning of the century the energy share of electric utilities was negligible, electric power generation in 1979 required 31% of the nation's energy inputs. Electricity cannot only be readily distributed to individual users, but, being a high-quality form of energy, equivalent, in thermodynamic terms to work, it can fulfill numerous energy needs. An energy



**FIGURE 2.2** *Energy Consumption by Type for Several Developed Nations, 1978. Hydropower and Nuclear Power Rated on a Fuel Equivalent Basis. Shaded areas indicate imports. (Reference 4.)*

cost is incurred, however, since on a national average, only about one-third of the energy of a fuel is converted to electrical energy.

Closely related to the coal to hydrocarbon transition is the development of the internal combustion engine at the end of the nineteenth century. The introduction of the automobile and the assembly line during the first decade of the twentieth century created a growing demand for petroleum products. The internal combustion engine used throughout the transportation sector now accounts for over one-half of the nation's petroleum consumption and approximately one-quarter of the nation's overall energy demand. Abundant (relative to demand) domestic petroleum reserves at the beginning of the century made the widescale use of automobiles feasible. The growth of this mode of transportation, however, has now significantly depleted the nation's petroleum reserves and necessitated large petroleum imports.

Indicated in Figure 2.2 is energy usage, by type of fuel, for several developed nations. Only a few of the developed nations have substantial fossil fuel reserves and, hence, most rely upon imports, primarily of petroleum fuels. Coal usage tends to be small except in those few nations with large reserves. On the basis of energy content, the world trade in coal is nearly insignificant compared to that of petroleum. For many of the developed nations, imported hydrocarbon fuels account for over half of the energy inputs.

The world reserves of hydrocarbon fuels are sufficient to maintain a growth in consumption for only one or two decades. The political disposition of the oil exporting nations, however, may further limit such growth. In the United States, the production of hydrocarbon fuels, including that of Alaska, has declined following a peak production in 1971. (In 1979, natural gas and petroleum production, assessed on an energy basis, was 12% below that of 1971.) Another energy transition may be on the horizon. Other energy inputs and/or increased efficiency in the use of energy will be needed if the production of hydrocarbon fuels in the United States continues to decline. Even if the world's oil production does not decline, a similar transition will probably be needed to provide for the world's growing population and to improve living standards.

## **2. COAL**

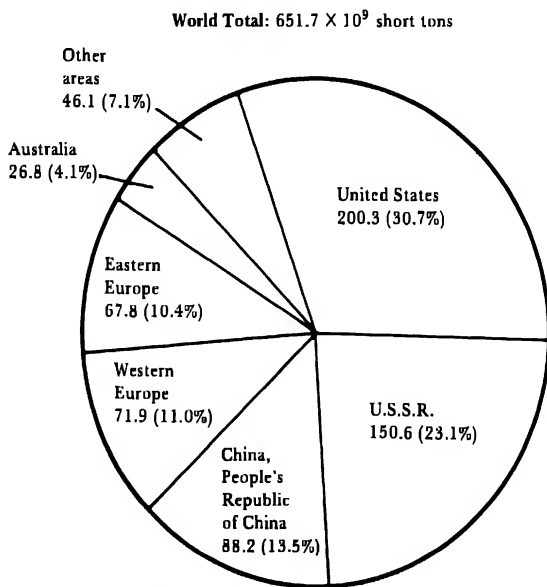
Most of the world's coal reserves was formed over 100 million years ago during the latter part of the Paleozoic Era. First, peat was produced from decaying vegetation that accumulated at the bottom of swamps. The formation of this low-energy fuel precedes that of the more desirable higher energy coals. Lignite, the lowest grade of coal, was formed when water, carbon dioxide, and methane were squeezed from the peat deposits by overbearing layers of soil and sand. A continuation of this basic coal-forming process eventually yielded the higher grades of coal, subbituminous and bituminous. Finally, when these coals were subjected to extremely high pressures, primarily the result of horizontal forces associated with mountain building, the coals were transformed to anthracite. Reserves of anthracite, the coal with the highest energy value (and generally the most desirable) are very limited. The longer the

**TABLE 2.1 Coal Classification**

	<i>Fixed carbon</i>	<i>Btu/lb</i>
Lignite	<40%	5,500–8,000
Subbituminous	40–60%	8,000–12,000
Bituminous	50–86%	11,000–15,000
Anthracite	86–98%	13,000–16,000

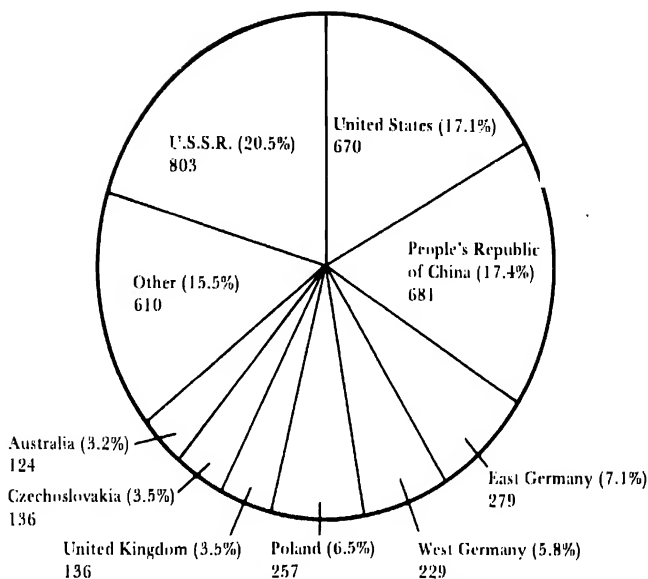
original organic material was subjected to the coal-forming process, the higher its carbon content and hence the higher its energy value [5]. Table 2.1 summarizes the characteristics of the various coals.

The world's recoverable reserves of coal are very large. M. King Hubbert has placed the original in-place reserves within 1000 ft of the earth's surface at 2 trillion metric tons [6–11]. Based on the 1978 world consumption rate of about 3.6 billion metric tons, this reserve would last in excess of 500 years. Even if all energy were



**FIGURE 2.3 Estimated Recoverable Reserves of Coal for the World, 1974 ( $10^9$  Short Tons) (Reference 2).**

World Total:  $3,925 \times 10^6$  short tons

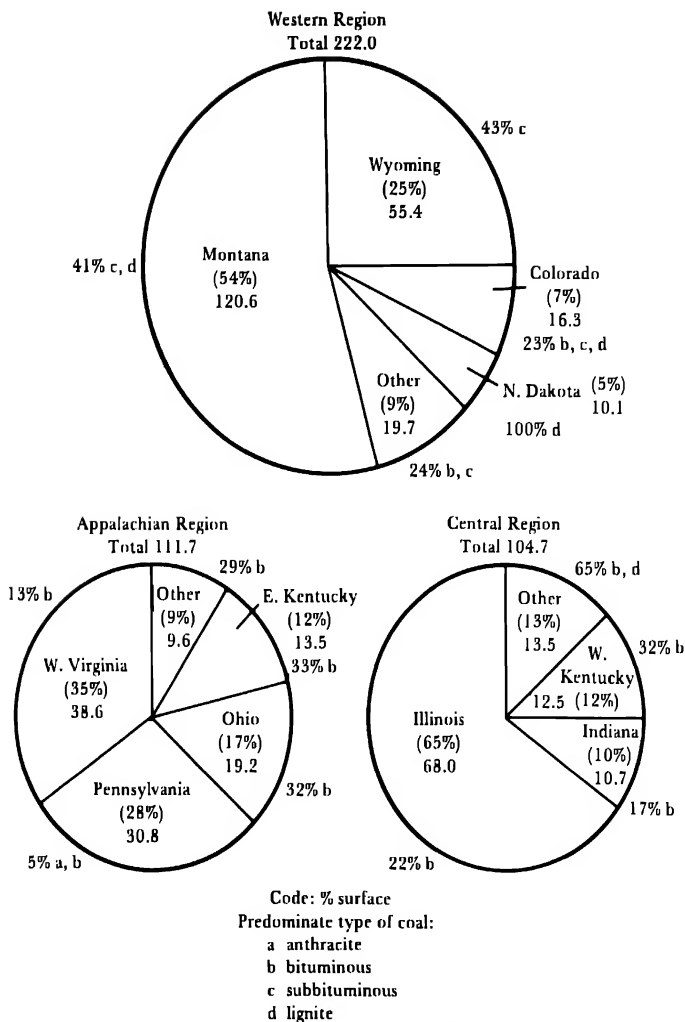


**FIGURE 2.4 International Coal Production, 1978 ( $10^6$  Short Tons)**  
(Reference 2).

to be derived from coal, these reserves would, again based on the 1978 energy consumption rate, last for about 180 years. If coal to a depth of 6000 ft is included, the reserve nearly quadruples ( $7.6 \times 10^{12}$  metric tons, according to Hubbert). Coal production, at least in the foreseeable future, will not be limited by insufficient reserves.

Another estimate for the world's reserves of coal, published by the U.S. Department of Energy, is presented in Figure 2.3 while production by the major coal producing nations is indicated in Figure 2.4 [2]. The recoverable reserves indicated in Figure 2.3 are less than one-third of that estimated by Hubbert. Such differences arise from the manner in which "recoverable" is defined.\* While both estimates are for reserves within 1000 ft of the earth's surface, it is the minimum acceptable thickness of the coal beds to be included that affects the estimates. Hubbert

\*Hubbert's estimate is for the original in-place recoverable reserve, that is, it includes coal already extracted, while that of Figure 2.3 is for coal remaining as of 1974. Since only a small fraction of the total reserve, regardless of the estimate, has been mined, this alone does not account for the large difference.



**FIGURE 2.5 Demonstrated Coal Reserve Base by State, 1976 (10<sup>9</sup> Short Tons). Depending Upon the Coal Field, 40 to 90% of the Reserve Base Is Considered Recoverable (Reference 2).**

(following Averitt) considers beds of anthracite and bituminous coal with a minimum thickness of 28 in. and beds of subbituminous coal and lignite with a minimum thickness of 5 ft. The data of Figure 2.3 appears to be based on a minimum thickness of 42 in. for anthracite and bituminous coal and 10 ft for the lower grade coals [4]. Based on the estimate of Figure 2.3 and the overall world production rate of Figure 2.4, the world's coal reserves are sufficient for 166 years (static index). However, from 1975 to 1978 world coal production increased by 8.5%, an average growth rate of 2.7%/y. If this growth in coal production and consumption continues, the reserves would last 63 years (exponential index).

Over one-quarter of the world's recoverable coal reserves are located in the United States (a slightly smaller fraction if Hubbert's estimate is utilized). Figure 2.5 shows the geographical distribution of the nation's coal reserve base and the dominant type of coal available in each region [2]. The "reserve base" of Figure 2.5 is about twice the U.S. portion of the recoverable reserves of coal of Figure 2.3. Only 40 to 90% of the in-place coal reserve, depending upon the particular deposit, is considered recoverable. Also of importance in assessing the economic value of a particular coal reserve is the sulfur content of the coal and the method by which it can be mined (Figure 2.6). The sulfur of the coal combines with oxygen during combustion, producing sulfur oxides which not only pose a significant health hazard but also react with water vapor producing acid rain. The lower the sulfur content of the coal, the lower the emission levels are without the use of emission controls (scrubbers). When emission controls are employed, the smaller quantities of sulfur oxides from the lower sulfur coals are easier to remove. Not surprisingly, coal consumers are willing to pay a premium for low-sulfur coals.

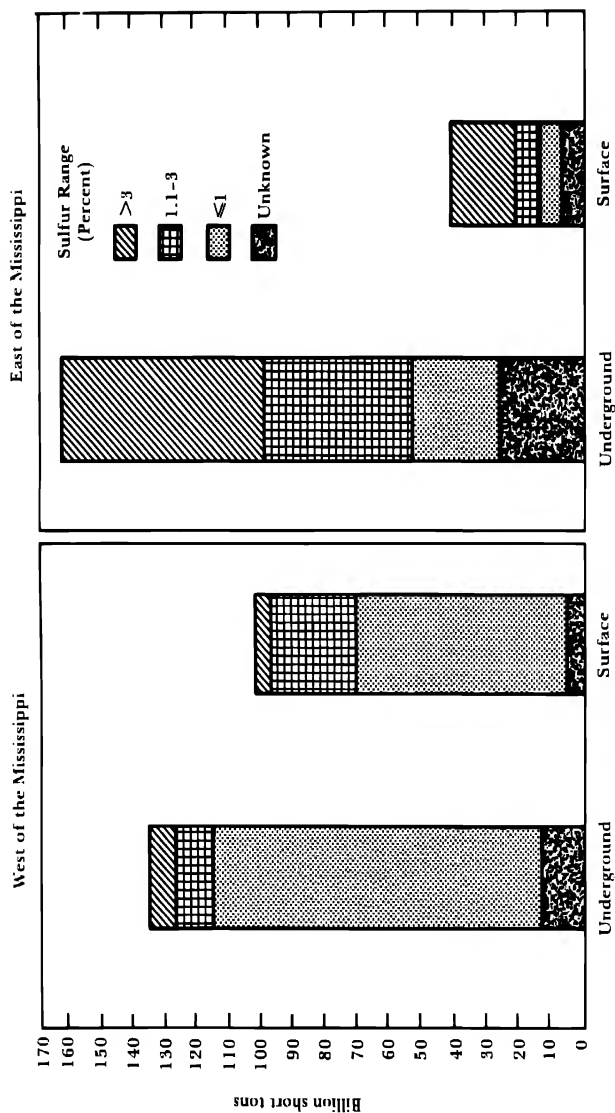
In the United States, approximately 60% of the coal is obtained from surface strip mines and the remainder from underground mines. The portion of coal obtained from surface mines has been steadily increasing: in 1950, only one-quarter of the coal was so produced. Not only is surface mining of coal less expensive, but the hazards associated with underground mining, high accident rates and black lung disease, are also avoided.

The flow diagram of Figure 2.7 indicates production and consumption rates for coal (19.3% of total energy consumption) in the United States [2, 12]. While coal accounted for 46% of the energy inputs of electric utilities, it accounted for 60% of the fossil fuels used in generating electrical energy. The next largest use of coal was for producing coke to supply the iron and steel industry. Coal is also used by industries for raising process steam and for high-temperature direct heat needs. A very small quantity of coal is used for water and space heating by the residential and commercial sectors, and almost none is used by the transportation sector. The petroleum and natural gas inputs of the consuming sectors are included in Figure 2.7, with the energy value of these inputs expressed in a coal equivalent.\*

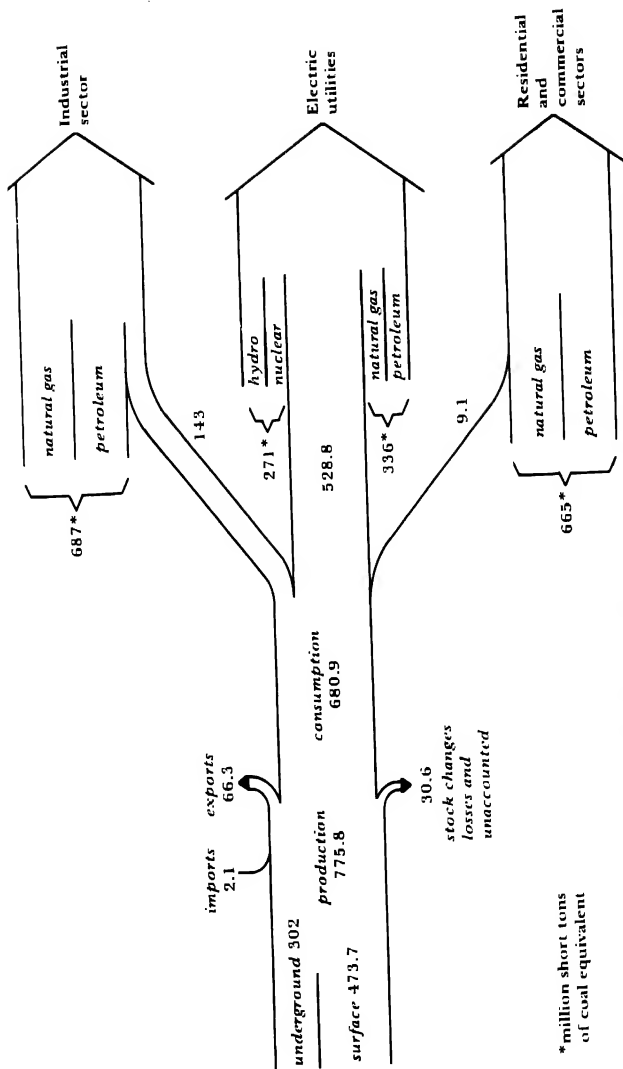
Proposed national energy policies have tended to be premised on more fully utilizing the nation's extensive coal reserves. Increased coal consumption, it is

\* An energy value of  $22.1 \times 10^6$  Btu/short ton was used in order to be consistent with the average energy values of the coal production portion of the diagram.





**FIGURE 2.6** U.S. Coal Reserve Base by Sulfur Content and Potential Method of Mining (Reference 2).



**FIGURE 2.7 U.S. Coal Production and Consumption, 1979. Units of Million Short Tons (Reference 2).**

proposed, could make up for the decline in the domestic production of hydrocarbon fuels and reduce the nation's dependence upon imported petroleum. While coal may be converted to liquid and gaseous fuels (synfuels) which could then be substituted for petroleum and natural gas presently obtained from conventional sources, coal can also be directly substituted for hydrocarbon fuels in some applications.

Coal is already extensively used by electric utilities, but its use could be further increased to reduce the consumption of hydrocarbon fuels. The energy conversion efficiency is comparable for all fossil fuels. While coal-fired generating facilities tend to be more expensive than those using petroleum or natural gas, coal, per unit of energy, has and is expected to remain less costly than hydrocarbon fuels, particularly imported petroleum. Coal could also be substituted for a portion of the hydrocarbon fuels used by the industrial sector. Based on data for 1974, approximately one-quarter of the hydrocarbon fuel consumption by this sector is for high-temperature direct heat requirements and nearly one-quarter for raising process steam. Furthermore, the cogeneration of electrical energy along with producing process steam (generally saturated steam with a temperature of 200°C or less) could, as is the case in Europe, be more widely adopted [13-15]. For cogeneration, high-temperature steam produced by a boiler is first used to turn a steam turbine connected to an electric generator. The turbine is designed so that the exhausted steam is suitable for the process steam need. As is often the case for an optimally designed thermodynamic system, the two tasks can be accomplished together with less fuel than when done individually.

If coal is substituted for the hydrocarbon fuels used by electric utilities and for half of those fuels used by the industrial sector (for process steam and direct heat), coal consumption would double and hydrocarbon consumption would decline by 26% (1979 consumption patterns). Even a partial conversion from hydrocarbon fuels to coal in these sectors could significantly reduce the demand for hydrocarbon fuels.

Usually the residential and commercial sectors are overlooked when assessing the potential for expanding the use of coal. Energy efficient, clean burning, convenient furnaces suitable for small residential and commercial demands are not available. Nevertheless, coal could be used indirectly through district heating systems [16], where a central station with a coal-fired boiler is used to heat water that is distributed by an underground pipe system to individual consumers. Heat is transferred by a heat exchanger to heat water and for space heating, and the cooled water of the district system is returned to the central station. As in the case of process steam, the cogeneration of electricity at the central station would increase the energy efficiency of the overall system. In many cities, district heating could be achieved by modifying older but often conveniently located electric generating facilities. While large investments are necessary to convert these power plants and install hot water distribution systems, these investments would be probably no greater than those needed to construct coal gasification facilities that might otherwise be needed. One set of authors estimates that half of the U.S. population could be economically served by district heating systems [16].

Coal, at one time, was the main fuel used by railroads. The energy inefficient coal-burning steam engines have now been replaced by more efficient diesel engines.

Electrified railroads, common in Europe but rare in the United States, could further reduce petroleum usage—assuming that the electrical energy is derived from coal-burning facilities. However, railroads account for only 3.7% of the transportation sector's energy usage in the United States (1974) and hence less than 1% of the nation's overall energy consumption [12]. The saving of petroleum fuels would be increased if a portion of the freight presently carried by trucks were transferred to railroads and if passenger travel were expanded.

As it has been shown, increased coal production could be helpful in alleviating the financial and political problems of importing petroleum and in offsetting decreasing domestic production of hydrocarbon fuels. But the production and use of coal has its own set of problems. Coal mining, particularly from underground mines, has long been recognized as a very hazardous occupation. Even though fatality and injury rates have declined since the provisions of the Coal Mine Health and Safety Act of 1969 became effective, they are still high—.36 fatalities and 38 nonfatal injuries per million employee-hours in 1977 [17].\* The accident rate for underground mines is more than  $2\frac{1}{2}$  times that for surface mines. Considering that employee productivity for surface mines is three times that for underground mines, the injury rate (both fatal and nonfatal) per ton of coal recovered from underground mines is about eight times that for surface mines. Underground mine injury rates, however, vary widely from mine to mine, from a low of 8 per million employee-hours (U.S. Steel, 1977) to rates in excess of 100 (Eastern Associated and Westmorland) [17]. Recognizing that the overall accident rate for underground mines could be significantly reduced, Congress passed the Federal Mine Safety and Health Amendment Act of 1977. Many miners have also been afflicted with black lung disease (pneumoconiosis), the effect of long exposures to high levels of coal dust in mines. With increased ventilation, as required by the Coal Mine Health and Safety Act, the frequency of black lung disease is expected to be drastically reduced.

Surface mining is not only safer than underground mining but worker productivity is also high. Furthermore, coal reserves that can be strip mined, particularly those of the West, are desirable because their sulfur content is low. In areas with sufficient rainfall, restoration of the land is possible after the coal is mined [18]. Even though it is not generally feasible to restore arid Western lands to their original condition, rehabilitation is possible so that the land's surface value is not entirely sacrificed [19, 20]. Restoration or rehabilitation requires that the surface soil be saved and that the mining of the coal be carried out in a systematic and careful fashion. It is the intent of the Surface Mining Control and Reclamation Act of 1977 to minimize the long-term environmental impact of coal mining.

The combustion of coal usually results in more air pollution than where hydrocarbon fuels, particularly natural gas, are used. In order to improve air quality in heavily polluted regions, the construction of new coal-burning facilities was restricted and the conversion of coal-burning facilities to oil was encouraged during the 1960s and 1970s. On the U.S. East Coast, electric utilities switched to oil, thus increasing

\* Since average miner productivity is about 15 tons per day, there are .21 fatalities and 22 non-fatalities for each million tons of coal mined.

the nation's dependence upon foreign imports. Utility companies in Southern California pursued a different approach: they participated in the construction of coal-fired generating facilities in the Southwest (Arizona, New Mexico, and Utah). The large accessible reserves of coal in this region minimized transportation requirements, albeit that an extensive electric transmission line system was required. It was originally assumed that the unpolluted air of the region would be able to absorb the emissions of the new generating facilities. This has not been the case since air pollution has become a major problem for the entire region [21-23]. Similar plans for constructing large generating facilities in the Northern Plains region, primarily in Montana and Wyoming, are being questioned [24, 25]. On the one hand, regions with these mine-mouth power plants may be viewed as exporters of electric power. On the other hand, they can be viewed as a means by which consuming regions export their air pollution.

A key issue in increasing coal consumption involves the effectiveness of technologies for reducing the emissions of coal-fired boilers. It is the issue of flue-gas scrubbers. Scrubbers typically utilize a slurry of limestone,  $\text{CaCO}_3$ , to combine with the sulfur oxides of the combustion-produced gases before the gases are vented to the atmosphere. Prior to 1977, sulfur oxide emission limits could often be achieved simply by using low-sulfur content coals. However, in order to further reduce emission levels and improve ambient air quality, emission standards implemented in 1977 require that the "best available control technology" be used regardless of the coal's sulfur content. A consensus on the effectiveness (and even the desirability) of scrubbers for coal-fired electric generating facilities is lacking [26].

Coal was once viewed as a cheap fuel, but this is no longer the case. In order to increase mine safety and to minimize the environmental impact, the cost of mining coal has significantly increased. Also, expensive equipment to minimize the air pollutant emissions resulting from the use of coal is now required. However, as the price of hydrocarbon fuels continues to increase, the increased cost of using coal may become less significant.

Even if the most advanced technologies are utilized, the overall environmental impact of an increase in coal consumption is likely to be substantial. Sulfur oxide emissions tend to combine with the moisture of the air to produce acid rain. Only a reduction in these emissions, not their curtailment, is anticipated. Also, the unavoidable carbon dioxide emissions for coal combustion are, per unit of thermal energy, considerably greater than those for hydrocarbon fuels. Increased atmospheric levels of carbon dioxide could significantly affect the global climate (Chapter 3). Hence, the problem associated with an expansion in the use of coal should not be minimized [27-29].

### **3. ESTIMATING CRUDE OIL RESERVES**

Petroleum products such as gasoline and heating oil are obtained through a refining process from both crude oil and liquids produced by natural gas wells. In 1979 crude oil provided for 83% of the domestic petroleum production and natural gas liquids

accounted for the remaining 17%. Since crude oil is the main source of petroleum products, a reliable estimate of remaining reserves of crude oil is important. Reserves of crude oil, on the basis of energy content, are considerably less than those of coal. The decline in the production of crude oil in the United States, following a peak in 1970, is very likely the result of the nation's crude oil reserve's having been significantly depleted. Based upon generally accepted estimates, more than half of the recoverable crude oil from the lower 48 states has been extracted, and hence the decline in production is likely to continue. On a global basis, however, the untapped oil reserves are still large, sufficient for increasing production rates. But should global consumption continue to increase, production will be limited by dwindling reserves around the turn of the century.\*

While it is an estimate of the quantity of oil or other resource that may yet be extracted that is of importance, it is generally more convenient to estimate the initial in-place reserve. That remaining is thus the initial quantity less that which has already been extracted. Exponential growth in the rate at which an energy resource is utilized or extracted has been common when reserves are large, that is, at the beginning of an extraction cycle.

$$P(t) = P_0 e^{at}$$

The quantity  $P(t)$  is the energy rate, that is, power at time  $t$ . The total cumulative energy produced by a particular resource is simply the time integral of the power over all previous values of time.

$$\begin{aligned} E(t) &= \int_{-\infty}^t P(t') dt' \\ &= \frac{1}{a} P_0 e^{at} \quad (\text{for exponential growth}) \end{aligned}$$

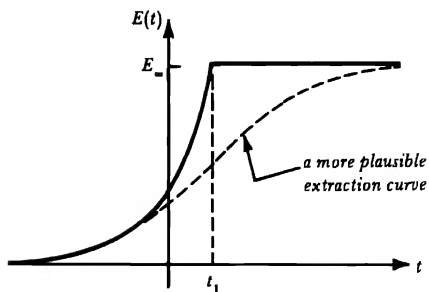
A finite resource with an energy value of  $E_\infty$  will be completely depleted at time  $t_1$  (Figure 2.8). As already discussed (Chapter 1), continued exponential growth until a resource is entirely depleted is highly unlikely. A more plausible energy extraction curve is also indicated in Figure 2.8. While the production rate, the slope of the extraction curve, tends to increase exponentially at the beginning, a peak in production occurs before the reserve is totally depleted. Following its peak, the production rate declines because of the growing scarcity of the remaining reserve [the reserve that remains at any time is  $E_\infty - E(t)$ ].

A logistic function results not only in a plausible curve for the extraction and eventual depletion of a finite resource, but it is also a mathematically convenient function.

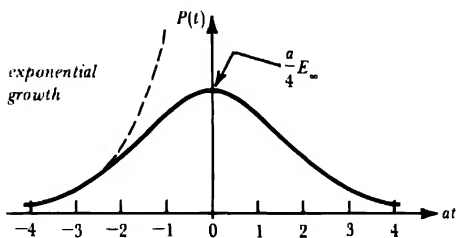
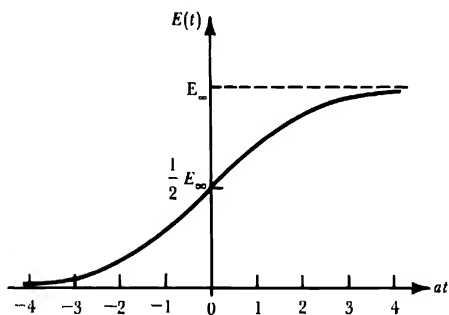
$$E(t) = \frac{E_\infty e^{at}}{1 + e^{at}} = \frac{E_\infty}{1 + e^{-at}}$$

---

\* The procedure followed and data utilized in this section are from M. King Hubbert, Reference 6. This readily available Senate Document is highly recommended for anyone wishing to gain not only a better background in estimating crude oil reserves but also an understanding of the methodology used in treating depletable fuel resources. References 7-11 are other papers by Hubbert.



**FIGURE 2.8** A Cumulative Energy Extraction Curve for Exponential Growth.



**FIGURE 2.9** A Logistic Energy Curve and Its Derivative.

For small values of time ( $t$  much less than zero), the behavior of both the energy function and its derivative, power, approximate that of exponential growth.

$$E \approx E_{\infty} e^{at}$$

$$P = \frac{dE}{dt} \approx aE_{\infty} e^{at} \quad t \ll 0$$

The time derivative of the logistic function for energy, the energy extraction rate, may be shown to be symmetrical about the time origin.

$$P = \frac{dE}{dt} = \frac{d}{dt} \left( \frac{E_{\infty}}{1 + e^{-at}} \right)$$

$$= \frac{aE_{\infty} e^{-at}}{(1 + e^{-at})^2} = \frac{aE_{\infty}}{(e^{at/2} + e^{-at/2})^2}$$

$$P(0) = \frac{a}{4} E_{\infty}$$

For a logistic function with the above time dependence, the resource is half depleted at  $t = 0$  at which time its derivative, the extraction rate, is the maximum (Figure 2.9). If it is desired that the peak extraction rate occur at a different time ( $t_0$ ), it is simply necessary to replace  $t$  with a new quantity  $t - t_0$  in the above expression.

The coefficient of the exponent,  $a$ , is the approximate growth rate during the early extraction period for a resource. The higher the growth rate, the greater the peak extraction rate for a logistic function. Its "width," or time interval for the large extraction rate, is correspondingly reduced. To demonstrate this, consider the energy extraction rate curve of Figure 2.10 in which a rectangular function of width  $\tau$  for the extraction curve is introduced. For convenience, the width,  $\tau$ , will be defined such that the rectangle's area is equal to the total reserve,  $E_{\infty}$ .

$$\frac{a}{4} E_{\infty} \tau = E_{\infty}$$

$$\tau = \frac{4}{a}$$

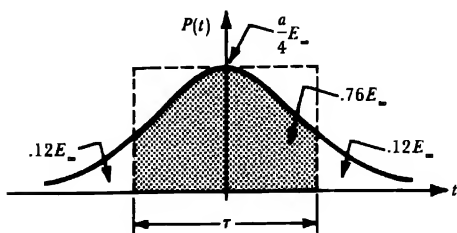
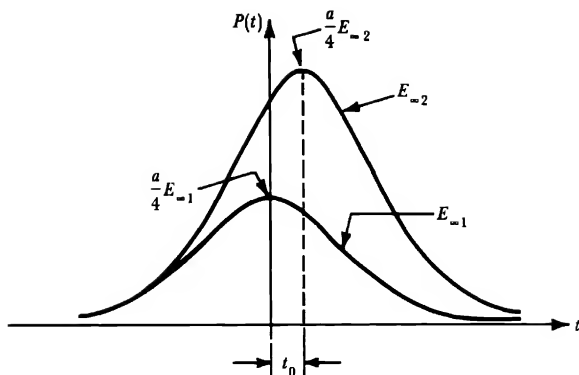


FIGURE 2.10 Energy Extraction for a Logistic Curve.





**FIGURE 2.11** *Energy Extraction Rates for Different Energy Reserves.*

For  $t = \pm \tau/2$  ( $at = \pm 2$ ), the height of the energy extraction rate curve for the logistic energy function is equal to .42 of its peak value. The area under the curve in this interval is  $.76E_{\infty}$  and each "tail" of the extraction rate curve ( $t < -\tau/2$  and  $t > \tau/2$ ) accounts for only 12% of the total resource usage. Except for small values of the exponential coefficient,  $a$ , almost all of a resource is depleted in a relatively short interval (76% in 80 years for an exponential coefficient,  $a$ , corresponding to  $5\%/y$ ).

Two extraction curves, corresponding to different resource values,  $E_{\infty 1}$  and  $E_{\infty 2}$ , are indicated in Figure 2.11. If the exponential coefficients of the curves are the same, the early extraction rates ( $t \ll 0$ ) can be made approximately equal by displacing one of the curves in time by a value of  $t_0$ . The following is obtained.

$$E_1(t) = \frac{E_{\infty 1}e^{at}}{1 + e^{at}} \approx E_{\infty 1}e^{at}$$

$$E_2(t) = \frac{E_{\infty 2}e^{a(t-t_0)}}{1 + e^{a(t-t_0)}} \approx E_{\infty 2}e^{a(t-t_0)}$$

$$E_{\infty 1} = E_{\infty 2}e^{-at_0}$$

$$t_0 = \frac{1}{a} \ln(E_{\infty 2}/E_{\infty 1})$$

When the second resource base is equal to twice the first ( $E_{\infty 2} = 2E_{\infty 1}$ ), then the time difference,  $t_0$ , is equal to the doubling time associated with a growth rate of  $a$ .

$$t_0 = (1/a) \ln 2 = t_D$$

Thus, a doubling of the resource base results in an energy extraction rate curve with

a peak delayed by only one doubling time and a peak value twice that of the original extraction curve.

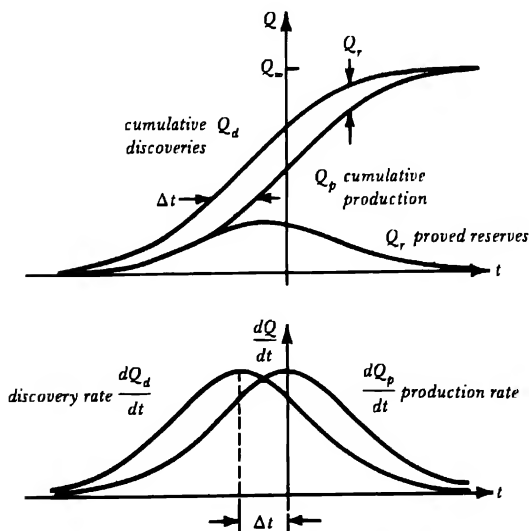
Quantities associated with domestic crude oil production have tended to follow that predicted by a set of logistic curves. Therefore, a prediction of future behavior based upon a continuation of the same trend is not unreasonable, though it should be treated as no more than a plausible extrapolation of past events. In the case of domestic crude oil production, however, other evidence, obtained from exploratory drilling records, gives added credibility to the prediction premised on the use of logistic functions.

To begin, it will be assumed that the cumulative production of crude oil,  $Q_p$  (expressed in barrels), can be approximated by a logistic curve.

$$Q_p = \frac{Q_\infty}{1 + e^{-at}} \text{ bbl}$$

Since  $Q_p$  approaches  $Q_\infty$  for large values of time,  $Q_\infty$  is the ultimately recoverable reserve of oil. The production rate, expressed in barrels per year, is the time derivative of  $Q_p$ .

$$\frac{dQ_p}{dt} = \frac{aQ_\infty}{(e^{-at/2} + e^{at/2})^2} \text{ bbl/y}$$



**FIGURE 2.12** *Discovery and Production Cycles Based Upon Logistic Curves.*

Production is preceded by the discovery of oil in which discoveries add to that normally referred to as proved reserves. Eventually, at the conclusion of the exploration process, cumulative discoveries will be equal to  $Q_\infty$ . In the United States, cumulative discoveries of crude oil have tended to precede cumulative production by a fixed time interval.

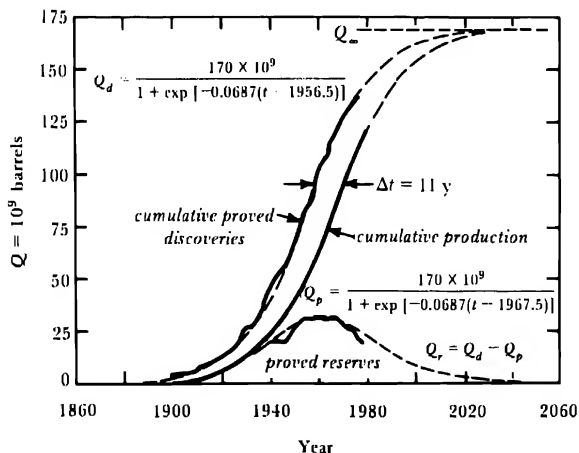
$$Q_d = Q_p(t + \Delta t)$$

Hence, if production follows a logistic curve, so too must discovery,  $Q_d$ , and the discovery rate,  $dQ_d/dt$ , precedes the production rate,  $dQ_p/dt$ , by the same interval,  $\Delta t$  (Figure 2.12). To the extent that a formulation based upon logistic curves is valid, the discovery rate serves as a preview of the forthcoming production rate.

Proved reserves, at a given time, are that quantity of oil which has been discovered but has not yet been produced. It is the reserve base utilized for formulating production decisions. Mathematically, the proved reserve,  $Q_r$ , is the difference between the cumulative discovery and production quantities.

$$Q_r = Q_d - Q_p$$

The quantity of proved reserves,  $Q_r$ , is thus equal to the vertical separation of the discovery and production curves. (Their horizontal separation is equal to  $\Delta t$ .) For logistic curves, the peak in the curve for the proved reserve may be shown to occur

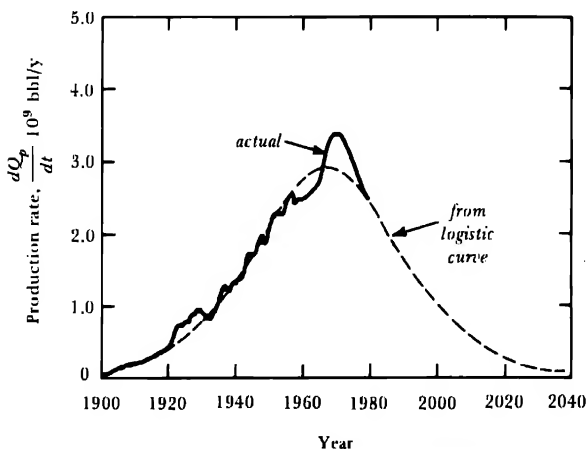


**FIGURE 2.13** *M. King Hubbert's Matching of Logistic Curves to Domestic Cumulative Crude Oil Discoveries and Production. Solid Lines Are Actual Quantities (Lower 48 States) (References 2 and 6).*

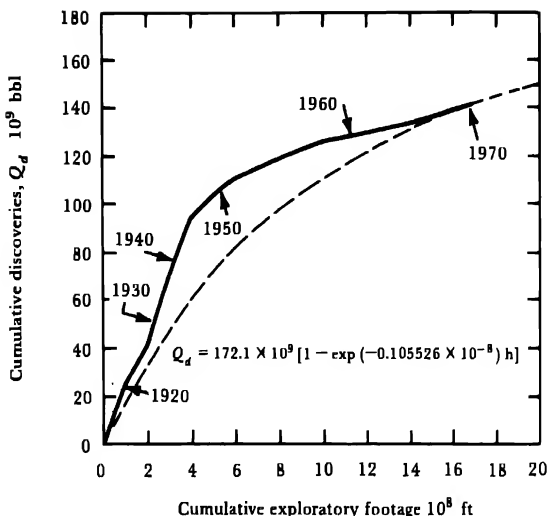
midway between the peaks of the curves of discovery and production rates (at  $t = -\Delta t/2$ ). It is also symmetrical about its peak.

M. King Hubbert has been an authoritative observer and commentator on the extraction of oil and other natural resources for many years. He has been instrumental in showing that oil discoveries and production in the United States have followed logistic curves. An early extrapolation of crude oil production, based upon data up to 1962, has come exceedingly close to what actually occurred in the following years. A more recent set of logistic curves fitted to data up to 1972 is indicated in Figure 2.13 (data through 1979 has also been added to these curves). The time difference for the logistic curves,  $\Delta t$ , was determined to be 11 years, the initial growth rate (exponential coefficient) was 6.87%, and the value of 170 billion barrels for  $Q_\infty$  provided the best fit to the actual quantities. The data applies only to the continental United States, excluding the Alaska North Slope discovery which added 10 billion barrels to proved reserves in 1969. As of 1972, proved reserves were declining; a recognizable peak had occurred.

The value of  $Q_\infty$  obtained by the fitting of logistic curves, 170 billion barrels, corresponds to a peak in the production rate near the end of the 1960s. In Figure 2.14 the actual and predicted production rates are indicated. Actual production peaked in 1970 and by 1979 (lower 48 states) it fell to the same value it had in 1961. The extrapolated production rate curve, based upon data prior to 1972, has done remarkably well in predicting the production decline of 1972 to 1979. Based upon the continued good fit of the discovery and reserve curves over this latter period, the theoretically derived curve for the production rate might be expected



**FIGURE 2.14** Crude Oil Production Rate (Lower 48 States) (References 2 and 6).

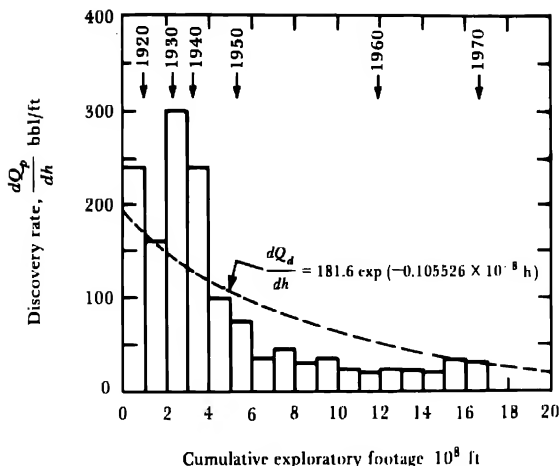


**FIGURE 2.15** Cumulative Crude Oil Discoveries as a Function of Exploratory Drilling. Compiled by M. King Hubbert (Lower 48 States) (Reference 6).

to be a reasonable projection for at least the next decade. A further decline in production is predicted.

Through fitting logistic curves to the discovery and production data for crude oil, an estimate of ultimate reserves is obtained. Another estimate, independent of that above, may be obtained from exploratory drilling data. Exploratory wells, if successful (over 70% tend to be dry holes), result in new reserves of oil and/or natural gas. From drilling records and concurrent reserve data, Hubbert has constructed a cumulative record of oil discoveries as a function of cumulative drilling activity (Figure 2.15). Since initial estimates of new reserves are usually low, that is, the amount of oil ultimately obtained from newly discovered oil fields tends to exceed that initially reported as reserves, the discovery curve has been appropriately corrected (the "α correction"). The oil ultimately produced, or in the case of recent discoveries, that expected to be recovered, is referenced to the drilling when the reserve was first discovered. A plot of the rate of discovery per foot of drilling is indicated in Figure 2.16 in which  $10^8$  ft increments were used for calculating rates. Except for being based upon finite increments, this curve is the derivative of the curve of Figure 2.15.

Early exploratory drilling done before about 1945 (less than  $4 \times 10^8$  ft of cumulative drilling) was very productive. More recently, the discovery rate, per foot of drilling, has been only 10 to 15% of that achieved earlier. This decline has persisted despite the introduction of new, extremely sophisticated, exploratory



**FIGURE 2.16 Average Discovery Rate of Crude Oil per Foot of Exploratory Drilling. Compiled by M. King Hubbert (Lower 48 States) (Reference 6).**

techniques. To estimate  $Q_x$  from this data, it is necessary to extrapolate the curves of Figures 2.15 and 2.16 to account for future discoveries. For a particular geographic area, an upper limit, based upon its geological character, may be set for the amount of drilling that could be productive. For the continental United States, including the offshore continental shelf, a limit of 5 billion ft has been generally accepted. The discovery rate (barrels per foot) for the exploratory wells that are yet to be drilled is unknown. If the recent discovery rate of about 30 bbl/ft were to be maintained for all drilling yet to be done ( $1.7$  to  $5.0 \times 10^9$  ft interval), it would yield 100 billion barrels of new oil, resulting in 240 billion barrels as the ultimate quantity discovered,  $Q_x$ . This estimate, however, assumes no further decline in the discovery rate ( $dQ_d/dh$ ). To obtain a more realistic projection, Hubbert approximated the discovery rate curve of Figure 2.16 by an exponentially decaying function.

$$\frac{dQ_d}{dh} = 181.6 \exp(-0.105526 \times 10^{-8} h)$$

This expression was chosen to give the actual discovery rate for  $h = 1.7 \times 10^9$  ft (30.2 bbl/ft) and to simultaneously account for the cumulative discoveries at this cumulative drilling footage. The cumulative discoveries are obtained by integrating the discovery rate.

$$Q_d = 172.1 \times 10^9 [1 - \exp(-0.105526 \times 10^{-8} h)]$$

A value of 172.1 billion barrels is obtained for  $h = 5 \times 10^9$  ft (essentially the same

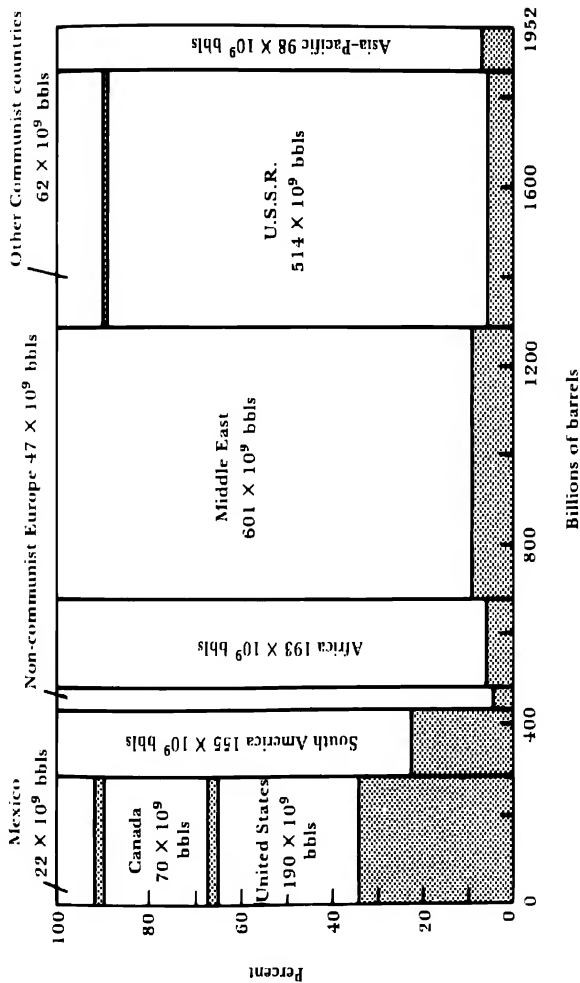
as for  $h = \infty$ ). In the interval of 1.7 to  $5 \times 10^9$  ft, new discoveries of only  $28.6 \times 10^9$  bbl are predicted. Values of initial reserves, obtained by the use of logistic curves and obtained through drilling data based upon the above exponential drilling approximation, are remarkably close. It will be noted, however, that a plot of predicted discoveries, based upon the exponential fit of the drilling discovery rate, provides a poor fit for the data of Figure 2.15. A better approximation for this curve yields an even lower value for ultimate reserves (Problem 9).

At the conclusion of 1979, total cumulative crude oil production (lower 48 states) was approximately 115 billion barrels. If  $Q_\infty$  is indeed 170 billion barrels, only 55 billion barrels remain to be produced. Since proved reserves (lower 48) were about 20 billion barrels (1979), only 35 billion barrels remain to be discovered. Even if estimates of the crude oil that may be obtained from Alaska are included, the resultant estimate of yet-to-be discovered domestic crude oil is considerably smaller than that generally accepted before the mid-1970s [30-33]. The authors of *Project Independence* [34], as well as those of the Ford Foundation's energy study, *A Time to Choose* [35], used much larger estimates of reserves, an initial reserve of 590 billion barrels. Prior to 1961, the average drilling discovery rate was 118 bbl/ft. The 590 billion barrel estimate is derived from the rather unrealistic projection of continuing this discovery rate for all future exploratory drilling (the discovery rate since the 1960s, however, has averaged 30 bbl/ft). Having accepted this very large estimate, the authors of these studies could project not only a substantial increase in the production rate but also that the increased rate could be maintained for several decades.

While in principle a fitting of logistic curves could be used to estimate world crude oil reserves, considerably less data is available. The discovery rate, the precursor of the production rate, has not reached a peak. Furthermore, the exploratory drilling record is insufficient for the type of extrapolation that was used to estimate discovery in the United States. However, Hubbert, following Jodry, has estimated initial reserves of crude oil in the world, based primarily upon geological considerations (Figure 2.17). As of 1978, only about one-fifth of the 1952 billion barrel estimated reserve had been produced. The midpoint of the cumulative production curve, 926 billion barrels, which would correspond to a peak in the production rate (assuming a logistic curve), has not been reached.

#### 4. PETROLEUM

Since petroleum products are derived not only from crude oil but also from liquids obtained from natural gas wells, projections of domestic petroleum production are larger than those for crude oil. Estimates of ultimately recoverable quantities of petroleum, including the Alaska reserves, are given in Table 2.2. The Alaskan reserves augment those of the lower 48 states by nearly 25%. But if overall U.S. petroleum production follows a logistic curve, the Alaskan discovery delays the peak production by only three years. The peak petroleum production rate, assuming the same exponential coefficient as that which fit crude oil production for the lower 48



**FIGURE 2.17** Estimates of Initial Reserves of Crude Oil. Shaded Portions Represents Quantities Already Consumed. M. King Hubbert (Based on Jodry) (Reference 6).



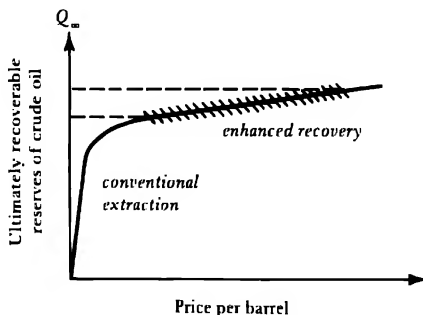
**TABLE 2.2 Ultimately Recoverable U.S. Reserves of Petroleum**

	<i>Billion barrels</i>		
	<i>Conterminous U.S.</i>	<i>Alaska</i>	<i>Total</i>
Crude oil	170	43	213
Natural-gas liquids	34	5	39
Total	204	48	252

Reference 6.

(6.87%/y) is increased by 25% by the Alaskan reserves. A peak of  $4.3 \times 10^9$  bbl/y would be expected for the total reserves of Table 2.2. The peak in production of  $4.1 \times 10^9$  bbl/y which occurred in 1970 was very close to that predicted by a logistic curve.\*

Ultimate recoverable reserves of crude oil, and hence petroleum, will be increased with the development of enhanced recovery techniques. Crude oil tends to be found both within porous rocks and between the fractures of rocks. Because oil is very viscous, only about one-third of the initial in-place oil resource can be extracted by conventional techniques. Several hundred billion barrels of oil thus remain in U.S. wells even after "exhaustion" occurs. If the remaining oil is heated with steam or by an underground combustion process, its viscosity will be decreased, thereby increasing its ability to flow. Also, carbon dioxide can be injected into one well to force the oil to flow toward another well. Another enhanced recovery technique uses detergentlike chemicals to displace oil held in tight formations. The Congressional Office of Technology Assessment estimates that for crude oil prices of

**FIGURE 2.18 Ultimately Recoverable Crude Oil,  $Q_\infty$ , as a Function of Price.**

\* The peak occurred shortly after the Prudhoe Bay discovery but before the construction of the Alaska oil pipeline.

\$30/bbl (1977 prices) or more, 25 to 50 billion additional barrels of oil may eventually be economically recovered by these techniques [36]. Ultimately, recoverable reserves of crude oil,  $Q_{\infty}$ , depend to an extent upon the effort expended to recover the oil, that is, upon the highest price that the oil may fetch (Figure 2.18). Although for the above estimate the marginal increase in reserves with price is small, some have argued that with higher prices an increased production rate, surpassing that of the 1970 peak, could occur around 1990 [37]. An upper limit to price may be dictated by petroleum consumers, since for many needs it is cheaper to introduce energy saving investments (insulation, for example) than to buy high-priced fuel that would otherwise be needed.

The U.S. reserves of petroleum that remain are not only limited but also considerable more difficult to extract than those utilized in the past. In 1968 Atlantic Oil and British Petroleum discovered reserves of approximately 10 billion barrels of oil at Prudhoe Bay in Alaska. In order to tap this oil field, a pipeline stretching 800 miles from Prudhoe Bay on the Beaufort Sea in the north to the southern port of Valdez as well as tanker loading facilities at Valdez had to be constructed. The construction of a 48-in. diameter pipeline carrying hot oil ( $60^{\circ}\text{C}$ ) across permafrost (permanently frozen ground) through remote arctic regions with winter temperatures of  $-50^{\circ}\text{C}$  and lower was unprecedented [38-40]. Half the pipeline had to be placed on elevated piles, many equipped with ammonia filled thermopiles to minimize the melting of the permafrost. Before proceeding with construction, numerous political, social, and environmental problems had to be resolved for the pipeline that was to be located primarily on public lands (500 miles on federally owned land and 250 miles on state-owned). It was also necessary to satisfy the provisions of the newly enacted National Environmental Protection Act.

When the pipeline was first proposed in 1969, its estimated cost was \$900 million. As a result of extensive engineering studies, it was completely redesigned before construction began in 1973. When completed in 1977 by Alyeska, the pipeline construction corporation owned by a consortium of oil companies, the pipeline's cost had reached \$7.7 billion. In addition to the investment in the pipeline, \$6 billion was expended on developing the Prudhoe oil field. The total investment, including interest, for producing and for delivering the oil at Valdez had reached \$15 billion. This investment, a record industrial undertaking, preceded the delivery of the first barrel of oil at Valdez in July 1977. While the pipeline was designed so that by adding more pumping stations its capacity could be increased to  $2 \times 10^6$  bbl/day, its initial capacity was  $1.2 \times 10^6$  bbl/day.\* Only a small increase in capacity is envisioned during the first few years of the 1980s. As of 1980, not only had the oil field not been sufficiently developed for supplying more oil, but the capacity of West Coast refineries was inadequate [41]. Some Alaska oil is being shipped to the East Coast which requires the additional expense of loading the oil from the supertankers that bring it from Valdez to smaller tankers that can use the Panama Canal.

\* For the \$15 billion investment, a 15% annual return results in approximately \$5 for each barrel of oil delivered at Valdez based upon the initial pumping capacity.



chains: each carbon atom is attached to its neighbor by a single bond. All other carbon bonds are with hydrogen atoms. A few of the normal and branched paraffin molecules are shown in Figure 2.21.\* The normal paraffin (*n*-paraffin) molecules have no branches, that is, they can be represented symbolically by a straight chain of carbon atoms. Each end carbon atom is connected by its electron bonds to three hydrogen atoms, while all other carbon atoms are connected by their two remaining bonds to two carbon atoms. Hence the chemical formula for molecules of the paraffin family is  $C_nH_{2n+2}$  in which *n*, an integer, is the number of carbon atoms. As indicated in Figure 2.21 for the smaller molecules, paraffin molecules may be branched. Branched molecules, known as *isomers*, have the same formulas as their nonbranched relatives, but owing to their branching, have different chemical properties. Isobutane and butane, for example, both have the chemical formula of  $C_4H_{10}$ . The number of branched combinations increases rapidly as the number of carbon atoms is increased. For  $C_7H_{16}$  (not indicated in Figure 2.21), there are ten branched combinations, whereas for  $C_8H_{18}$  there are 22. The density of a gas at a given temperature and pressure is proportional to the molecular weight of the molecules (Chapter 4). Boiling point temperatures (those for atmospheric pressure are given in Figure 2.21) tend to be related to vapor densities and hence the molecular weight of the molecules. A mixture of paraffin molecules can be separated into groups of molecules with comparable molecular weights by selectively distilling the mixture.

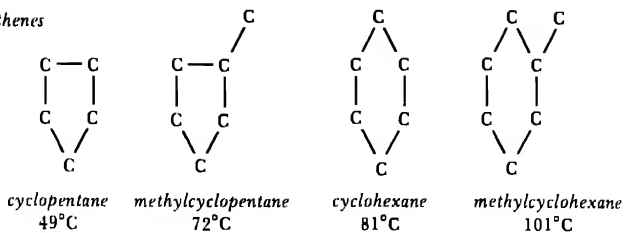
Paraffins, however, are not the only family of hydrocarbons found in crude oil. A few of the smaller molecules of other hydrocarbon families are given in Figure 2.22. The naphthenes consist of carbon atoms joined in a ring of usually five or six atoms. Owing to this cyclic arrangement, elementary naphthene molecules have two fewer hydrogen atoms than a paraffin molecule with the same number of carbon atoms. Their general formula is  $C_nH_{2n}$ . Olefins form a family of molecules which have a double bond between a pair of carbon atoms. Cyclic olefins are also found in crude oil. Owing to their double bond, olefins tend to be chemically more reactive than paraffins and naphthenes. Diolefins have two double bonds and the acetylenic hydrocarbons have a triple bond between a pair of carbon atoms.

The last important family of hydrocarbons consists of the aromatics. These have at least one ring of six carbon atoms with three double bonds. Even though the aromatics are chemically less active than those with double bonds and triple bonds, they are extensively used as "building blocks" by the chemical industry. As with the paraffins, families of larger molecules are formed from the basic molecules indicated in Figure 2.22.

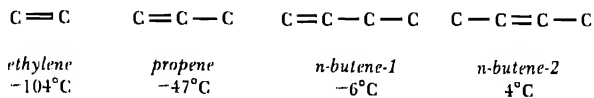
The classification of a crude oil based upon its chemical composition would be unwieldy: the concentration of hundreds of unique compounds would need to be ascertained. Crude oil analyses are usually in terms of the yields obtained for a succession of progressively higher distillation temperatures. Separating the constituents by boiling points tends to separate the compounds according to their molecular weights, but not according to the different hydrocarbon families. A

\*To simplify diagrams of hydrocarbon molecules, only bonds between carbon atoms are customarily indicated.

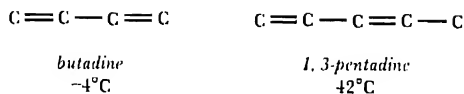
### Napthenes



### Olefins



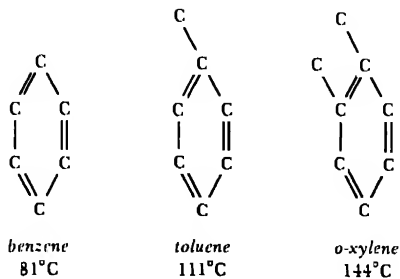
### Diolefins



### Acetylenic hydrocarbons



### Aromatics



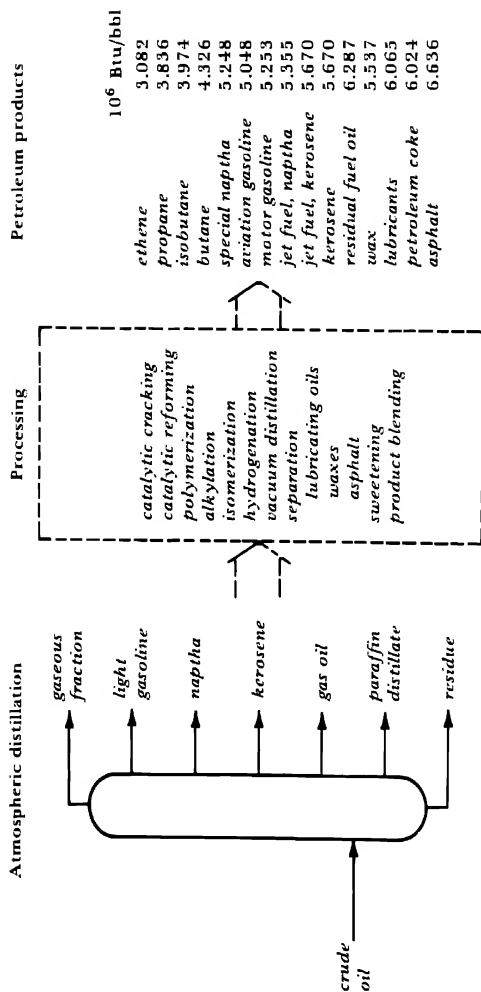
**FIGURE 2.22 The Simpler Molecules of Various Hydrocarbon Families (Boiling Points for Atmospheric Pressure).**

distillation process is also the first step of the refining process (Figure 2.23). By means of a distillation tower which has a vertical temperature gradient, various fractions of the crude oil are condensed and extracted. The lightest fractions, gases at atmospheric pressure, are removed from the top of the tower (the coolest region), while the heaviest fractions, the large hydrocarbon molecules, are removed as a residue from the bottom of the tower. Distillation merely sorts the hydrocarbon fractions according to their boiling points. Each fraction contains hydrocarbon compounds of comparable molecular weights, but since they belong to several families they have varied chemical properties. The yields of the various fractions are determined by the composition of the crude oil.

In a modern refinery, numerous sophisticated refining processes are employed to transform the distillation products into desired petroleum products. These processes not only allow for a choice in the products to be produced but they also permit, within rather wide limits, variations in their yields. Distillation, at or near atmospheric pressure, is the first step of the refining process. Heavy products, the residues, have high boiling temperatures, too high to be conveniently separated at atmospheric pressure. Since their boiling temperatures are lower at reduced pressures, they can be separated by a vacuum distillation process. The yield of heavy products, large hydrocarbon molecules, is generally much greater than desired. At elevated temperatures and pressures, large hydrocarbon molecules will "crack" into smaller molecules, a process known as *cracking*. In the presence of suitable catalysts, cracking of heavy hydrocarbons can be accomplished at moderate temperatures (around 500°C) and at atmospheric pressure. Modern refineries utilize a cracking process in which fluidized solid catalysts are mixed with the heavy hydrocarbon feed stocks. Mixtures of alumina and silica compounds are commonly used as a catalyst. While the catalysts need only be present for cracking to occur (catalysts do not directly take part in the reaction), they become coated with hydrocarbons and must be continually regenerated by burning off the hydrocarbon coating at elevated temperatures. The products formed by the cracking process can then be separated by distillation.

Listed in Figure 2.23 are other processes used to obtain various refined petroleum products. Catalytic reforming is used to convert hydrocarbons of one family to those of another family. In producing gasoline, catalytic reforming is used to obtain aromatics from other hydrocarbons since aromatics tend to increase the octane number of the fuel. In a refinery, polymerization is used to combine gaseous olefinic molecules with each other, thus producing more valuable liquid hydrocarbons. Closely related to polymerization is *alkylation*, the adding of an alkyl group to a compound. Refineries use alkylation to make olefins react with iso-paraffins in order to obtain larger iso-paraffins. Polymerization and alkylation of light hydrocarbons are both used to increase gasoline yields. Another process, isomerization, is used to convert nonbranched paraffins to branched paraffins since the latter not only result in higher octane gasolines but can also be readily transformed into other refinery products.

To produce valuable heavy hydrocarbons such as lubricating oils and waxes, it is necessary to separate the hydrocarbons from the asphalt of the residue (obtained

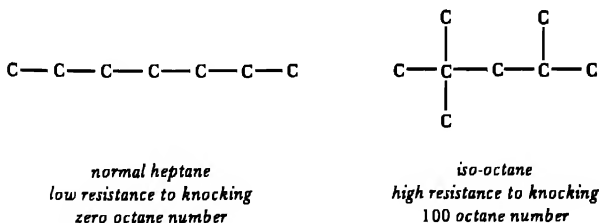


**FIGURE 2.23** *Petroleum Refining and Products (Reference 2).*

from the bottom of the distillation tower).<sup>\*</sup> Crude oil, depending upon its source, may contain sulfur compounds. Crude oils high in sulfur content are referred to as *sour* owing to their odor. The process of removing sulfur compounds is called *sweetening*.

The products of the various refining processes are used either as feeds for other processes or are blended together to produce fuels with specified characteristics. Only a few refinery products, such as propane and butane, are basically pure chemical compounds. Gasolines, for example, consist of a blend of hydrocarbon liquids which are blended to achieve a specific performance in an internal combustion engine. They consist of hydrocarbons with individual boiling points in the 10 to 230°C range. A relatively high vapor pressure is necessary for proper carburetion. Too high a vapor pressure, however, results in excessive evaporative losses. Also important for gasolines is a resistance to preignition. Preignition of the fuel-air mixture in an internal combustion engine results in "knocking." Pure iso-octane is an ideal fuel since it has a very high resistance to knocking, whereas normal heptane is a very poor fuel (Figure 2.24). An octane number, defined in terms of a mixture of these hydrocarbons, is used to specify the resistance of a gasoline to knocking. For these hydrocarbons, the octane number is the percentage of iso-octane. An octane number of 100 corresponds to 100% iso-octane. The octane number of any gasoline blend is defined in terms of the iso-octane/normal heptane mixture that has the same resistance to knocking.<sup>†</sup> Since the likelihood of preignition increases as the compression ratio of an engine increases, fuels with higher octane ratings are necessary for higher compression engines.

The octane number of gasoline depends upon its blend of hydrocarbons. A high concentration of aromatics and highly branched paraffins tends to result in a



**FIGURE 2.24** *Hydrocarbons Used for Specifying the Octane Number of Gasolines.*

<sup>\*</sup> While lubricating oils, waxes, and asphalt are nonfuel petroleum products, they are, by convention, included in energy consumption statistics. Refineries also produce hydrocarbons used by the chemical industry which are similarly included in energy statistics.

<sup>†</sup> The octane number is determined by a single-cylinder variable-compression ratio test engine. The number usually specified is the average of that obtained by two methods: the research octane method (ROM) for mild operating conditions and the motor octane method (MOM) in which severe conditions are simulated.



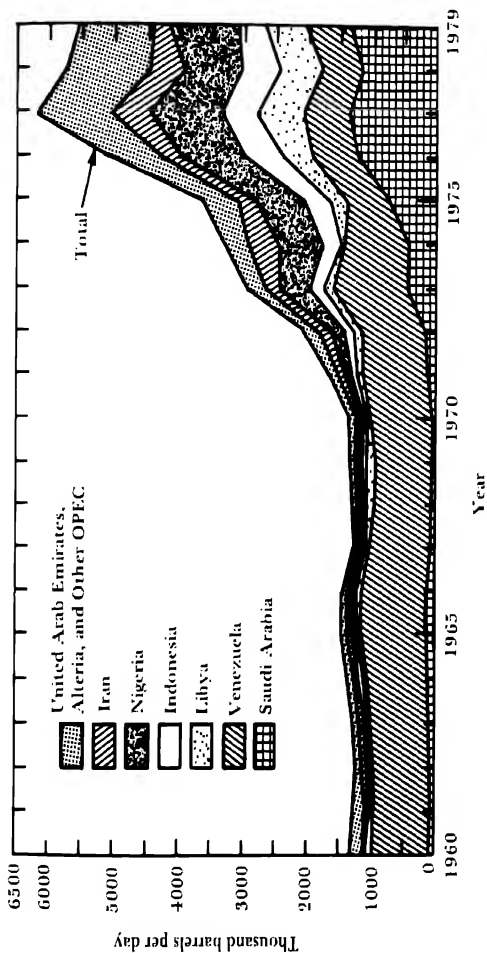
high octane number. Additives of tetraethyl lead or tetramethyl lead (less than .1%) are also used to increase octane ratings. To prevent cylinder deposits of lead oxides from forming during combustion, scavengers such as ethylene dibromide and ethylene dichloride must also be added to the fuel. However, in vehicles with catalytic converters unleaded fuels must be used since lead additives poison catalytic converters.

The energy contents of several refined petroleum products are indicated in Figure 2.23. Variations between different products are due not only to their different constituents but also to variations in their densities. The barrel, as well as the gallon, is a volume measure. These variations can be important. For example, federally mandated motor vehicle fuel efficiency standards are based upon a miles per gallon specification. Based on a volume measure, diesel fuel (a distillate oil) has an 11% higher energy content than gasoline. Therefore, for comparable engine efficiencies, an 11% better volumetric fuel efficiency rating is achieved by using diesel oil. However, per unit of mass, the energy content of diesel fuel and gasoline is nearly equal.

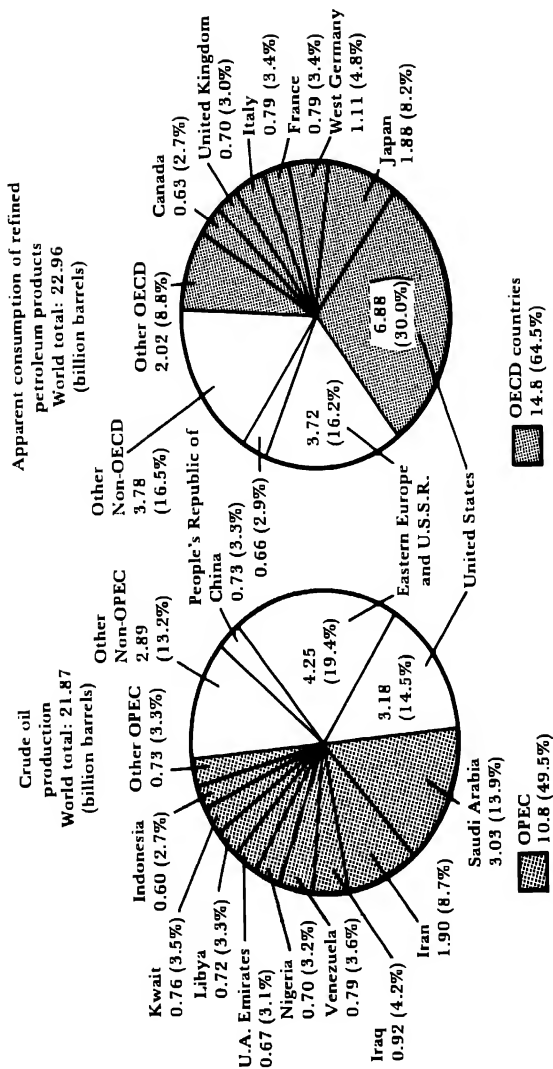
It has already been shown that petroleum consumption by electric utilities and the industrial sector could be reduced by converting some of these facilities to use coal. The transportation sector, however, accounts for over half of the nation's petroleum consumption. Furthermore, gasoline, used primarily by passenger cars, accounts for 73% of the petroleum used by this sector. Not only has the number of registered vehicles been steadily increasing (doubling in less than two decades), but compared with vehicles used in other developed nations, they are considerably less fuel efficient. A doubling in average fuel efficiency, through the use of lighter weight and lower power vehicles, could significantly reduce petroleum consumption even if no change in driving patterns were to occur. Since 1978 gasoline consumption has been decreasing; consumption in 1982 was 12% less than it was in 1978. A trend toward reduced usage, attributable to higher prices, has begun. While a transition to more fuel-efficient vehicles as mandated by the Energy Policy and Conservation Act of 1975 involves little change in overall transportation patterns, modal changes (increased intracity bus use and intercity rail travel, for example), could further reduce petroleum consumption.

Over the 1970 to 1980 decade, the United States became increasingly dependent upon imports of crude oil and refined petroleum products. Most of the imports are from nations forming the OPEC (Organization of Petroleum Exporting Countries) cartel (Figure 2.25). The dependence upon OPEC is even greater than indicated in Figure 2.25 since most of the imported refined petroleum products are from Caribbean nations which secure their oil from OPEC.

The growing petroleum imports prior to 1979 might be viewed as another energy transition. Imports in 1979 accounted for 45% of petroleum consumption, whereas prior to 1949 the United States was a net exporter of petroleum. As for the previously discussed energy transitions, that from wood to coal and from coal to hydrocarbons, the new energy source, imported petroleum, did not replace domestically produced petroleum products but augmented them. Even though domestic production nearly doubled from 1950 to 1970, imports accounted for 23% of total consumption in 1970. While domestic production declined by 10% from 1970 to 1979, overall consumption



**FIGURE 2.25 Petroleum Imported Directly from OPEC (Reference 2).**



**FIGURE 2.26 International Petroleum Supply and Disposition, 1978 (Reference 2).**

increased by 26%, necessitating a 142% increase in imports. The Arab oil embargo of 1973–1974 briefly reduced consumption and the large price increases levied during this period resulted in a growing balance of payments deficits. Following 1978, however, overall consumption of petroleum has been significantly reduced—consumption in 1982 was 19% less than in 1978. As a result, imports in 1982 were 43% lower than in 1977.

Figure 2.26 provides an international perspective on petroleum production and consumption. The United States, it will be noted, was the second largest petroleum producer in 1978—its production exceeded that of Saudi Arabia. It is primarily the industrialized nations (the 24 nations forming the OECD (Organization for Economic Cooperation and Development) community, the Eastern European block of nations, and the USSR) that account for over 80% of the petroleum consumption. The developing nations subsist on very small supplies of oil.

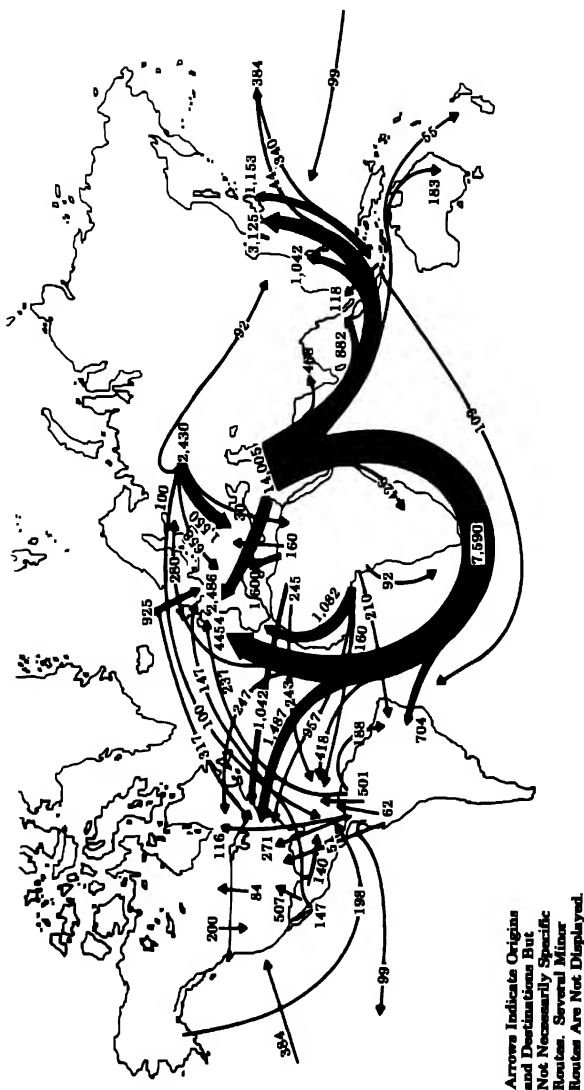
During the 1970s substantial quantities of oil were discovered in Mexico. While its 1978 production was relatively small,  $1.2 \times 10^6$  bbl/day, Mexico's proved reserves were 31.3 billion barrels at the end of 1979, slightly larger than those of the United States. Though estimates of recoverable crude oil vary, some believe that Mexico's reserve may eventually be found to be comparable to that of Saudi Arabia (166.5 billion barrels) [45]. The anticipated production increases could favorably affect the availability of crude oil for the United States. Even though imports from Mexico may be preferable to those from the Middle East, the balance of payments problem for the United States would remain unchanged.

The industrialized areas tend to be distant from regions with reserves sufficient for sustaining high levels of production (Figure 2.27). The international trade of oil requires globe-spanning ocean tankers. Not only is the number of tankers required very large but their size can also be immense, up to .5 million tons capacity (3.7 million barrels of crude oil).<sup>\*</sup> Over half of the world's crude oil production is shipped by ocean-going tankers; not surprisingly, this has had a significant environmental impact on the oceans. While the spill of approximately 100,000 tons of crude oil from the *Torrey Canyon* off the coast of Cornwall (England) in 1967 was perhaps the first to attract world attention, numerous equally damaging spills have since occurred. Furthermore, routine tanker operations, primarily ballasting and washing of tanks, have resulted in considerably larger releases of oil. The authors of a National Academy of Sciences Report estimate that the transportation of oil adds over 2 million metric tons of oil yearly to the oceans [47]. This oil, plus that from other sources, primarily petroleum laden industrial wastes which eventually enter the oceans, could be causing irreversible harm to the oceans' ecological systems.

## 5. NATURAL GAS

Natural gas obtained from reservoirs in the proximity of those of crude oil is referred to as *associated gas*; natural gas not found close to crude oil reservoirs is called

<sup>\*</sup> Noel Mostert provides an interesting perspective in *Super Ship* [46].



**FIGURE 2.27 International Crude Oil Trade, 1940 (Units of 10 bbl/d) (Courtesy of the Energy Information Administration 1982 Annual Energy Review April 1983).**

*nonassociated gas*. Even though methane is the main hydrocarbon constituent of natural gas, small quantities of other members of the paraffin family are generally present [48]. In addition, it contains varying quantities of noncombustible gases (nitrogen, hydrogen sulfide, and carbon dioxide) and traces of inert gases (argon and helium). The lighter paraffins are gaseous at atmospheric pressure while the heavier ones are liquid (Table 2.3). Dry natural gas (usually obtained from nonassociated fields) consists primarily of methane, a small quantity of ethane, and traces of propane. Natural gas obtained from associated fields is generally "wet," that is, it contains heavier hydrocarbons that are liquid at atmospheric pressure. Liquids are usually removed from the gas at the wellhead. The lighter of these liquids, a mixture of propane and butane, is known as *liquified petroleum gas* (LPG) while the heavier constituents are referred to as *natural gasoline*. The butane and propane obtained from natural gas wells are supplemented by that obtained from petroleum refineries and the combination is marketed as liquified petroleum gas. Natural gasoline is blended with refinery produced gasolines.

**TABLE 2.3 Principal Hydrocarbon Constituents of Natural Gas**

		Boiling point atmospheric pressure	
Methane	CH <sub>4</sub>	-162°C	} gaseous
Ethane	C <sub>2</sub> H <sub>6</sub>	-89°C	
Propane	C <sub>3</sub> H <sub>8</sub>	-42°C	
Iso-butane	C <sub>4</sub> H <sub>10</sub>	-12°C	
Butane	C <sub>4</sub> H <sub>10</sub>	-5°C	
Iso-pentane	C <sub>5</sub> H <sub>12</sub>	-28°C	} liquid
Pentane	C <sub>5</sub> H <sub>12</sub>	36°C	
Hexane	C <sub>6</sub> H <sub>14</sub>	69°C	
Heptane	C <sub>7</sub> H <sub>16</sub>	98°C	

The natural gas obtained when producing crude oil was at one time considered a nuisance and while it could be used for repressurizing oil wells, enhancing their oil production, it was often flared. Without a pipeline system for its distribution, the natural gas had little value as a fuel. In oil producing regions such as those of the Middle East that are remote from population centers, natural gas is still flared. Natural gas, primarily methane, is an extremely clean burning fuel. With the construction of a pipeline distribution system in the United States, natural gas became the main fuel for space and water heating of the residential and commercial sectors. The transition from coal to natural gas (or oil) by these sectors was the main factor in reducing urban air pollution caused by stationary sources. The heating value of natural gas depends primarily upon the concentration of the inert gases and ranges from about 850 to 1130 Btu/ft<sup>3</sup> (an average value of 1050 Btu/ft<sup>3</sup> or 1.1 MJ/ft<sup>3</sup> or 39.1 MJ/m<sup>3</sup>). The therm, 10<sup>5</sup> Btu, is frequently used for specifying natural gas usage and is equal to the energy value of approximately 100 ft<sup>3</sup> of gas.

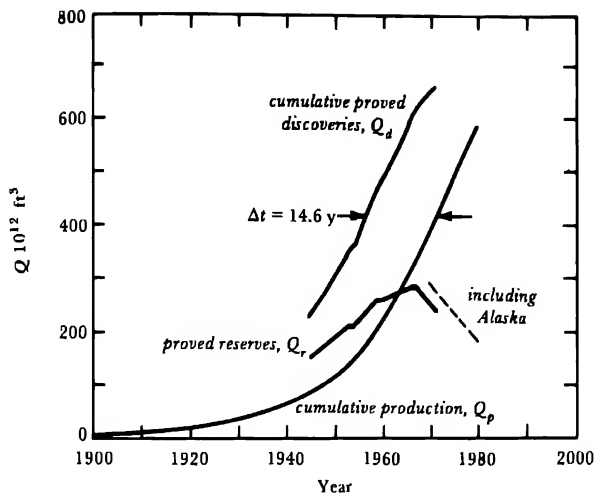
Prior to 1970, natural gas consumption in the United States increased very rapidly. The curves of Figure 2.28 (prepared by M. King Hubbert for data prior

(1972) are similar to those for crude oil and, hence, suggest a set of logistic curves to approximate the discovery and production of natural gas. It will be noted that a peak in proved reserves (even when those of Alaska are included) has occurred. Furthermore, the production rate peaked in 1971, cumulative production being 414 trillion  $\text{ft}^3$  at that time. Since the peak production rate for a logistic curve occurs when cumulative production is equal to half of  $Q_\infty$ , a value of 828 trillion  $\text{ft}^3$  is predicted for  $Q_\infty$  based upon this line of reasoning. M. King Hubbert has estimated 1050 trillion  $\text{ft}^3$ , a somewhat higher  $Q_\infty$  for the lower 48 states. The estimate was obtained from an examination of two sets of data. Projections of anticipated discoveries of natural gas, as a function of exploratory drilling footage (in a fashion similar to that discussed for crude oil) provided one basis for the estimate while the correlation between crude oil and natural gas discoveries provided the second. If an estimate of ultimately recoverable reserves of natural gas in Alaska are included in Hubbert's figures, an overall value of 1184 trillion  $\text{ft}^3$  is obtained for the entire United States.

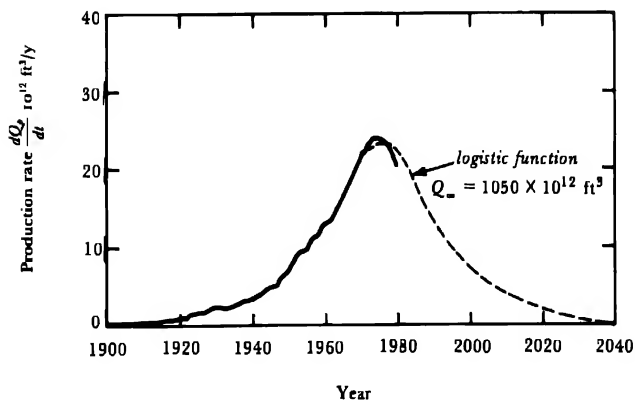
Figure 2.29 shows yearly production through 1979, as well as a theoretical production rate curve for a value of 1050 trillion  $\text{ft}^3$  for  $Q_\infty$  and an initial growth rate of 6.73%/y (from M. King Hubbert). The growth rate is the average rate for the period 1905 to 1957. Although this curve provides a projected production rate for only the lower 48 states, the contribution from Alaska which will augment future production will be limited by the capacity of the pipeline system utilized (an initial capacity of approximately 2 trillion  $\text{ft}^3/\text{y}$  is anticipated).

The regulation of natural gas prices provides a perspective on the difficulty of managing a depletable resource. As a result of a 1954 Supreme Court decision, the Federal Power Commission was henceforth required to regulate the price of natural gas, at the wellhead, for gas involved in interstate commerce. At the time of this ruling cumulative production had reached only 150 trillion  $\text{ft}^3$ . The production rate was nearly 11 trillion  $\text{ft}^3/\text{y}$  with 1.5 trillion  $\text{ft}^3/\text{y}$  being used for repressuring crude oil reservoirs and .72 trillion  $\text{ft}^3/\text{y}$  being flared. Marketed production, that is, consumption, was 8.74 trillion  $\text{ft}^3/\text{y}$ . Based upon a value of 1050 trillion  $\text{ft}^3$  for  $Q_\infty$ , 900 trillion  $\text{ft}^3$  remained to be produced, which based upon the 1954 production rate, was a static index of 82 years. However, generally accepted reserve estimates at that time were considerably larger, in excess of 2000 trillion  $\text{ft}^3$ , which corresponded to a static index on the order of 200 years. Since depletion of the reserve in the then foreseeable future did not appear likely, wellhead prices were set based purely upon extractive costs, 16¢/1000  $\text{ft}^3$  (approximately 16¢/MBtu). Not surprisingly, consumption increased significantly over the ensuing years, reaching a 1972 peak of 22.5 trillion  $\text{ft}^3/\text{y}$  while the wellhead prices were increased only slightly, to 20¢/1000  $\text{ft}^3$ . Even considering distribution costs, natural gas, on the basis of energy content, was the lowest priced of the fossil fuels. Considering its low pollutant emissions, it could be readily classified as an energy bargain.

Owing to increased production rates, cumulative production reached 438 trillion  $\text{ft}^3$  in 1972. Using a 2000 trillion  $\text{ft}^3$  for reserves, the remaining natural gas, based upon the 1972 production rate of 24 trillion  $\text{ft}^3/\text{y}$  (includes that used for repressuring wells), the static index was reduced to 65 years. Based upon the 1050



**FIGURE 2.28** Domestic Natural Gas Discoveries and Production. Compiled by M. King Hubbert (Reference 6).



**FIGURE 2.29** Domestic Natural Gas Production Rate and Predicted Future Production Rate. Compiled by M. King Hubbert (Reference 6).

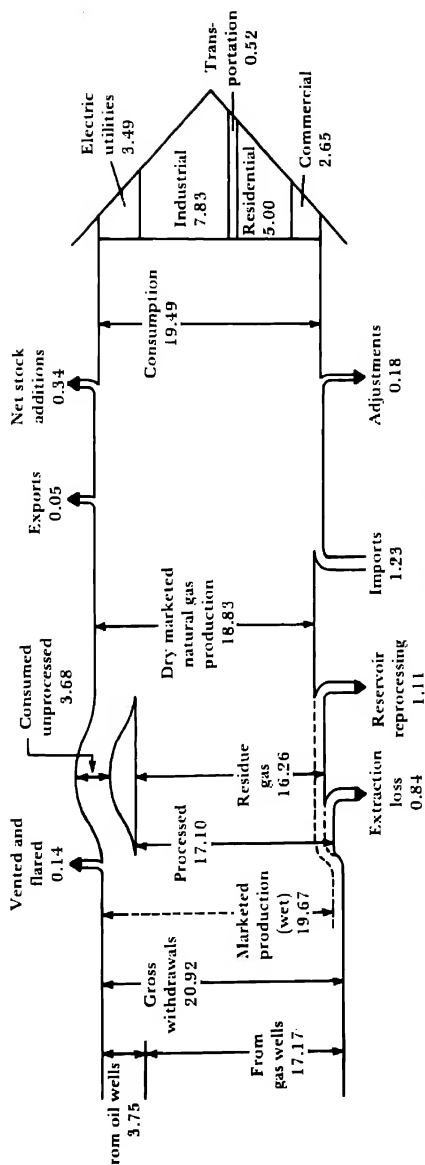


trillion  $\text{ft}^3$  for  $Q_\infty$ , a quantity that had come to be generally accepted by the end of the 1970s, the static index (as of 1972) was only 25 years. However, at the conclusion of 1979, cumulative production had reached 590 trillion  $\text{ft}^3$ , leaving only 460 trillion  $\text{ft}^3$ , or 44% of the initial lower 48 reserve. The static index for 1979 (20.9 trillion  $\text{ft}^3/\text{y}$ ) was only 22 years, a number that increases by only six years if the Alaska reserve is included.

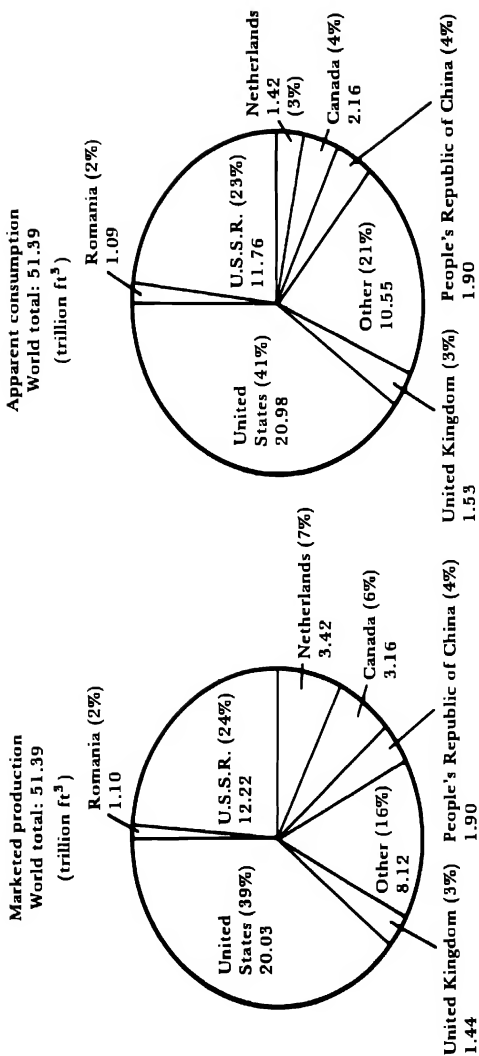
Both an overly optimistic estimate of ultimate reserves and a disregard of the compounding effect of growth led to the setting of prices that resulted in what may now be judged to have been a too rapid exhaustion of natural gas resources. Production and consumption of natural gas in 1979 are indicated in Figure 2.30. The consumption rate for 1982 was 22% below the 1972 peak, and if the logistic-derived production rate curve of Figure 2.29 should prevail, a further decline will occur. Furthermore, even if through additional drilling natural gas production should be increased, the increased production rate will be short-lived if the estimate of  $Q_\infty$  proves to be valid. Only if extensive new reserves, not accounted for in arriving at the 1050 trillion  $\text{ft}^3$  for  $Q_\infty$ , should be found, may an increased production rate be justified.

On an energy basis, natural gas accounts for over half of the direct fuel usage by the residential and commercial sectors. In these sectors it is used primarily for space and water heating. Prior to 1970 the average residential price was about  $\$1/10^6$  MBtu (Figure 1.13) and somewhat less for the larger commercial consumers. In light of the wellhead price of natural gas during this period, (16 to 20¢/MBtu), consumer prices reflected primarily distribution costs. Yearly heating costs prior to 1970 for a typical residence ( $10^8$  Btu/y) were only about \$100. While not negligible, heating costs were not important when compared to other expenses. Thus, homes constructed between 1950 and 1970 tended to have very little insulation if any. Based upon the then prevailing natural gas prices, no economic benefit would be derived from even the least costly investment to improve the thermal performance of buildings. A similar situation existed in the industrial sector with natural gas prices as low as 30¢/MBtu for large users. Fuel costs were a very small part of overall production expenses. The net result was an optimization of productive processes for extremely low-priced natural gas. Hence, this system is ill-suited for using natural gas at a wellhead price which has increased by a factor of ten and is anticipated to increase even more. Reoptimizing the nation's use of natural gas based upon present prices, or, more ideally, for prices anticipated over the life of the system, will require extensive investments for the retrofitting of buildings and for introducing more energy-efficient industrial processes. Such a reoptimization would, however, significantly reduce the demand for natural gas.

The United States is the largest producer and consumer of natural gas in the world, accounting for over 40% of global consumption (Figure 2.31). Approximately 5% of consumption, about 1 trillion  $\text{ft}^3$  for 1977, was imported by pipeline from Canada. A very small quantity, 11 billion  $\text{ft}^3$ , was imported as liquefied natural gas from Algeria. After the United States, the next largest producer and consumer was the Soviet Union. European nations form the third largest consuming group and accounted for 18% of world consumption. While the United Kingdom relies



**FIGURE 2.30 U.S. Natural Gas Production and Consumption, Trillion Cubic Feet per Year, 1979 (Reference 6).**



**FIGURE 2.31** *International Natural Gas Supply and Disposition, 1977 (Reference 2).*

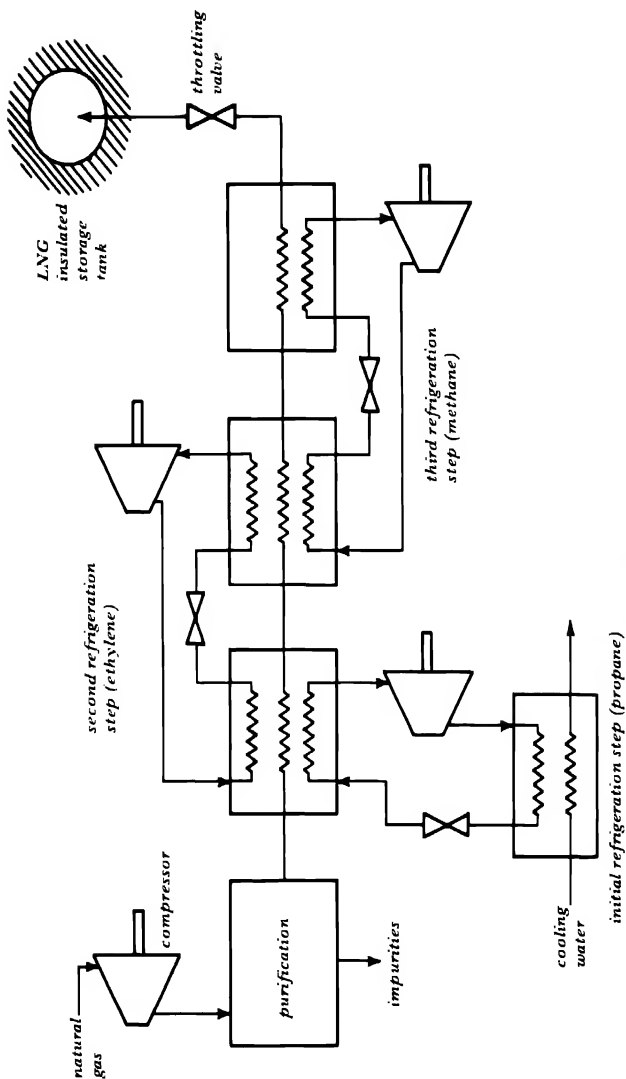
primarily on domestic production (it imports a small quantity from the North Sea), it is The Netherlands and the Soviet Union that are the main producers of natural gas consumed in Europe. Small quantities of liquefied natural gas are imported by France from Algeria and by Spain and Italy from Lybia.

A liquid fuel, liquefied natural gas (LNG), is obtained by cooling the natural gas at atmospheric pressure to a temperature of  $-162^{\circ}\text{C}$ . All hydrocarbon constituents, including methane which has the lowest boiling point, are then liquid. Liquefaction reduces the volume by a factor of approximately 600. Its density, as a liquid, is  $.43\text{ g/cm}^3$ , about half that of conventional liquid hydrocarbon fuels. On a volume basis, the energy value of liquefied natural gas is about 60% that of hydrocarbon fuels. Hence, useful quantities of liquefied natural gas can be stored in containers of reasonable size and shipped by ocean-going carriers [48-50].

Liquefaction may be accomplished with a succession of refrigeration cycles, each serving to reduce the temperature of the compressed natural gas (Figure 2.32). Impurities, especially those with relatively high boiling points, must first be removed since they tend to interfere with the operation of the heat exchangers. In addition to the cooling achieved by the conventional refrigeration cycles, added cooling is obtained by an irreversible throttling of the compressed natural gas: a cooling referred to as the *Joule-Kelvin effect*. This effect, also utilized by conventional refrigeration systems, occurs for certain conditions when a compressed gas is allowed to expand through a throttling valve. Special materials are needed to contain the liquefied gas, since, at its very low temperature, mild steels become brittle and disintegrate. Alloyed steels or aluminum alloys are used for parts that are in direct contact with the liquefied gas. In addition, well-insulated storage tanks with large volume to surface ratios are required to minimize the evaporation that occurs as a result of heat transfers. For stationary storage tanks, the gas that evaporates can be reliquefied; for ocean carriers, the gas is used for fueling the vessel.

Liquefied natural gas was first utilized by utilities for "peak shaving." When the demand for natural gas is less than that of the supply system, the excess gas is liquefied. When demand exceeds supply, such as on a cold winter day, the stored gas is rapidly re vaporized to provide for the peak demand. Revaporization is achieved with a heat exchanger in which water, if sufficient is available, is used for the vaporization (boiling) of the liquefied gas. (Water from a river used for this purpose would be cooled.) The first commercial peak shaving facility, located in Cleveland, began operation in 1941 and operated successfully until October 20, 1944, when a recently installed storage tank ruptured. The spilled liquefied gas, upon ignition, resulted in a fire storm which killed 130, injured 300, and left 1400 homeless.

Liquefied natural gas flows as an ordinary liquid with a low viscosity. However, being in contact with objects and the atmosphere (at the ambient temperature) the liquid is warmed causing it to evaporate. An inflammable mixture of natural gas and air (5 to 15% methane) is formed above the surface of the liquid. If ignited (it requires a very low ignition energy), the flame front readily traverses the inflammable layer thus engulfing the entire spill. Since combustion warms the remaining liquid, accelerating evaporation, it is almost impossible to extinguish the resultant fire. The best that can be done is to contain the spill and allow it to burn



**FIGURE 2.32** A Simplified Natural Gas Liquefaction System. The Refrigerants, Propane, Ethylene, and Methane, Have Successively Lower Boiling Points.

itself out. An inferior tank liner and the lack of an adequate dike surrounding the Cleveland facility were blamed for the disaster. Many are concerned that a similar or even worse accident might again occur [51-53].

It was not until the 1960s that new peak shaving facilities were constructed (the Cleveland facility was permanently closed after the 1944 disaster). By 1980, however, 59 such facilities were in operation throughout the United States [54]. Their combined liquefaction capacity was  $3.3 \times 10^8$  ft<sup>3</sup>/day and their storage capacity was in excess of  $6.5 \times 10^{10}$  ft<sup>3</sup>.<sup>\*</sup> Each peak shaving unit has its own revaporization facility which can rapidly supply natural gas to the utility system.

Only a very small portion of the 1977 U.S. imports of natural gas (5% of total natural gas consumption),  $1.1 \times 10^{10}$  ft<sup>3</sup>, was imported from Algeria as liquefied natural gas (.05% of total consumption). Most of the imports, nearly 1 trillion ft<sup>3</sup>, were imported from Canada by pipelines. Some envision that LNG imports from oil producing nations with large yields of associated gas might increase to one to two trillion cubic feet in the 1980s [55]. Liquefaction provides a means by which nations with large yields of associated gas might increase to one to two trillion cubic feet in the 1980s [55]. Liquefaction provides a means by which nations with large reserves of gas but with small internal demands can export their resource. Venezuela, Iran, Iraq, Saudi Arabia, United Arab Emirates, Algeria, Libya, Nigeria, and Indonesia have large reserves and small demands, but they are too remote from the areas of large consumption to transport gas by pipeline. Although present liquefaction capacity of the exporting nations is still small,  $5.7 \times 10^9$  ft<sup>3</sup>/day (1980), this capacity could be significantly expanded. Increased international shipments of liquefied natural gas by large specially built ocean carriers, however, also increases the likelihood of accidents. If a spill should occur as a result of a tanker accident, a devastating fire is almost certain and if the accident should occur while the carrier is at port, the resulting fire might spread to populated areas surrounding the docking facilities.<sup>†</sup>

An alternative to liquefaction for the international shipment of natural gas is to first convert it to methanol, CH<sub>3</sub>OH [57]. This involves a partial oxidation of methane with water to produce intermediate products of carbon monoxide and hydrogen. In the presence of suitable catalysts, the carbon monoxide and hydrogen will react at moderate temperatures and pressures to form methanol. While methane is converted to methanol, larger hydrocarbon molecules are transformed to other (heavier) alcohols. The mixture of alcohols, equally suitable as a fuel as pure methanol, is referred to as *mythl-fuel*. Mythl-fuels, if shipped by conventional crude oil carriers, present no greater hazard than the shipping of crude oil. Owing to its

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<sup>\*</sup> These quantities refer to the nonliquefied volume of natural gas (atmospheric pressure and ambient temperature).

<sup>†</sup> There is considerable uncertainty as to the likelihood of such an accident. One study has concluded that the risk of such an occurrence is very small [56]. This study, however, utilized an event-tree type of analysis similar to that used for assessing the safety of nuclear reactors. The Nuclear Regulatory Commission has deemed this type of analysis, applied to nuclear reactors, inadequate for policy decisions (see Chapter 6).

**TABLE 2.4 Summary of Reserves and Production of Fossil Fuels for the United States**

	$Q_{\infty}$ original in-place reserves	$Q_p$ cumulative production through 1979	$Q_{\infty} - Q_p$ remaining reserve	Percent of total energy	$\frac{dQ_p}{dt}$ production rate, 1979	Percent of total power
Coal	$245 \times 10^9$ short tons <sup>a</sup>	$48 \times 10^9$ short tons	$197 \times 10^9$ short tons $1.45 \times 10^{14}$ Wy <sup>d</sup>	77%	$776 \times 10^6$ short tons/y $5.7 \times 10^{11}$ W	28%
Petroleum, crude oil, and natural gas liquids	$252 \times 10^9$ bbl <sup>b</sup>	$138 \times 10^9$ bbl	$114 \times 10^9$ bbl $2.2 \times 10^{13}$ Wy	12%	$3.72 \times 10^9$ bbl/y $7.2 \times 10^{11}$ W	36%
Natural gas, dry	$1184 \times 10^{12}$ ft <sup>3</sup> /y <sup>c</sup>	$590 \times 10^{12}$ ft <sup>3</sup> /y	$594 \times 10^{12}$ ft <sup>3</sup> /y $2.1 \times 10^{13}$ Wy	11%  100%	$20.9 \times 10^{12}$ ft <sup>3</sup> /y $7.3 \times 10^{11}$ W	36%  100%

<sup>a</sup>Based upon 1974 estimated recoverable reserve of Fig. 2.3.

<sup>b</sup>From Table 2.2.

<sup>c</sup>Based upon  $Q_{\infty} = 1050 \times 10^{12}$  ft<sup>3</sup> for lower 48 states and  $134 \times 10^{12}$  ft<sup>3</sup> for Alaska.

<sup>d</sup>Average energy value of  $22.1 \times 10^6$  Btu/short ton.

hydroxyl radical (OH), the energy value of alcohols, on a per unit mass basis, is necessarily less than that of hydrocarbon fuels which contain no oxygen. On the basis of energy content, the cost of shipping mythl-fuels would be more than that of crude oil, but not necessarily more than that of shipping liquefied natural gas. Facilities to convert natural gas to mythl-fuels are more expensive and less energy efficient than for its liquefaction, but new, extremely expensive liquefied natural gas carriers would not be needed. (Tanker capacity is already in excess of that required for oil trade.) While mythl-fuels could, conceivably, be converted back to a gaseous state, they can be directly used. Mythl-fuels are extremely clean burning and can be used as a boiler fuel. They can also be blended with gasoline, up to 15% by volume, without significantly affecting vehicle performance.\* In addition, mythl-fuels can be used as chemical feed stocks.

## 6. COAL-BASED SYNTHETIC FUELS, OIL SHALE, AND TAR SANDS

Nearly three-quarters of the energy used in the United States is obtained from liquid and gaseous hydrocarbon fuels. Domestic production of these fuels declined during

\* Gasohol is a blend of alcohols produced from grains and gasoline.

the 1970s and if estimates of remaining reserves (Table 2.4) prove valid, production rates will very likely continue to decline. Petroleum and natural gas accounted for 72% of domestic fossil fuels production in 1979 while coal accounted for 28%. However, of the remaining reserves of U.S. fossil fuels, assessed on the basis of energy content, 23% are hydrocarbon fuels and 77% are coal. The production rates of the individual fuels are thus ill-matched to their remaining reserves.

Several opportunities for the direct replacement of petroleum and natural gas with coal have already been discussed. If, however, coal is converted to liquid and gaseous synthetic fuels, the use of coal could be even further expended. In addition, the use of synthesized fuels would necessitate few changes in the overall energy system. These conversion schemes are not new; gaseous fuels have been obtained from coal since the beginning of the nineteenth century and liquids have been produced since the beginning of the twentieth century [58, 59].\* However, to produce synthetic fuels at the rate now envisioned in the United States, several new processes are being developed. It is hoped that the new processes will be more energy efficient than the older conversion methods and that they will be more suited to producing large quantities of fuel such as proposed by the Energy Security Act of 1980 [60].

While the main energy-yielding component of coal is carbon, coal also contains some hydrogen. On a mass basis the hydrogen content is small, but on an atomic basis, coal has six to ten atoms of hydrogen for each ten atoms of carbon. Hydrocarbon fuels, however, have much larger hydrogen to carbon ratios. Methane has four hydrogen atoms for each carbon atom and petroleum averages nearly two atoms of hydrogen for each carbon atom. Therefore, to synthesize hydrocarbon fuels, hydrogen must be provided and the appropriate chemical bonds formed [48, 61, 62].

Coal gas, also known as *town gas*, was first produced commercially and used for lighting in London in 1812, and was introduced in the United States a few years later. To obtain this gas, coal is heated in the absence of air to drive off the volatile gaseous products, a process known as *carbonization*. Heat is provided by the combustion of coal outside the gasification vessel. Hydrogen and methane are the main constituents of the gas which has a moderately high energy value (500 Btu/ft<sup>3</sup>, one-half that of natural gas). To produce coal gas, oven temperatures of 550 to 750°C are required. When carbonization is carried out at a temperature of 900°C or higher, coke is formed which is used for refining steel. The gaseous products, known as *coke-oven gas*, are used for heating coke ovens. When carbonizing coal, only a small portion of the coal's energy is obtained as gaseous products, the carbonized coal (char) that remains as a residue represents up to 90% of the original energy. Since no reactions with oxygen or hydrogen other than that which the coal might contain occur, the above gasification is not considered a synthesis reaction.

The first synthesized gas, called *producer gas*, was introduced commercially the middle of the nineteenth century. This gas was obtained by the partial oxidation of coal with air.

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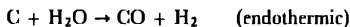
\* Manufactured gases, produced from coal as well as petroleum, have been a particularly important source of clean fuels in urban areas that lacked accessible supplies of natural gas.





The first reaction, exothermic (heat producing), is the combustion of carbon with oxygen; the second reaction, endothermic (heat absorbing), occurs if the amount of oxygen is insufficient for complete combustion. Carbon monoxide is the main combustible product of producer gas. Since air is used as a source of oxygen, the resultant product has a high nitrogen content. Producer gas also contains the volatile products of coal gas but, owing to the nitrogen content, has an energy value of only about 170 Btu/ft<sup>3</sup>.

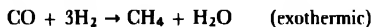
A subsequent improvement in the gasification of coal was the introduction of a product known as *water gas*. At elevated temperatures, hot carbon will react with steam to form hydrogen and carbon monoxide.



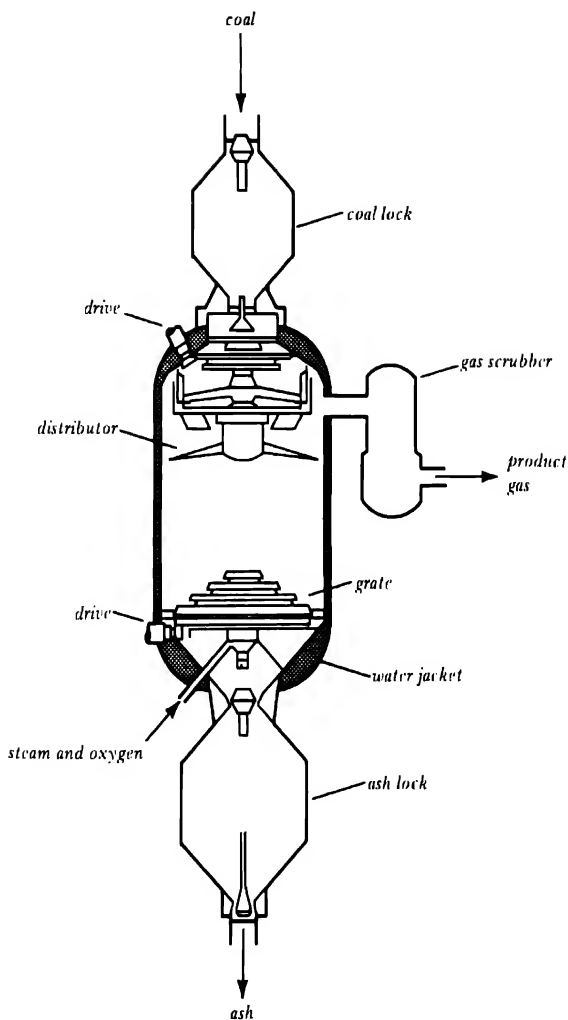
Water gas, which is relatively free of nitrogen, is obtained by the cyclic operation of a gasifier. First the coal is blown with air. Combustion occurs heating the coal bed while the combustion products, along with the nitrogen of the air, are vented to the atmosphere. The coal is then "steamed" to form carbon monoxide and hydrogen, water gas. Since this reaction is endothermic and cools the coal, the reaction must be soon terminated and the coal again blown with air to repeat the process. If the coal's temperature drops too low while being steamed, an undesired shift reaction occurs which increases the carbon dioxide content of the water gas.

An important development for producing synthetic fuels was the Lurgi pressurized gasifier introduced in 1936 (Figure 2.33). Lurgi gasifiers, using oxygen (as opposed to air), are used to produce what is known as *synthesis gas*. After the combustion-produced carbon dioxide is removed, the gas consists primarily of carbon monoxide and hydrogen. This gas can be further synthesized into methane, a substitute natural gas (SNG), or into liquid hydrocarbons. Lurgi gasifiers operate at pressures of up to 30 atm and large units (16 ft in diameter) can gasify up to 1000 tons of coal a day. Coal is charged intermittently into the air lock at the top of the gasifier. Both steam and oxygen enter at the bottom. Combustion, and hence the highest temperatures, occur just above the rotating grate at the bottom of the gasifier. Gasification, the carbon monoxide and hydrogen forming reaction, occurs in the middle of the reactor while volatile gases and moisture are driven from the coal at the top of the gasifier.

The raw synthesis gas obtained from a Lurgi gasifier after the carbon dioxide is removed consists of hydrogen, carbon monoxide, and methane. Two reactions are necessary to convert the synthesis gas to methane. Methanation is accomplished over a nickel-bearing catalyst at a moderate temperature of 260 to 370°C.

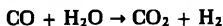


This exothermic reaction (by means of suitable heat exchangers) can be used for generating the steam required for gasification. For the entering gas to be completely converted to methane (and water), the molecular ratio of hydrogen to carbon



**FIGURE 2.33 A Lurgi Gasifier.**

monoxide must be three. That of raw synthesis gas is less than one. Methanation must therefore be preceded by a shift reaction in which some of the carbon monoxide of the synthesis gas is reacted with steam to increase the hydrogen content of the gas.



Shifting may be most readily effected by a high temperature reaction ( $\sim 400^\circ\text{C}$ ) followed by a lower temperature reaction. Catalysts are used to enhance the shift reaction.

Fourteen Lurgi gasifiers will be utilized by the Great Plains Coal Gasification Project located in Mercer County, North Dakota (Figure 2.34) [63]. In this project, the raw synthesis gas will be cooled to remove tar and dust. It will be again cooled and refrigerated after the shift reaction to remove impurities. The hydrocarbon liquids recovered will be sold, while phenols, used as fuel by the plant, will be recovered by solvent extraction (Phenosolvan unit). After methanation, the gas will be dried to remove the water produced by the methanation reaction and then compressed. A large air-separation plant will be required for producing the oxygen used by the Lurgi gasifiers. Also, since coal fines (small particles) are not suitable for Lurgi gasifiers, the fines will be separated from the coal and delivered to an adjacent coal-fired electric power plant. Based upon 1980 estimates, the plant is expected to cost \$1.5 billion and will produce  $137.5 \times 10^6 \text{ ft}^3/\text{day}$  of synthesized gas.\*

The Koppers-Totzek generator, another gasifier, was also developed in the 1930s. In this process, pulverized coal mixed with oxygen is injected into the gasifier vessel through burner heads. Steam is introduced around the heads, tending to surround the coal-oxygen combustion reaction. This gasifier operates at atmospheric pressure and requires only that the entering coal be pulverized. More recently, several other gasification processes have been studied. These units, referred to as *second-generation gasifiers*, are intended to handle very large quantities of coal and to operate at conventional natural gas line pressures ( $\sim 70 \text{ atm}$ ). They are also designed to reduce the amount of shifting and methanation required.

**BI-GAS Process (Bituminous Coal Research, Inc.).** This gasifier utilizes a two-stage pressurized reactor. The first stage is fueled by char recovered from the gaseous products and the second stage uses pulverized coal. Steam and oxygen are used for the gasification reaction. The advantage of this reactor over a Lurgi reactor is that the gases obtained from the char produced in the first stage interact in the lower temperature second stage forming increased quantities of carbon monoxide and hydrogen.

**Synthane Process (U.S. Bureau of Mines).** This gasifier is designed to utilize coking coals, coals unsuited for Lurgi gasifiers. A fluidized mixture of coal, oxygen, and steam are first injected into a moderate temperature gasifier to drive

\* The gas production rate of  $137.5 \times 10^6 \text{ ft}^3/\text{day}$  is equal to a thermal power of  $1.76 \times 10^6 \text{ kW}$ . The investment of \$1.5 billion is equivalent to \$840/kW of thermal power.



off volatile products. Concurrently, a noncaking char is produced that is then gasified in a high-temperature second-stage reactor.

***CO<sub>2</sub> Acceptor Process (Consolidated Coal Development).*** Calcined dolomite ( $\text{MgO} \cdot \text{CaO}$ ) along with pulverized coal, oxygen, and steam are used as the feedstock for the gasifier. The dolomite removes the combustion-produced carbon dioxide through a heat-producing exothermic reaction in which a carbonated product ( $\text{MgO} \cdot \text{CaCO}_3$ ), "spent dolomite," is formed. Char and the spent dolomite are removed from the bottom of the reactor and sent to a regenerator. In the regenerator, the heat produced by the combustion of the char liberates the carbon dioxide from the spent dolomite forming calcined dolomite which can then be reused. While the synthesis gas obtained is free of carbon dioxide, the gasifier, owing to the dolomite reaction, must be operated at a lower temperature than are other reactors, resulting in a slower rate of gasification.

***HYGAS Process (Illinois Institute of Gas Technology).*** This gasification process utilizes steam and hydrogen for the direct hydrogenation of coal. A synthesis gas that does not require shifting before methanation is obtained. Hydrogen may be produced through an auxiliary steam-char reaction. Alternatively, if a steam-iron process is used for producing hydrogen, air rather than oxygen can be used for the gasifier.

While other, even more advanced and potentially more efficient coal gasification schemes are being studied, these processes have not reached the pilot plant stage. Even though Lurgi gasifiers are less efficient than the other gasifiers, they have reached commercial acceptance.

Coal can also be gasified underground (in situ), thus eliminating the hazards of coal mining. The simplest in situ scheme requires the drilling of adjacent boreholes into the coal seam. For gasification, a passage must be formed (by hydrofracturing, for example) through the coal seam linking the boreholes. Combustion is initiated at one of the boreholes and air is pumped into the hole to sustain it. The hot combustion products, pushed toward the second borehole by the entering air, react with the coal, forming carbon monoxide. If ground water is available, hydrogen will be formed (shift reaction). For dry coal fields, steam can be injected with the air to sustain the hydrogen shift reaction. The crude synthesis gas produced by the underground reaction is collected at the second borehole. In situ gasification has attracted considerable interest (the earliest efforts were at the beginning of the century), but it has not yet achieved commercial acceptance.

Synthesis gas produced by Lurgi gasifiers can be converted to liquid fuels through the Fischer-Tropsch process. F. Fischer and H. Tropsch discovered this process in 1923 and the first commercial plant was constructed in Germany in 1935. Since the hydrogen to carbon ratio of petroleum is less than two, less shifting is necessary than when producing a substitute for natural gas. The mixture of petroleum products obtained from the Fischer-Tropsch reaction depends not only upon the hydrogen to carbon ratio of the synthesis gas but also upon the temperature and

pressure at which the synthesis occurs and upon the catalysts used to enhance the reaction.

In South Africa, petroleum products, primarily fuels suitable for transportation vehicles, are commercially produced by the Fischer-Tropsch synthesis process. The Sasol Two facility completed in 1979 (the initial smaller facility, Sasol One, operated successfully for 25 years) converts 40,000 tons of coal per day to 58,000 bbl/day of petroleum products [64]. The facility has 36 Lurgi gasifiers and a series of "Synthol" reactors using a specially developed catalyst. According to one estimate, a similar facility constructed in the United States would cost \$3.6 billion [65].\* Based upon this estimate, 17 such facilities, entailing an investment of \$61 billion, would be required to synthesize  $10^6$  bbl/day (5.3% of 1979 petroleum consumption). Furthermore, 250 million tons of coal per year (approximately one-third of the 1979 production rate) would be required. To produce  $3 \times 10^6$  bbl/day (16% of 1979 consumption) of synthetic fuel, domestic coal production would have to be doubled. A significant environmental and social impact cannot be avoided if synthetic fuels are to contribute significantly to the nation's hydrocarbon fuel supply [66-68].

A liquefaction process discovered in 1913 by F. Bergius avoids the gasification step. He discovered that gaseous hydrogen will directly react with coal at high pressures and temperatures in the presence of suitable catalysts. Bergius hydrogenation was extensively used in the 1930s and 1940s for producing petroleum in Germany. The feed stock for this process was a coal-oil slurry with the hydrogenation being accomplished in two successive stages. The first stage yielded "middle oils" with boiling points of 180 to 325°C. A temperature of nearly 500°C and a pressure of 70 atm or more, depending upon the coal, were required for the synthesis reaction. A steam reaction with the char recovered from the petroleum products was used to produce the hydrogen gas required. The second stage which operated at a somewhat lower pressure and temperature relied upon a vapor-phase hydrogenation that converted the oils obtained from the first stage to lighter hydrocarbons including gasoline.

Several advanced liquefaction schemes have been proposed and are being studied. These advanced schemes depend upon the direct transfer of molecular hydrogen from a slurry oil to the coal. Not only are lower temperatures and pressures required for hydrogenation, but higher overall conversion efficiencies are also anticipated. Processes being developed by various corporations include the following [61]:

H-Coal Process (Hydrocarbon Research)

Exxon Donor Solvent (Exxon)

Solvent Refined Coal-II (Gulf Oil)

Catalytic Coal Liquids (Gulf Oil)

Clean Fuels from Coal (Lummus Corp.)

\* Since the output of this facility, 58,000 bbl/day, is equivalent to a thermal power of  $4 \times 10^6$  kW, its investment cost per unit of thermal power is \$887/kW.

## Consolidation Synthetic Fuels (Consolidated Coal Company)

Synthoil and COSTEAM (Pittsburgh Energy Research Center, administered by the U.S. Department of Energy)

While it is envisioned that one or more of these advanced processes will eventually be utilized in producing synthetic liquid fuels, considerable further development (as of 1980) will be required before they are commercially acceptable. Furthermore, based upon numerous estimates, the costs of synthesizing hydrocarbon fuels from coal are expected to remain high [64, 69-71].

Even though a synthesis process is not required, hydrocarbons obtained from oil shale are frequently referred to as *synthetic fuels*. Oil shales contain organic substances, kerogen and bitumen. The mineral host is not a shale but marlstone, its principal constituents being dolomite, calcite, and quartz. The kerogen, which contains 90% or more of the hydrocarbons of the ore, is composed of very large molecules that have atomic weights of several thousand. To remove the kerogen imbedded within the mineral matrix of the oil shale, the shale must be first crushed and then heated in a retort to a temperature on the order of 500°C. During the retorting process, the chemical bonds of the kerogen are ruptured and the smaller hydrocarbon molecules are distilled. The distillate, known as *crude shale oil*, must be further processed before it can be handled by conventional crude oil refineries.

The hydrocarbon concentration of oil shale is usually specified in terms of the quantity of oil recovered from the shale when it is retorted. The richest shales yield no more than 100 gal of oil per ton while the yield of most shales is fewer than 30 gal. The energy value of a ton of oil shale which yields 30 gal of oil is only about  $4 \times 10^6$  Btu, that is, one-sixth that of high-rank bituminous coals or one-third that of lignites. Therefore, very large quantities of shale must be processed to obtain significant quantities of oil and since only a small portion of the shale is organic, the quantities of spent or waste shale are similarly large.

The extraction of hydrocarbons from oil shale is not a new development despite the recent interest in exploiting this technology [72-78]. In Scotland, an oil shale industry was established in 1859 which operated on a commercial scale until 1962. Peak production reached approximately 1.7 million bbl/y. In Sweden, an oil shale industry, initiated in 1920, eventually reached a peak production rate of 550,000 bbl/y. However, with the depletion of its reserves and the availability of imported oil, production ceased in 1966. France, Spain, Germany, South Africa, and Australia have also had small oil shale industries.

In the United States, hydrocarbon-rich oil shales of the Green River formation of Colorado, Utah, and Wyoming were discovered in 1874. It is estimated that this formation contains 418 billion barrels of oil locked in shales which would yield over 25 gal of oil per ton. In addition, shales which yield 10 to 25 gal per ton are believed to contain 1400 billion barrels of oil. For a comparison, the proved crude oil reserves of the entire Middle East were 362 billion barrels at the end of 1979.\* Extracting

\*The estimated initial reserves for the Middle East, which include some yet to be discovered, were 601 billion barrels (see Figure 2.17).

oil from shale, however, is considerably more difficult than extracting it from conventional formations. The first significant U.S. interest in processing oil shale came between 1915 and 1930. At least 25 retorting processes were developed to the pilot plant stage. But, with the 1930 discovery of a large oil field in eastern Texas, U.S. interest in oil shale waned. Since the Arab oil embargo of 1973-1974, there has been a renewed interest in utilizing the nation's oil shale reserves.

The commercial production of shale oil has been based upon the underground mining of shale and above-ground retorts. However, underground, in situ, retorting of the shale is also being studied. For a true in situ process, the shale must be sufficiently permeable to recover the oil by underground retorting. In a modified in situ process, some of the shale is first mined to create a large room and then explosives are used to fracture the roof, thus filling the room with broken shale. Air and fuel gas are injected through a borehole at the top, ignited, and the crude shale oil is recovered from a sump at the bottom of the room. While underground retorting is expected to be less expensive and to have a smaller environmental impact than above-ground retorting, the latter processes are better understood and further developed. It is expected that underground mines and large surface retorts will initially be used in the United States.

The Nevada-Texas-Utah retort, developed by the NTU Co. in 1925, has been extensively tested and modified by the U.S. Bureau of Mines facility at Rifle, Colorado. In this batch-type retort, which resembles early coal gasifiers, a retort is filled with crushed shale. Combustion is initiated at the top of the retort by a gas burner and is sustained by the air blown into the retort. As combustion proceeds downward in the retort, the kerogen of the shale below the combustion zone is decomposed. The crude shale oil and gases move downward and are recovered at the bottom of the retort. Although 12,000 bbl of oil have been produced by the Bureau of the Mines by this type of retort, the batch-type retort is not considered suitable for the large quantities of oil shale that are expected to be processed in the United States.

For a large oil shale industry with production rates of 50,000 bbl/day or more, continuously fed retorts are thought to be necessary. While several continuously fed retorts are being developed, none sufficiently large for commercial operation has been constructed (as of 1980). As is the case for coal gasification and liquefaction, several advanced processing schemes are being studied and developed for oil shale.

***Pharaho Retort (Pharaho Development Corp., a consortium of energy companies).*** Crushed shale is fed into the top of the retort and spent shale is removed from the bottom. Combustion, sustained by air blown into the retort, occurs in the middle with the kerogen retorting reaction occurring above the combustion zone. The vaporized shale oil and gases are removed from the top of the retort. The Pharaho retort can also be heated indirectly by the combustion of the shale-produced gases.

***Union Retort (Union Oil Co.)*** This indirectly heated retort is unique in that crushed shale is pushed into the bottom of the retort by a "rock pump." The oil and gas liberated from the shale flow downward heating the upward-moving



shale. A high oil yield is anticipated for this retort, but the maintenance of mechanical rock pumps may complicate the operation of the retort.

***Superior Oil Indirectly Heated Retort (Superior Oil).*** Shale processing is designed to recover not only oil but also sodium-bearing minerals. A horizontal, doughnut-shaped retort utilizes a traveling grate to move the shale. The good temperature control necessary for recovering the minerals can be maintained with this type of retort.

***TOSCO II (Colony Development, a joint venture).*** The TOSCO process, named after the Oil Shale Company which did some of the early development work, utilizes hot ceramic balls to heat the finely crushed shale feed stock. Today, the gases obtained from the shale are used for heating the ceramic balls, but it is anticipated that the less valuable char of the spent shale will eventually be used. Even though the oil recovery rate is very high, the ball separator and heater make the system considerably more complex than other retorts. Furthermore, the spent shale is in the form of a very fine carbon-bearing powder which makes its disposal more difficult.

Regardless of the retorting process utilized, a large-scale development of oil shale would have a major social and environmental impact.\* Leaching the spent shale by runoff is considered a major environmental hazard. The alkaline waters, if not impounded, would significantly degrade both surface and ground waters of the region. Even for in situ retorting of the shale, leaching by ground waters may be a problem. Both the mining and retorting of the shale would result in vast quantities of air pollutants, thus degrading the air quality of a large region. Since water is required, as much as four barrels for each barrel of oil, an oil shale industry in the arid West would compete with other water users, primarily agriculture. Furthermore, the small communities of the region may not be able to sustain the population influx required by a large-scale oil shale industry. Because of the overall impact of a large oil shale industry, many are opposed to its development [6, 80, 81].†

Closely related to the processing of oil shale is the recovery of crude oil from tar sands. Tar sands (also known as oil sands) consist of a porous sand which contains a mixture of bitumen and water. The crude oil potential of U.S. tar sands (primarily in the Uinta Basin of Utah) is placed at about 30 billion barrels, but Canadian

\*It has been proposed that a production rate as high as  $5 \times 10^6$  bbl/day of crude shale oil could be achieved in the Piceance Basin of Colorado. A gigantic open pit mine, 2 to 3 mi in length, 1 mi wide, and 1000 ft deep would be required for a production rate of only  $10^6$  bbl/day [79].

†It no longer seems likely that a major oil shale industry will emerge during the 1980s. Exxon, which became a major partner (60%) of Tosco in 1980, decided to withdraw from the venture in 1982 [82, 83]. Tosco also terminated the venture by exercising an option of its partnership agreement to sell out to Exxon. Exxon estimated that their 50,000 bbl/day facility would cost in excess of \$5 billion when completed. Only the Union Oil Company venture is being actively developed (as of 1983); all other ventures have been put "on hold." Union Oil estimates that its relatively small facility will produce 10,000 bbl/day and will cost \$550 million. This implies an investment cost of nearly \$800 per kilowatt of thermal power supplied by the shale oil so produced.

reserves have been placed at 1350 billion barrels [84, 86]. The Canadian reserves are in Northeastern Alberta, with the largest, the Athabasca deposit, account for 869 billion barrels. Other reserves of tar sands are found in Columbia and Venezuela. Compared to estimated reserves of conventional liquid hydrocarbons, those of tar sands are immense. As is the case for oil shale, however, these crude oil estimates may be misleading because their recovery appears to be severely limited by other factors.

The Athabaskan tar sands consist of 8 to 12% bitumen, the remainder being sand and mineral-rich clays along with water. Bitumen, a semisolid which must be thermally processed to yield a synthetic crude oil, is several hundred to a thousand times more viscous than conventional petroleum. Furthermore, the highly abrasive nature of the tar sands makes mining difficult. While about two tons of tar sands must be mined to produce one barrel of synthetic crude oil, up to three tons more of overburden must be removed. After being surface-mined, the tar sands are mixed with a hot-water alkaline solution to produce a slurry. The slurry is sent to separation cells in which the sand of the slurry sinks to the bottom. The bitumen, skimmed from the surface, is then coked (heated) to obtain the liquid hydrocarbons as well as roke and gases which are used to fuel the tar sands plant.

Techniques for recovering oil from the Canadian tar sands are fairly well developed. Great Canadian Oil Sands, Ltd. (96% owned by Sun Company, Inc., of Philadelphia) began operation in 1968 at a site north of Ft. McMurray, Alberta, and has been producing about 50,000 bbl/day. The synthetic crude oil can be refined in a conventional refinery. Great Canadian Oil Sands became financially profitable in the last years of the 1970s, after operating at a loss for eight years. Syncrude (a consortium of Imperial Oil Ltd., Gulf Oil Canada, Ltd., and Canadian-Cities Service, Ltd.) constructed a plant just north of the Great Canadian Oil Sands facility in the late 1970s. This plant produces 100,000 bbl/day of synthetic crude oil.\*

Technology for the surface mining and processing of the Athabaskan tar sands is well developed, but, unfortunately, less than 10% of the tar sands reserves can be recovered in this fashion. The bulk of the reserves will have to be recovered by in situ techniques that have yet to be developed. For bitumen to flow through the pore system of the sand, it must be heated to about 150°C. Heating has been attempted with steam, hot water, combustion, and with electrical resistance currents (resistance heating).

The technology used in Canada does not appear to be well-suited for the U.S. tar sands reserves of Utah. Not only is the composition of the tar sands different, but because of the nature of the formations, in situ recovery will also probably be required. Water constraints further limit recovery options.

\* Since \$2.3 billion were needed to build the facility, the plant's investment cost per unit of equivalent thermal output power was \$329/kW. It is estimated that now (1982) \$5.1 billion would be required to build a similar plant, an investment cost of \$729/kW [84].

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## PROBLEMS

1. It is convenient, when doing both algebraic manipulations and numerical evaluations, to express the logistic function and its derivative in terms of hyperbolic functions.

- (a) Show that the following is valid:

$$\begin{aligned} E(t) &= E_{\infty}(1 + e^{-at})^{-1} \\ &= \frac{1}{2}E_{\infty} + \frac{1}{2}E_{\infty} \tanh(at/2) \\ \frac{d}{dt}[E(t)] &= \frac{1}{4}aE_{\infty} \cosh^{-2}(at/2) \end{aligned}$$

- (b) For small values of  $t$  (as  $t \rightarrow -\infty$ ), an exponential approximation is valid for the derivative of the logistic function.

$$\frac{d}{dt}[E(t)] = aE_{\infty}e^{at}$$

Determine the range of  $at$  for which the error is no greater than 10%. What is the range of a 5% error?

- Following the notation of the text, the quantity  $P(t)$  is the derivative of a logistic function  $E(t)$ .
  - Determine the values of  $at$  for which the derivative of  $P(t)$  is a maximum (its inflection points).
  - What is the fraction of the total resource,  $E_{\infty}$ , extracted over the time interval between the inflection points?
- The rate of increase of a quantity such as  $P(t)$ , the extraction rate for a logistic function, is its derivative. Its fractional growth rate is the derivative divided by  $p(t)$ :

$$\text{growth rate} = \frac{1}{P(t)} \frac{d}{dt} P(t)$$

- Obtain an expression for the growth rate for a logistic curve. Show that the growth rate approaches  $a$ , the exponential coefficient, as  $t$  becomes very small ( $t \rightarrow -\infty$ ). Show that it is equal to  $-a$  as  $t$  becomes very large ( $t \rightarrow \infty$ ).
  - At what value of  $at$  is the growth rate equal to  $.5a$ ? At what value is it equal to  $-.5a$ ?
  - What is the total quantity extracted over the time interval determined by the result of part (b)?
- Assume that the total energy consumption rate for the world can be approximated by a straight line (Figure 1.4) through the following points:

1900	$.4 \times 10^{12}$ W
1970	$6.5 \times 10^{12}$ W

- What is the growth rate for this consumption function? What is, in watt-years, the total quantity of energy consumed prior to 1970?
- Assume the following original in-place reserves of fossil fuels for the world:

Coal	$2 \times 10^{12}$ metric tons ( $28 \times 10^6$ Btu/metric ton)
Crude oil	$2 \times 10^{12}$ bbl
Natural gas	$3.7 \times 10^{14}$ m <sup>3</sup>

What is the value expressed in watt-years of  $E_\infty$  which corresponds to these reserves of fuels?

- (c) Assume that cumulative consumption for the world can be approximated with a logistic function having values of  $E_\infty$  and  $a$  already determined. In what year is the consumption rate a maximum? What is its value?
  - (d) When in the future is consumption equal to that of 1970? What is the quantity of energy consumed up to that time?
5. One estimate of the initial energy value for the U.S. coal reserves is a coal equivalent of  $390 \times 10^9$  metric tons ( $28 \times 10^6$  Btu/ton). The energy production rate for this resource in 1980 corresponded to a coal equivalent of  $550 \times 10^6$  metric tons/y.
- (a) Approximately  $40 \times 10^9$  metric tons were produced prior to 1980. Assume exponential growth in production for this period. What was the growth rate? Assume that future production is that predicted by a logistic function. When will, based upon the pre-1980 exponential pattern, a peak in production occur? What is the peak production rate?
  - (b) Assume production increases at a  $3\%/y$  rate for small values of cumulative production. Obtain a logistic function which yields the correct production rate for 1980. When does the rate peak and what is its value? Repeat for a  $5\%/y$  growth rate.
6. The U.S. population has tended to increase according to that predicted by a logistic function.

$$P(t) = \frac{P_\infty}{1 + e^{-at}}$$

This approximation predicts an ultimate population of  $P_\infty(t \rightarrow \infty)$ . Over the early years of the nation (the first half of the nineteenth century) the average population growth rate was  $3\%/y$ . The nation's population was  $80 \times 10^6$  in 1900 and  $220 \times 10^6$  in 1980.

- (a) Obtain a value for  $P_\infty$  and the year at which the population was  $.5P_\infty$ .
  - (b) In what year does the population reach  $.9P_\infty$ ? When does it reach  $.95P_\infty$ ?
  - (c) What is the percentage growth rate predicted by the logistic function for 1980?
  - (d) In what year is the percentage growth rate equal to  $.2\%/y$ ?
7. Both the cumulative production and discovery of a particular resource are given by a logistic function. The cumulative discoveries precede production by  $10y$ , the exponential coefficient of the logistic function is  $.05/y$ , and  $Q_\infty$  has a value of 100.

$$Q_p = \frac{Q_\infty}{1 + e^{-at}}$$



$$Q_d = \frac{Q_\infty}{1 + e^{-a(t+\Delta t)}}$$

Proved reserves are the difference between cumulative discoveries and production.

$$Q_r = Q_d - Q_p$$

- (a) When does the peak discovery rate occur? When does the peak production rate occur?
- (b) Show that the peak in proved reserves,  $Q_r$ , occurs for  $t = -\Delta t/2$ . (Since

$$\frac{dQ_r}{dt} = \frac{dQ_d}{dt} - \frac{dQ_p}{dt}$$

the value of time for which a zero occurs for the derivative of  $Q_r$  may readily be obtained.)

- (c) Using sufficient calculated points to obtain a reasonably accurate curve, plot the functions  $Q_d$ ,  $Q_p$ , and  $Q_r$ .
- (d) A static index is often used to specify proved reserves. Calculate and sketch this function:

$$Q_r \left( \frac{dQ_p}{dt} \right)^{-1}$$

When it is equal to 10y?

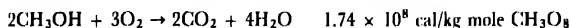
8. An expression, developed by M. King Hubbert, for the cumulative crude oil production (lower 48 states) is given in Figure 2.13.
  - (a) What is the production rate (barrels per year and barrels per day) in 1980 based upon this expression?
  - (b) The expression will be used to predict future production. What is the production rate for the years 2000 and 2020?
  - (c) What will be the cumulative crude oil production from 1980 to 2020?
  - (d) What is, in the year 2020, the quantity of crude oil that remains to be produced?
9. Cumulative discoveries of crude oil,  $Q_d$ , are indicated in Figure 2.15. The discoveries are to be approximated by an expression of the following form:

$$Q_d = Q_\infty(1 - e^{-ah})$$

- (a) What is the initial discovery rate for this expression ( $dQ_d/dh$  for  $h = 0$ )?
  - (b) What was the average discovery per foot of exploratory drilling prior to 1940?
  - (c) Obtain an expression for  $Q_d$  which yields the result of part (b) and the correct value of  $Q_d$  for 1970. What is the value of  $Q_\infty$  for this approximation?
10. Assume that the U.S. energy consumption rate remains unchanged for the period 1980 to 2020 at a value of  $2.6 \times 10^{12}$  W. Cumulative production of crude oil (lower 48 states) follows the logistic curve of Figure 2.13. Natural gas liquids contribute an additional 20% to total petroleum production and  $2 \times 10^4$  bbl/day

of crude oil are obtained from Alaska. Cumulative production of natural gas is given by the logistic curve of Figure 2.28 ( $a = .0673/\text{y}$ ,  $Q_\infty = 1050 \times 10^{12} \text{ ft}^3$ ). In addition, assume that  $4 \times 10^{12} \text{ ft}^3/\text{y}$  are obtained from Alaska starting in 1990.

- (a) Determine the portion of the nation's total energy usage rate supplied by petroleum and by natural gas for the years 2000 and 2020.
  - (b) What is the fraction of energy consumed in the 1980 to 2000 interval supplied by each of these hydrocarbon fuels? What is the fraction for the period 2000 to 2020?
11. Assume that, as a result of a very effective energy conservation effort, the U.S. energy consumption rate is reduced from  $2.6 \times 10^{12} \text{ W}$  in 1980 to  $10^{12} \text{ W}$  in 2020. Exponential decline occurs, that is, the yearly rate of decline is constant. Repeat Problem 10 for this consumption pattern.
12. Both natural gas and coal can be converted to methanol, a liquid fuel that could be substituted for petroleum fuels used for transportation. The following combustion reactions occur for methanol and heptane:



- (a) Determine the heat of combustion (joules and British thermal units) per kilogram of each fuel.
- (b) Why would the heat of combustion for methanol, on a mass basis, be expected to be about half that of heptane and other petroleum hydrocarbon fuels?
- (c) The density of methanol is  $.79 \text{ g/cm}^3$  and that of heptane is  $.69 \text{ g/cm}^3$ . What is the per liter energy value of these fuels? What is their per barrel energy value (42 gal/bbl)?

## CHAPTER 3

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# *Terrestrial Limitations*

### 1. SOLAR RADIATION

As is well known, life on earth is completely dependent upon the energy radiated by the sun. Not only were our fossil fuel reserves formed as a result of the incident radiation, but the very evolutionary process that has brought us to our present concern about energy resources could not have occurred without the sun's energy. All biological processes are either directly or indirectly dependent upon photosynthesis.

At the earth's average distance from the sun ( $R_{SE} = 1.49 \times 10^{11}$  m), the radiation density,  $S$ , is  $1353 \text{ W/m}^2$  [1]. The radiation density  $S$  is often referred to as the *solar constant*. It is the power density of the sun's radiation outside the earth's atmosphere. Solar radiation outside the earth's atmosphere as well as at the earth's surface is often specified in terms of langleys, a measure of energy density. One langleys is defined as an energy density of one calorie per square centimeter. A power density of  $1353 \text{ W/m}^2$ , that is,  $S$ , is thus  $.1353 \text{ W/cm}^2$  which is equal to  $.0323 \text{ cal/cm}^2\text{-s}$  or  $1.94 \text{ cal/cm}^2\text{-min}$ . Expressed in langleys, the solar constant is therefore 1.94 langleys per minute. This is the radiation power density available to energize solar cells on a satellite outside the earth's atmosphere. Consequently, solar cells with an efficiency of 10% are capable of producing 135 W electrical power for an incident area of one square meter. This high-power density forms the basis of a proposed satellite collector scheme which would transmit the energy thus collected to the earth by means of a microwave beam [2].

Even at the earth's surface, power densities of greater than half the solar radiation density are not uncommon. While a satellite can eliminate nighttime

interruptions, many experimenters have produced useful earth-based collectors. Houses have been heated, working heat engines have been constructed, and electricity has been generated with solar devices. Commercially produced hot water heaters are common in many arid and tropical countries. A professional society has been founded which publishes an excellent journal, *Solar Energy* [3]. Solar energy will undoubtedly be our ultimate energy resource.

To the extent that it behaves as a black-body radiator, the apparent surface temperature of the sun may be calculated using the Stefan-Boltzmann radiation law. For an ideal black body, the power radiated per unit surface area is proportional to the fourth power of its temperature.

$$\text{Power radiated/m}^2 = \sigma T^4$$

where  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

The total power radiated by the sun may be obtained by multiplying by its total surface area,  $4\pi R_S^2$ . Its temperature will be designated as  $T_S$ .

$$\text{Power radiated by sun} = 4\pi R_S^2 \sigma T_S^4$$

where  $R_S = 6.95 \times 10^8 \text{ m}$  (radius of sun)

Knowing the solar constant  $S$ , that is, the radiated power density at the earth's average distance from the sun, one may determine the total power radiated by the sun. For the calculation, one need only imagine a sphere with a radius equal to the distance of the earth from the center of the sun,  $R_{SE}$ , that has the sun at its center. Since each square meter of such a sphere accounts for a radiated power of  $S$  watts, the total radiated power is simply  $S$  times the area of the sphere,  $4\pi R_{SE}^2$ .

$$\text{Power radiated by sun} = 4\pi R_{SE}^2 S$$

where  $R_{SE} = 1.49 \times 10^{11} \text{ m}$  (distance of earth to sun)

This result may be equated to the one obtained above for the total power radiated by the sun.

$$4\pi R_S^2 \sigma T_S^4 = 4\pi R_{SE}^2 S$$

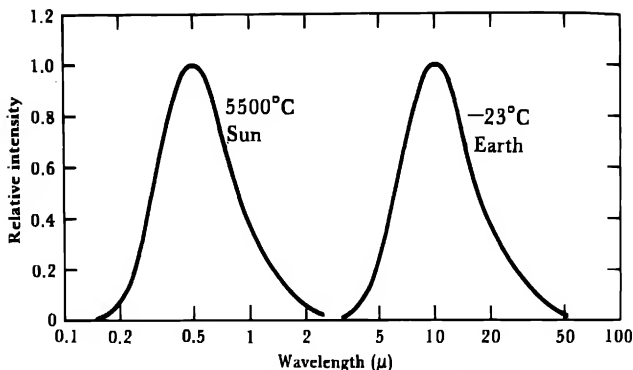
$$T_S^4 = \frac{R_{SE}^2 S}{R_S^2 \sigma}$$

$$T_S = (R_{SE}/R_S)^{1/2} (S/\sigma)^{1/4}$$

$$T_S = 5755 \text{ K} (5482^\circ\text{C})$$

The temperature is obtained by introducing the appropriate numerical values. Spectral measurements indicate a temperature of approximately  $5500^\circ\text{C}$ .

While the power radiated by the earth-atmosphere system is approximately equal to that absorbed, a change in the wavelength of the radiation occurs. The spectral distribution of a black-body radiator is dependent upon its temperature. The power density for each unit of wavelength is given by Planck's radiation formula.



**FIGURE 3.1** *Relative Radiation Intensity for the Earth and the Sun.*

$$E_{\lambda} = \frac{2\pi c^2 h}{\lambda^5 (e^{hc/\lambda kT} - 1)} \text{ W/m}^3$$

where  $c = 3 \times 10^8$  m/s (velocity of light)  
 $h = 6.625 \times 10^{-34}$  J-s (Planck's constant)  
 $k = 1.38 \times 10^{-23}$  J/K (Boltzmann constant)

The above expression may be somewhat simplified by combining two groups of constants.

$$E_{\lambda} = \frac{c_1}{\lambda^5 (e^{c_2/\lambda T} - 1)} \text{ W/m}^3$$

where  $c_1 = 3.74 \times 10^{-16}$   
 $c_2 = 1.44 \times 10^{-2}$

It should be noted that  $E_{\lambda}$  is a power density ( $\text{W/m}^2$ ) per unit wavelength. A plot of the relative amplitude of  $E_{\lambda}$  for a temperature of  $5500^{\circ}\text{C}$  is given in Figure 3.1. The maximum radiation occurs for a wavelength of  $.5 \mu$ . This wavelength, which corresponds to blue light, is very nearly in the center of the  $.4$  to  $.7 \mu$  visible spectrum. Our eyes, not surprisingly, have evolved so that they respond to the most intense portion of the sun's spectrum.

The wavelength corresponding to the peak of the spectral density curve may be found by differentiating the expression for  $E_{\lambda}$  with respect to  $\lambda$ . Setting this result equal to zero yields the desired wavelength,  $\lambda_{\max}$ . A particular simple result is obtained, namely, the product of  $\lambda_{\max}$  and the temperature,  $T$ , which is a constant.

$$\lambda_{\max} T = 2.898 \times 10^{-3} \text{ m K}$$

This is known as *Wien's displacement law*. The higher the temperature of a body, the

shorter the wavelength (or the higher the frequency) of its radiation. It may easily be verified that this expression does yield a value of  $.5\mu$  for  $\lambda_{max}$  when  $T$  is equal to 5723 K (5500°C).

The earth-atmosphere system also behaves as a black-body radiator, reradiating energy into space. In addition to the curve for the sun in Figure 3.1, a curve corresponding to a temperature of 250 K ( $-23^{\circ}\text{C}$ ), the average radiative temperature of the earth, is shown. The peak of this curve occurs for a relatively long wavelength,  $11.6\mu$ , which is in the infrared portion of the radiation spectrum.

## 2. THE SOLAR ENERGY BALANCE OF THE EARTH

The earth-atmosphere system absorbs a portion of the incident energy of the sun and, in turn, radiates this energy into space, establishing an energy balance. In addition to the energy received from the sun, the surface of the earth is warmed by a heat flux from its interior that results, primarily, from a decay of radioactive isotopes. The tides, which are a consequence of the earth-moon system, result in viscous friction, another energy input that affects the surface energy balance. Tidal friction dissipates a minute fraction of the energy of the otherwise conservative earth-moon system [4]. A further contribution to the surface energy balance is the heat generated through use of fossil and nuclear fuels. While a portion of the solar energy incident upon the earth produces fossil fuels, the rate at which the fuels are formed is completely negligible compared to the present consumption rate.

To the extent that the earth is not changing, the heat radiated by the surface of the earth is equal to the sum of the heat inputs. If an equality did not exist, a warming or cooling of the earth's surface would necessarily result. The earth is changing, or evolving, only at an extremely slow rate. Fossil fuels, representing stored energies, are being released. Biological evolution or complexification, which is a very slow process which, except for hominids, required millions of years for significant changes to occur. Consequently, a change over a one-year period is, for all practical purposes, imperceptible. For establishing an energy balance, the condition at the end of a given year appears about the same as at the beginning of the year. Essentially the same quantity of biological mass and chemical deposits are contained in the earth's inventory. While solar energy is necessary for photosynthesis, a dynamic equilibrium (homeostasis) is achieved, in which a net generation of new material is negligible.\*

Incident solar energy is the dominant energy input. While the other inputs are not completely negligible, they will be ignored in the initial analysis. The radiation absorbed by the earth depends upon the earth's cross-sectional area perpendicular to the sun's flux, that is,  $\pi R_E^2$ . A fraction of the incident energy,  $\tau$ , is reflected. Hence, the fraction absorbed is  $1 - \tau$ .

\*The net world population growth could be considered an exception although this does not necessarily imply an increase in the total biomass of the earth.

$$\text{Absorbed power} = (1 - r)\pi R_E^2 S$$

where  $R_E = 6.378 \times 10^6 \text{ m}$  (mean radius of earth)

The earth, as mentioned, behaves as a black-body radiator. The radiative temperature of the earth-atmosphere system determines the actual power radiated by the earth. This temperature, approximately 250 K (Figure 3.1) is considerably less than the average surface temperature of approximately 286 K (13°C). To express the radiated power density in terms of the earth's surface temperature, a relative emissivity constant,  $\epsilon$ , which has a value of less than one, is frequently used.

Using  $T_E$  for the earth's temperature, the power radiated by each square meter is the following:

$$\text{Power density} = \epsilon \sigma T_E^4 \text{ W/m}^2$$

Assuming that the surface temperature of the earth changes very little from day to night, the earth may be considered a spherical radiator with a uniform temperature. The power radiated is thus obtained by multiplying the power density by the total surface area of the earth,  $4\pi R_E^2$ .

$$\text{Radiated power} = 4\pi R_E^2 \epsilon \sigma T_E^4$$

For equilibrium, the powers absorbed and radiated must be equal.

$$(1 - r)\pi R_E^2 S = 4\pi R_E^2 \epsilon \sigma T_E^4$$

$$(1 - r)S = 4\epsilon \sigma T_E^4$$

Both the reflectivity and emissivity constants are average values that depend upon the cloud coverage and the atmospheric composition. According to various sources, the average reflectivity is between .3 and .4 [5-8]. By specifying  $r$ , the average power absorbed and radiated by each square meter of the earth's surface may be calculated. For a value of .35 for  $r$ , the following is obtained.

$$\begin{aligned} \epsilon \sigma T_E^4 &= \frac{1}{4}(1 - r)S \text{ W/m}^2 \\ &= \frac{1}{4}(1 - .35) 1353 \\ &= 220 \text{ W/m}^2 \end{aligned}$$

Since  $T_E$  is approximately 13°C or 286 K, the average value of  $\epsilon$  may be determined.

$$\begin{aligned} \epsilon &= \frac{220}{\sigma T_E^4} = \frac{220}{(5.67 \times 10^{-8})(286)^4} \\ &= .580 \end{aligned}$$

The significance of the other inputs to the energy balance of the earth may be determined now by calculating the change in temperature that results when they are included. Munk and MacDonald [9] estimate that the input due to tidal friction is  $2.4 \times 10^{12} \text{ W}$ , that is, approximately one-third of the heat generated by fuels.

Dividing by the earth's surface area,  $4\pi R_E^2$ , an average power density for tidal friction is obtained.

$$\begin{aligned}\text{Surface area} &= 4\pi R_E^2 \\ &= 4\pi(6.378 \times 10^6)^2 = 5.11 \times 10^{14} \text{ m}^2 \\ P_{\text{tides}} &= \frac{2.4 \times 10^{12}}{5.11 \times 10^{14}} = .005 \text{ W/m}^2\end{aligned}$$

Tidal energy is thus only .0021% of that absorbed and radiated by the earth. Van Herzen [10] estimates the average heat flow from the earth's interior due to conduction to be .063 W/m<sup>2</sup>. The contribution of volcanoes and hot springs averaged over the surface of the earth adds approximately 1% to this value. This results in a total heat input of  $3.25 \times 10^{13}$  W which is 0.29% of the solar input. The societal input of  $8.1 \times 10^{12}$  W, or .016 W/m<sup>2</sup>, lies between the tidal and thermal inputs.

Since the above contributions are small, they can be treated as a perturbation of the original result. If  $P_E$  (W/m<sup>2</sup>) is the input for which an effect is desired, the following is obtained.

$$\frac{1}{4}(1-r)S + P_E = \epsilon\sigma T_E^4$$

The left-hand side is the total input power density, and the right-hand side is the radiated power density. The solar input, the first term, is 220 W/m<sup>2</sup>.

$$\begin{aligned}\epsilon\sigma T_E^4 &= \frac{1}{4}(1-r)S \left[ 1 + \frac{4P_E}{(1-r)S} \right] \\ &= \frac{1}{4}(1-r)S(1 + \delta)\end{aligned}$$

$$\delta = 4P_E/(1-r)S$$

The fraction,  $\delta$ , is thus the ratio of the input being considered to the solar input.

$$T_E = \left[ \frac{(1-r)S}{4\epsilon\sigma} \right]^{1/4} (1 + \delta)^{1/4}$$

The first term in the right-hand side of the above equation is the earth's nominal temperature, that is, the surface temperature calculated ignoring the input  $P_E$ . It will be designated as  $T_{E_0}$  ( $= 286$  K). Since  $\delta$  for the inputs being considered is very small ( $\ll 1$ ), the second term may be approximated by the first term of its Taylor series expansion.

$$\begin{aligned}(1 + \delta)^{1/4} &\simeq 1 + \frac{1}{4}\delta \\ T_E &= T_{E_0} \left( 1 + \frac{1}{4}\delta \right) \\ T_E - T_{E_0} &= \Delta T_E = \frac{1}{4}\delta T_{E_0}\end{aligned}$$



Since  $\delta$  is known for each of the inputs considered, the expectant temperature effect may readily be calculated.

	<i>Tides</i> .005 W/m <sup>2</sup>	<i>Thermal</i> .0636 W/m <sup>2</sup>	<i>Man</i> .016 W/m <sup>2</sup>
$\delta$	.000021	.00029	.000073
$\Delta T_E$	.0015 K	.02 K	.0052 K

The above results certainly justify ignoring these inputs for the solar energy balance.

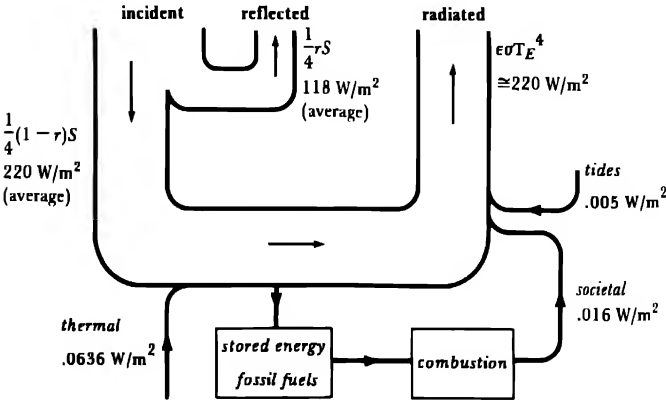
While still small, societies' contribution to the energy balance could, if it continues to grow, significantly modify the earth's surface temperature. An increase of only one degree of the average temperature may be sufficient to significantly modify the climate of the earth. Societies' power input for a one-degree change may readily be determined.

$$\delta = \frac{4\Delta T_E}{T_{E_0}} = .014 \qquad (\Delta T_E = 1 \text{ K})$$

$$P_E = 3.08 \text{ W/m}^2$$

This implies an energy consumption rate about 200 times the present rate. While it is highly unlikely that such an enormous consumption rate will ever result, growth at a 5%/y rate would reach this level in one century.

The solar balance, it should be emphasized, imposes an absolute limit to the rate of use of energy derived from fuel. While a one-degree change may not be the precise limit, an ultimate limit does exist. This limit is totally independent



**FIGURE 3.2** *Solar Energy Balance—Average Power per Square Meter of the Earth's Surface.*

of all other effects, that is, it exists even for pollutionless (including thermal) generation. The only exception is directly derived solar energy. For this source, useful work is obtained from the incident radiation before it heats the earth-atmosphere system. The output work that eventually results in heat restores the heat balance. Figure 3.2 illustrates the significant energy flows.

Although it is very unlikely that energy consumption will modify the overall average temperature of the earth by direct thermal heating, local heating can occur. For example, New York City had a 1970 population density of 24,673 people per square mile [11] or 7193 per km<sup>2</sup>. While the per capita energy consumption rate for the United States is approximately 12 kW (Table 1.2), only a fraction of this energy is directly consumed. If only the residential and a small portion of the transportation sector is considered (see Figure 1.8), an estimate for local consumption of one-quarter of the overall per capita consumption is not unreasonable. Three kilowatts per person results in a localized power density for New York of 21.58 W/m<sup>2</sup>. This is nearly 10% of the incident solar energy and would be expected, by itself, to appreciably modify the local temperature. If the solar balance equation held for this local area, it would predict a temperature rise of nearly 7 K (12 F°). A densely populated area, however, modifies its climate in a number of ways. Particulate matter, carbon dioxide, and water vapor emissions all can affect the thermal energy balance. Buildings and other structures modify air flow patterns. While direct thermal effects can be significant for highly developed and densely populated urban areas, other climatic modifications may be equally significant.

### 3. SENSITIVITY TO CHANGES IN THE REFLECTIVITY AND EMISSIVITY CONSTANTS

Both the reflectivity and emissivity constants affect the earth's temperature. Small changes in either of these constants may have a profound climatic effect.

An expression for temperature (in which the effect of nonsolar energy inputs is ignored) may be obtained from the energy balance of the previous section.

$$T_E = \left[ \frac{(1 - r)S}{4\epsilon\sigma} \right]^{1/4}$$

For a differential change in  $r$  and  $\epsilon$ , a corresponding change in  $T_E$  results.

$$dT_E = \frac{\partial T_E}{\partial r} dr + \frac{\partial T_E}{\partial \epsilon} d\epsilon$$

The partial derivative of the temperature with respect to the reflectivity,  $r$ , will first be evaluated.

$$\frac{\partial T_E}{\partial r} = \frac{1}{4} \left[ \frac{(1 - r)S}{4\epsilon\sigma} \right]^{-3/4} \frac{-S}{4\epsilon\sigma}$$

This expression may be simplified by multiplying the numerator and denominator of the last term by  $(1 - r)$ .

$$\begin{aligned}\frac{\partial T_E}{\partial r} &= -\frac{1}{4} \left[ \frac{(1-r)S}{4\epsilon\sigma} \right]^{-3/4} \frac{(1-r)S}{4\epsilon\sigma} \frac{1}{(1-r)} \\ &= -\frac{1}{4} \left[ \frac{(1-r)S}{4\epsilon\sigma} \right]^{1/4} \frac{1}{(1-r)}\end{aligned}$$

Since a change from the nominal temperature of  $T_{E_0}$  is desired, the derivative should be evaluated for this condition. The quantity to the one-fourth power is equal to  $T_{E_0}$ .

$$\frac{\partial T_E}{\partial r} = -\frac{T_{E_0}}{4(1-r)} \quad (\text{evaluated for } T = T_{E_0})$$

In a like fashion, the second partial derivative may readily be evaluated.

$$\begin{aligned}\frac{\partial T_E}{\partial \epsilon} &= \frac{1}{4} \left[ \frac{(1-r)S}{4\epsilon\sigma} \right]^{-3/4} \left[ \frac{(1-r)S}{4\sigma} \right] \frac{-1}{\epsilon^2} \\ &= -\frac{1}{4} \left[ \frac{(1-r)S}{4\epsilon\sigma} \right]^{1/4} \frac{1}{\epsilon} \\ &= -\frac{T_{E_0}}{4\epsilon} \quad (\text{evaluated for } T = T_{E_0})\end{aligned}$$

The following is thus obtained for the differential change in temperature.

$$dT_E = -\frac{T_{E_0} dr}{4(1-r)} - \frac{T_{E_0} d\epsilon}{4\epsilon}$$

While this was obtained for differentials, the result is approximately valid for small but finite increments. The temperature expression is, after all, well behaved except for  $r$  near one and  $\epsilon$  very small. Neither of these cases is of physical interest.

$$\Delta T_E \simeq -\frac{T_{E_0} \Delta r}{4(1-r)} - \frac{T_{E_0} \Delta \epsilon}{4\epsilon}$$

To gain a perspective as to the sensitivity of the temperature to small changes in the reflectivity or emissivity, the necessary change in each constant that results in a one-degree temperature change will be calculated. First, the reflectivity will be considered.

$$\begin{aligned}\Delta r &\simeq -\frac{4(1-r)}{T_{E_0}} \Delta T_E = \frac{4(1-r)}{T_{E_0}} \\ \Delta r &\simeq -\frac{4(1-.35)}{286} = -.00909\end{aligned}$$

A nominal value for  $r$  of .35 or 35% was used. As indicated by the above result, a change of less than 1% in the reflectivity results in a one-degree temperature change. If the reflectivity is maintained constant, a similar result is obtained for a change in emissivity.

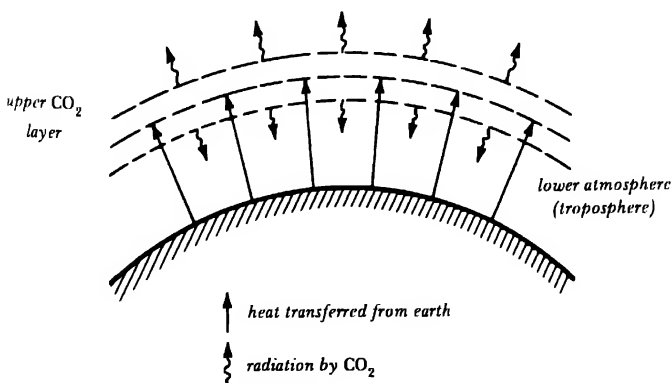
$$\Delta \epsilon \simeq - \frac{4\epsilon}{T_{E_0}} \Delta T = - \frac{4(.580)}{286}$$

$$\simeq - .00836$$

Again, a change of less than .01 for the emissivity can result in a one-degree temperature change. While these results depend upon the particular values of  $\epsilon$  and  $\tau$  used for the solar balance, they may be shown to be rather insensitive to the values of  $\epsilon$  and  $\tau$ . This may be verified by assigning different values to  $\tau$ , calculating the corresponding values of  $\epsilon$ , and then evaluating the resultant changes.

Both reflectivity and emissivity are dependent upon the atmospheric composition. Pollutants can affect both these quantities. The reflectivity depends upon the behavior of the atmosphere for short wavelength radiation, that is, radiation in and near the visible spectrum.

Pollutants, especially particulates, affect visibility and hence the reflectivity of the earth-atmosphere system. Individual particles, depending upon their optical properties, can increase either the absorption or reflection of incident solar energy ( $\Delta \tau$  could be either positive or negative). A decrease in the overall reflectivity, assuming all other conditions remain unchanged, would tend to increase the earth's surface temperature. Emissivity depends upon the behavior of the atmosphere for long wavelength, infrared radiation. The rate at which energy is radiated from the earth is strongly dependent upon the carbon dioxide content of the lower atmosphere since carbon dioxide absorbs infrared radiation. The radiative mechanism, known as the *greenhouse effect*, is illustrated in Figure 3.3. Thermal energy is transferred from the earth to the lower atmosphere, the troposphere, by a variety of processes which include radiation, conduction, convection, and the evaporation and condensation of



**FIGURE 3.3** *The Carbon Dioxide Greenhouse Effect.*

water vapor.\* The energy absorbed by the carbon dioxide of the troposphere is radiated outward to space as well as inward to the earth. The returning infrared radiation thus tends to result in a higher average temperature at the earth's surface than would occur if the atmosphere were devoid of carbon dioxide. An increase in the carbon dioxide content of the atmosphere enhances this warming or greenhouse effect. The change in emissivity,  $\Delta\epsilon$ , is thus negative for this effect. The combustion of fossil fuels results in an unavoidable release of carbon dioxide to the atmosphere (see the next section).

#### 4. CARBON DIOXIDE GENERATION

The combustion of fossil fuels necessarily results in the release of carbon dioxide. While the emission of sulfur and nitrogen oxides can in practice be reduced and in theory eliminated, the same is not true for carbon dioxide. The oxidation of carbon or hydrocarbons is the very process through which the heat energy is obtained from fossil fuels.

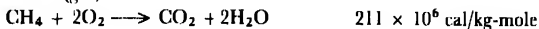
For carbon, the oxidation process is well known.



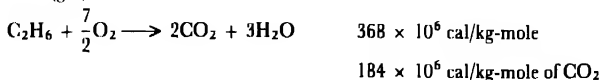
The heat of combustion of each kilogram-mole of carbon, that is, 12 kilograms, is  $94 \times 10^6$  cal. One kilogram-mole or 44 kg of carbon dioxide is also produced. Therefore, each kilogram of carbon has a potential energy content of  $7.83 \times 10^6$  cal. One metric ton of carbon has a heat equivalent of  $7.83 \times 10^9$  cal or  $31 \times 10^6$  Btu ( $252 \text{ cal} = 1 \text{ Btu}$ ). An average heat value of  $28 \times 10^6$  Btu for coal therefore infers a carbon content of 90%. This is an upper limit since a small quantity of heat results from the oxidation of the noncarbon impurities.

The combustion of liquid and gaseous hydrocarbons results in the release of both carbon dioxide and water vapor. Tabulated below are a few of the reactions for constituents of hydrocarbon fuels.

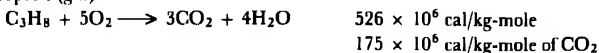
Methane (gas)



Ethane (gas)

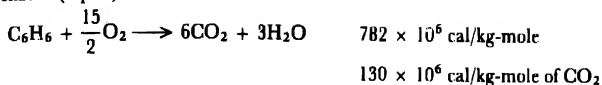


Propane (gas)

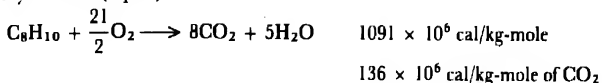


\* Budyko provides an excellent discussion of the effect of these mechanisms on the earth's heat balance [12].

**Benzene (liquid)**



**Ethylbenzene (liquid)**



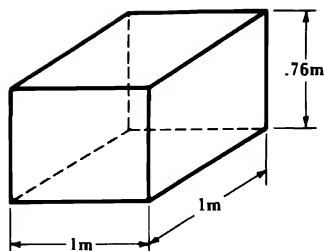
For the above reactions, carbon (coal) results in the greatest quantity of carbon dioxide for each unit of energy derived.

Knowing the composition of each fuel and the rate at which it is consumed, the emission of carbon dioxide may be determined. Rather than trace through the statistical data necessary for such a calculation, an estimate of carbon dioxide emissions will be obtained using the total energy consumption rate ( $8.1 \times 10^{12}$  W for 1978). If energy were produced wholly from the combustion of coal, the heat of combustion for carbon could be used. One mole of carbon dioxide would thus be produced for each  $94 \times 10^6$  cal of heat energy. This will be assumed to obtain an estimate of carbon dioxide emission. Comparing the heat of combustion for each mole of carbon dioxide produced by the hydrocarbons listed, the result of the simplified calculation can be seen to be within a factor of two of that which a more precise calculation would yield.

Dividing the energy per mole of carbon dioxide by the mass of a mole, 44 kg, the energy for each kilogram of carbon dioxide is obtained as  $2.14 \times 10^6$  cal or  $8.94 \times 10^6$  J. For a power of  $8.1 \times 10^{12}$  W(J/s),  $9.06 \times 10^5$  kg/s of carbon dioxide will be generated. On a yearly basis, this results in  $2.9 \times 10^{13}$  kg or  $2.9 \times 10^{10}$  metric tons. A more detailed calculation taking into account the carbon dioxide yields for hydrocarbon fuels results in a somewhat smaller yearly quantity of carbon dioxide (Problem 10).

Carbon dioxide is stored in both the atmosphere and the oceans. While it is soluble in water, the bulk of that stored in the oceans is in the form of carbonates. The present atmospheric concentration of carbon dioxide is approximately .033% or 330 ppm by volume. Since the atomic weight of carbon dioxide is 44 and that of air is approximately 29, the atmosphere is .050%  $\text{CO}_2$  by mass. Standard atmospheric pressure of sea level is .76 mHg, that is, the force on each unit area of the earth's surface is equal to the weight of a mercury column .76 meter high with a cross section of one unit area.

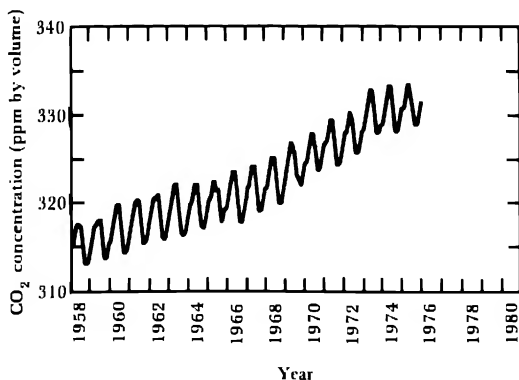
For a one-square meter area, the mass of the mercury may readily be calculated. The volume of the column of Figure 3.4 is  $.76\text{m}^3$ . Since one kilogram of water occupies a volume of one liter or  $10^{-3}\text{m}^3$ , the column would contain a mass of 760 kg if filled with water. Therefore, the mass of mercury (specific gravity of 13.6) is 10,336 kg. Of this rather impressively large mass of atmosphere for each square meter of the earth's surface, 5.18 kg is carbon dioxide. This results, when multiplied by



**FIGURE 3.4** *Mercury Column with a Mass Equal to That of the Atmosphere for a One-Square Meter Surface Area.*

the surface area of the earth ( $5.11 \times 10^{14} \text{ m}^2$ ), in  $2.65 \times 10^{15} \text{ kg}$  or  $2.65 \times 10^{12}$  metric tons of carbon dioxide. The  $2.9 \times 10^{10}$  metric tons of carbon dioxide calculated on the basis of the oxidation of carbon is greater than 1% of the atmospheric carbon dioxide. A more detailed calculation yields a yearly quantity somewhat less than 1%. If the world trend of increased usage of fossil fuels continues, the carbon dioxide produced yearly by combustion will exceed 1% of that contained in the atmosphere. If the combustion-produced carbon dioxide remained in the atmosphere, the atmospheric concentration would increase by about 1% per year.

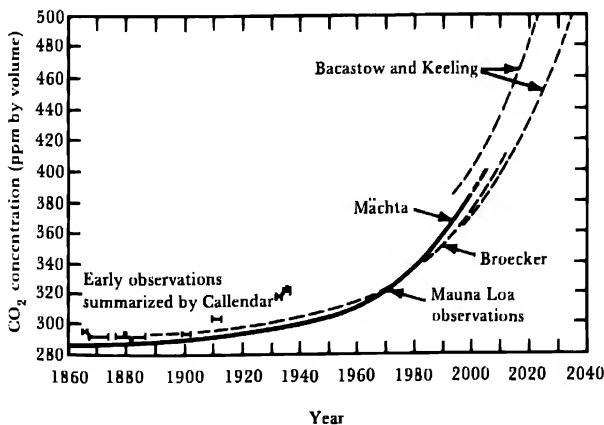
As shown by Figure 3.5, atmospheric carbon dioxide has been steadily increasing although at a slower rate than that which would occur if all combustion-produced carbon dioxide remained in the atmosphere [13]. The rate of increase corresponds to approximately half of the combustion-produced carbon dioxide. Natural processes



**FIGURE 3.5** *Atmospheric Carbon Dioxide Concentration at Mauna Loa, Hawaii (Reference 13).*

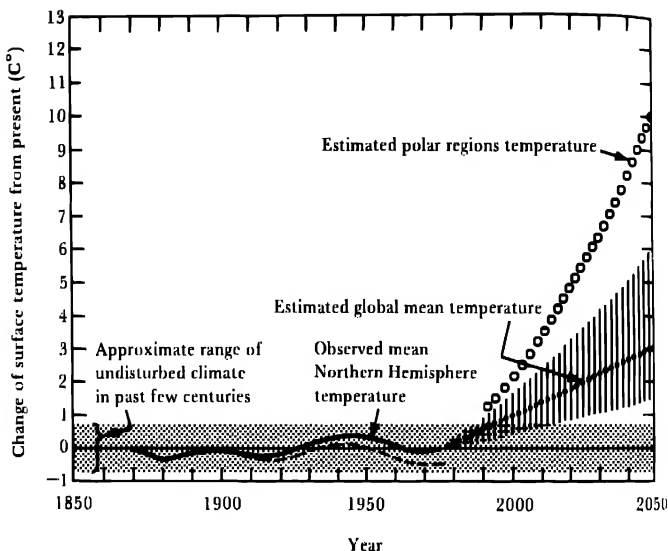
result in continual exchange of atmospheric carbon dioxide with living materials and with the oceans of the earth, a mechanism frequently referred to as the *carbon cycle*. Since an increase in the carbon dioxide concentration could result in a significant global climatic change, considerable effort has been directed toward gaining a better understanding of the effect of added releases of carbon dioxide on the earth's carbon cycle [14–18]. The rates at which carbon dioxide is absorbed by the surface layers of the ocean and by living materials appear insufficient to account for the "missing" portion of the combustion-produced carbon dioxide. Furthermore, other human activities, the clearing of land for agriculture and the burning of forest materials or allowing the cut materials to decay, also affect the atmospheric concentrations of carbon dioxide.

If approximately half of the combustion-produced carbon dioxide continues to remain in the atmosphere, the concentration of atmospheric carbon dioxide will continue to increase even if the rate of energy usage in the world remains unchanged. Increased energy usage rates, however, will accelerate the buildup. Figure 3.6 illustrates the possible effect of a 4 to 5%/y growth in the use of fossil fuels [19]. A shift from hydrocarbon fuels to coal as hydrocarbon reserves are depleted further increases the rate at which carbon dioxide is released. The combustion of coal, primarily carbon, results in 124% more carbon dioxide released per unit of thermal energy than the use of natural gas, primarily methane. Coal also produces more carbon dioxide than do the other hydrocarbons. Furthermore, if liquid or gaseous fuels are synthesized from coal, additional carbon dioxide is released because of the energy inefficiencies of various synthesizing processes.



**FIGURE 3.6** *Atmospheric Carbon Dioxide Concentration from 1860 to 1975 and Estimates of Future Trends. Compiled by Kellogg (Reference 19).*





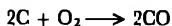
**FIGURE 3.7** *The Mean Surface Temperature Record for the Northern Hemisphere from 1850 to the Present and the Course It Might Have Taken Without the Addition of Combustion-Produced Carbon Dioxide (dashed line). The Estimates of Future Trends in Surface Temperature Are of Kellogg (Reference 19).*

Climatic changes considerably greater than the short-term fluctuations (100-year time scale) that have occurred before human activities had a significant global impact are now likely [19-31]. Not only will increased atmospheric levels of carbon dioxide result in a gradual warming of the earth but the effects of other activities are also expected to accelerate this process (Figure 3.7) [19]. While a warming could eventually melt the polar ice caps, the significant effect in the next couple of centuries could be the modification of precipitation patterns. Added rainfall in presently arid regions could result in new agricultural areas while other currently agriculturally productive regions may become too dry to sustain their present yields.

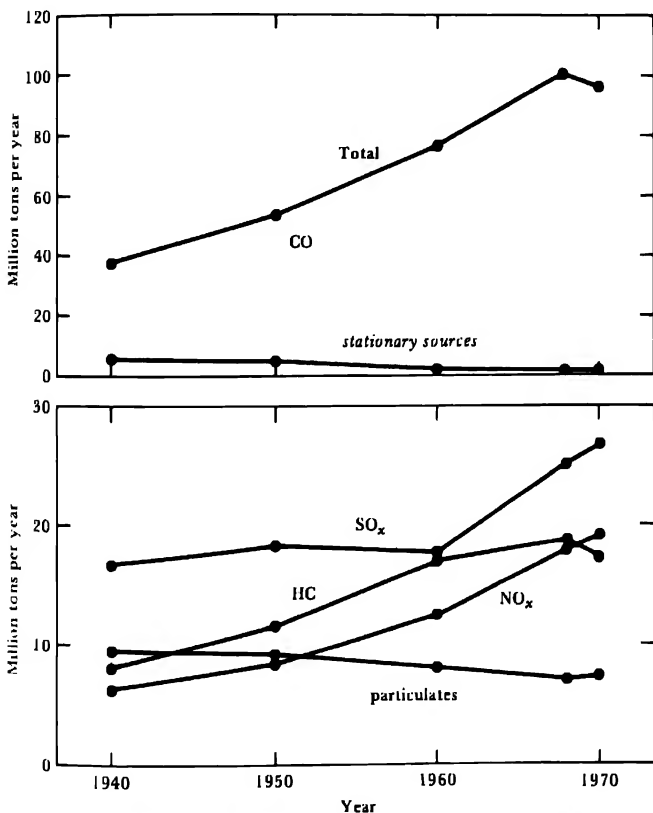
Long-term climatic changes could be triggered by present energy usage patterns. The level of fossil fuel usage which results in an "acceptable" climatic change is not known. Furthermore, since the time constant for changes may be several decades, irreversible processes may be triggered before their full effects are known.

## 5. ATMOSPHERIC POLLUTION

Fossil fuel combustion, in addition to producing carbon dioxide, results in the emission of numerous undesirable and biologically harmful compounds. Incomplete combustion of fuels yields carbon monoxide.



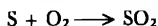
The above reaction for coal is also an energy inefficient process, since its heat of combustion is less than 30% of that which is obtained for complete combustion.



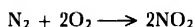
**FIGURE 3.8** U.S. Pollutant Emissions of Energy Sources, 1940 to 1970 (Reference 33).

Incomplete combustion of hydrocarbons (gas and liquids) also results in excessive quantities of carbon monoxide. Incomplete combustion of petroleum, as is frequently the case for internal combustion engines, results in the emission of a variety of complex hydrocarbon compounds.

Fuel impurities also contribute to undesirable emissions. Sulfur, an impurity found both in coal and petroleum, reacts with oxygen at high temperatures producing oxides of sulfur such as the sulfur dioxide of the following reaction.



Noncombustible constituents may result in the emission of undesirable particulate matter such as the smoke emission frequently associated with the combustion of coal. Furthermore, since air (79% nitrogen) is the oxygen source for combustion, the formation and subsequent emission of nitrogen oxides, especially at elevated combustion temperatures, is a common occurrence.



The internal combustion engine is the dominant source of nitrogen oxides.

A substantial portion of emissions, as indicated by the data of Table 3.1, is the result of energy-related fuel combustion [32]. Energy conversion processes (transportation and stationary fuel combustion) account for 85% of the carbon monoxide, 48% of the particulates, 85% of the sulfur dioxide, 46% of the hydrocarbon and 96% of the nitrogen oxide emissions. Emissions associated with energy usage thus significantly affect air quality. Based upon energy and natural resource usage, a reasonable estimate for total planetary emissions would be three times the amounts of Table 3.1. Since the dispersal of air pollution is unaffected by national boundaries, the emission of pollutants often results in global problems and concerns.

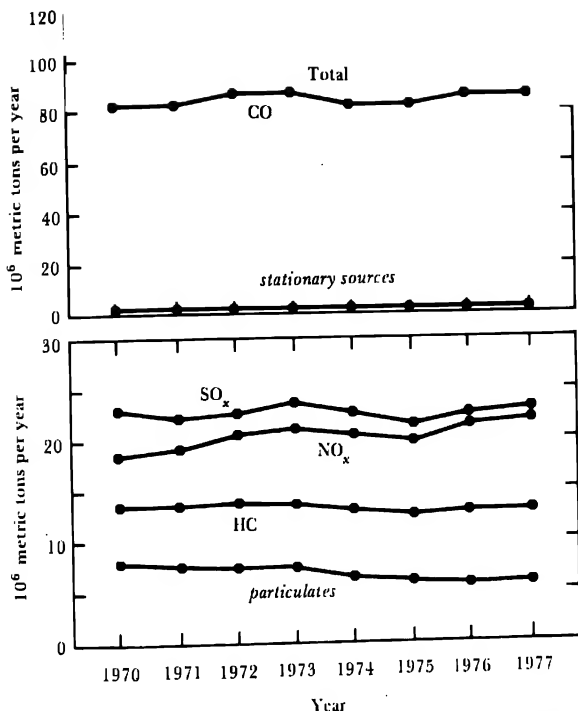
Prior to 1970, estimated total emissions by the United States tended to increase steadily from one year to the next (Figure 3.8). Although transportation accounts for only one-quarter of the total U.S. energy consumption, most of the carbon monoxide, hydrocarbon, and nitrogen oxide emissions are due to it.

**TABLE 3.1** *Estimated Emissions of Air Pollutants by Mass for the United States in 1977*

Source	(In million metric tons per year)				
	CO	Particulates	SO <sub>x</sub>	HC*	NO <sub>x</sub>
Transportation	85.7	1.1	.8	11.5	9.2
Fuel combustion in stationary sources	1.2	4.8	22.4	1.5	13.0
Industrial processes	8.3	5.4	4.2	10.1	.7
Solid waste disposal	2.6	.4	—	.7	.1
Miscellaneous	4.6	.7	—	4.5	.1
<b>TOTAL</b>	<b>102.4</b>	<b>12.4</b>	<b>27.4</b>	<b>28.3</b>	<b>23.1</b>

\*Volatile organic compounds.

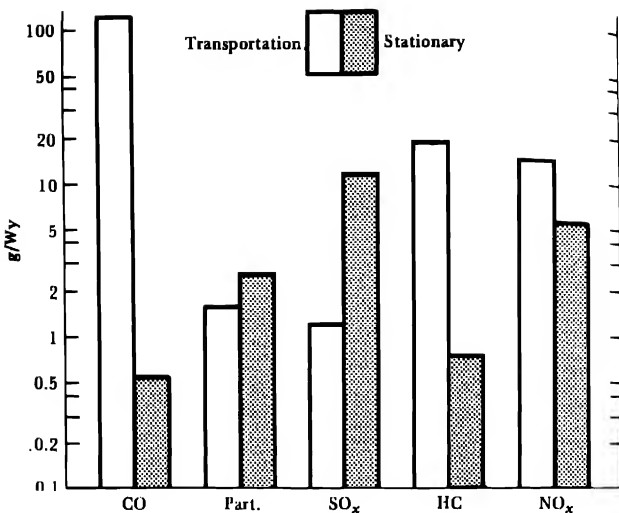
SOURCE: Data from Reference 32.



**FIGURE 3.9** *U.S. Pollutant Emissions from Energy Sources, 1970 to 1977. Data of This Graph Should Not Be Compared to That of Figure 3.8 Because Different Statistical Methods Were Employed (Reference 32).*

As indicated by the graphs of Figure 3.9, emission rates have changed little from 1970 to 1977: NO<sub>x</sub> emissions increased slightly and particulate emissions declined somewhat. Since overall energy consumption increased by 14.3% over this period, emissions per unit of energy have tended to decline. The decline may be attributed to the emission restrictions adopted as a result of the Clean Air Act of 1970 (PL 91-604). As more stringent restrictions are adopted, further declines are anticipated.

The data of Figure 3.10 provide a measure of the average pollutant emissions of stationary and mobile sources per unit of energy consumption. To obtain this data, the total 1977 energy consumption was allocated between transportation (25.9%) and stationary sources (74.1%). While only petroleum is used for transportation, all three fossil fuels are used in stationary sources. Since natural gas



**FIGURE 3.10 1977 Average Emissions per Unit of Energy.**

combustion tends to produce very few pollutants, coal and petroleum combustion emissions per unit of energy are considerably higher than the average values of Figure 3.10.

Not all pollutants are equally harmful, that is, an index based simply upon the sum of all emissions (so many total tons per year) would have limited meaning. Based upon the Environmental Protection Agency standards presented in Table 3.2 [34], the harmful effect of carbon monoxide for a specified mass density is considerably less than that of other pollutants. While averaging times for the individual pollutants differ, mass concentrations for carbon monoxide 30 to 40 times greater than the harmful quantities of other pollutants are tolerable. On a mass basis, the harmful effects of particulates, sulfur dioxide, hydrocarbons, and nitrogen dioxide are comparable. As a consequence, air quality indices which include the effect of a mixture of pollutants need to be based upon a weighted average of the pollutants.

The background level of carbon monoxide ("clean" air) is estimated to be .01 to .2 mg/m<sup>3</sup> (.01 to .2 ppm) [35]. Emissions due to natural sources are relatively insignificant. While atmospheric removal processes are not well understood, the removal rate has been sufficient to prevent an accumulation of carbon monoxide.

Inhaled carbon monoxide combines with the hemoglobin of the circulating blood in the same manner as oxygen. The affinity of hemoglobin for carbon monoxide, however, is over 200 times its affinity for oxygen. Therefore, relatively small concentrations of carbon monoxide can result in significant quantities of carboxyhemoglobin, COHb. Each addition 7 mg/m<sup>3</sup> of atmospheric carbon

**TABLE 3.2 National Ambient Air Quality Standards (NAAQS)**

<i>Pollutant</i>	<i>Averaging time</i>	<i>Primary standard levels</i>	<i>Secondary standard levels</i>
Particulate matter	Annual (geometric mean) 24 h <sup>b</sup>	75 $\mu\text{g}/\text{m}^3$ 260 $\mu\text{g}/\text{m}^3$	60 $\mu\text{g}/\text{m}^3$ 150 $\mu\text{g}/\text{m}^3$
Sulfur oxides	Annual (arithmetic mean)  24 h <sup>b</sup>  3 h <sup>b</sup>	80 $\mu\text{g}/\text{m}^3$ (.03 ppm) 365 $\mu\text{g}/\text{m}^3$ (.14 ppm)	— — 1300 $\mu\text{g}/\text{m}^3$ (.5 ppm)
Carbon monoxide	8 h <sup>b</sup>  1 h <sup>b</sup>	10 $\text{mg}/\text{m}^3$ (9 ppm) 40 $\text{mg}/\text{m}^3$ (35 ppm)	10 $\text{mg}/\text{m}^3$ (9 ppm) 40 $\text{mg}/\text{m}^3$ (35 ppm)
Nitrogen dioxide	Annual (arithmetic mean)	100 $\mu\text{g}/\text{m}^3$ (.05 ppm)	100 $\mu\text{g}/\text{m}^3$ (.05 ppm)
Ozone	1 h <sup>b</sup>	235 $\mu\text{g}/\text{m}^3$ (.12 ppm)	235 $\mu\text{g}/\text{m}^3$ (.12 ppm)
Hydrocarbons (nonmethane) <sup>a</sup>	3 h (6 to 9 a.m.)	160 $\mu\text{g}/\text{m}^3$ (.24 ppm)	160 $\mu\text{g}/\text{m}^3$ (.24 ppm)

<sup>a</sup>A nonhealth related standard used as a guide for ozone control.

<sup>b</sup>Not to be exceeded more than once per year.

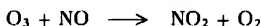
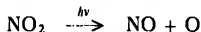
SOURCE: Reference 34.

monoxide will, as a result of an exposure of approximately 12 h, increase the carboxyhemoglobin blood level by 1% (the normal level for nonsmokers is .5%). No human health effects have been observed for carboxyhemoglobin blood levels of less than 1%. At a level of 2.5% some impairment in time-interval discrimination is observed, at 3% the visual acuity and relative brightness threshold can be affected, and at 5% a lower performance in psychomotor tests can result. Blood carboxyhemoglobin concentrations greater than 5% place an extra burden on heart disease patients. High carboxyhemoglobin blood concentrations result either from relatively long (12 h) exposures to low levels of carbon monoxide or short exposures to very high levels. The highest levels, not surprisingly, occur in the vicinity of heavily utilized freeways.

Unpolluted ("clean") air has rarely less than several hundred particles of matter for each cubic centimeter, while concentrations in very polluted air may reach  $10^5/\text{cm}^3$  [36]. The smallest particles may be as minute as molecules whereas the largest may have diameters of 500  $\mu$  ( $500 \times 10^{-6}$  m). Particles resulting from combustion normally have diameters less than one micron. Particulate matter, especially those with sizes comparable to the wavelength of light, tend to scatter sunlight, thus reducing the visible radiation. Particulates reduce average visible

illumination of some cities by one-third or more. An atmospheric concentration of  $100 \mu\text{g}/\text{m}^3$ , a quantity only one-third greater than the primary particulate standard, reduces sunlight by 5%. The toxicity of particulates for humans depends upon the particle's composition and size. The greatest penetration of surfaces within the respiratory system is the result of small (less than one micron) particles. Smoke is a particularly toxic form of particulates, since it is usually accompanied by high levels of sulfur oxides. A synergistic effect between the sulfur oxides and the particulates results in a considerably greater toxicity than the combined effect of either pollutant acting alone [37]. Particulates that result in the conversion of sulfur dioxide to sulfuric acid are especially irritating. Because of this synergistic reaction, considerations of the biological effects of either pollutant individually have little meaning. For concentrations of  $715 \mu\text{g}/\text{m}^3$  (25 ppm) of sulfur dioxide accompanied by smoke with a particulate concentration of  $750 \mu\text{g}/\text{m}^3$ , an increased daily death rate may occur, with lower combined concentrations possibly resulting in increased illness and absenteeism ratio and in additional respiratory diseases. At concentrations as low as  $115 \mu\text{g}/\text{m}^3$  (0.4 ppm) of carbon dioxide and  $160 \mu\text{g}/\text{m}^3$  of smoke, an increase in mortality from bronchitis and lung cancer may occur.

Another synergistic effect occurs between nitrogen oxides and hydrocarbons—the constituents responsible for photochemical smog. The reactions that result in smog are complicated and not fully understood [38–42]. Nitrogen dioxide,  $\text{NO}_2$ , absorbs ultraviolet light and forms nitric oxide,  $\text{NO}$ , and atomic oxygen,  $\text{O}$ . Atomic oxygen combines with molecular oxygen,  $\text{O}_2$ , forming ozone,  $\text{O}_3$ , which in turn reacts with nitric oxide, converting it back to nitrogen dioxide.



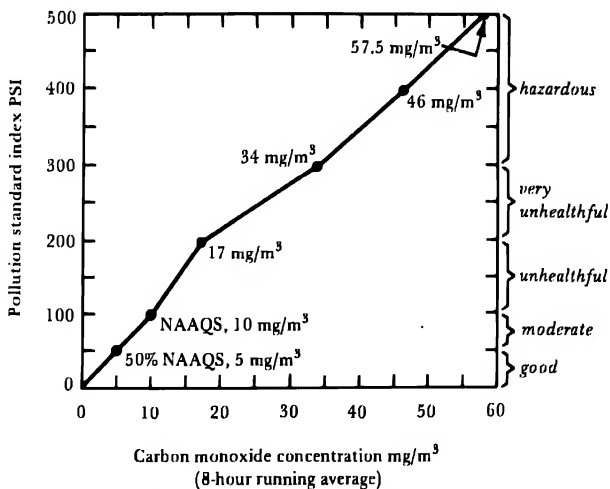
An intermediate product of this reaction, atomic oxygen, reacts with hydrocarbons forming a series of complex hydrocarbon compounds, including formaldehyde, ketones, and peroxyacyl nitrates (PAN). Sulfur dioxide, if present, can also interact with these compounds, forming additional toxic pollutants. The intensity of these effects is primarily dependent upon the initial hydrocarbon–nitrogen dioxide ratio. A significant reduction in nitrogen dioxide emissions can therefore substantially reduce the formation of subsequent products that constitute photochemical smog. Ozone provides a measure of photochemical activity of smog, since it is an intermediate product of the  $\text{NO}_2$ – $\text{NO}$  reaction and hydrocarbon reactions. It is the major component of photochemically produced oxidants. Concentrations as small as  $200 \mu\text{g}/\text{m}^3$  (1 ppm) result in eye irritation while concentrations one-half this value can damage sensitive species of vegetation.

The health effects due to a combination of air pollutants are difficult to isolate from effects due to nonrelated causes. A significant correlation between health effects and air pollution, nevertheless, has been established. Respiratory diseases, not surprisingly, are most closely associated with high levels of air pollution. Further-

more, Lave and Seskin [43] suggest that the average pollution levels may be considerably more significant than occasional peak levels.

A Pollutant Standard Index (PSI) based upon National Ambient Air Quality Standards (Table 3.2) is now used throughout the United States for reporting and tabulating pollutant levels. The index for a particular pollutant is defined as 100 when the pollutant's concentration is equal to the national standard. Other values for the index (usually referred to as PSI values) are indicated in Table 3.3 [34]. A dimensionless index with a range of 0 to 500 is used to indicate the harmfulness of the five "criteria pollutants" of Table 3.3. The PSI definition implies that an index of 200 for each of the pollutants is equally harmful. A straight-line interpolation, as indicated for carbon monoxide in Figure 3.11, is used for ascertaining PSI values for pollutant levels that fall between those indicated in Table 3.3 [44].

A complete specification of the pollutant levels for the five pollutants requires five PSI values. Often only one pollutant has a particularly high concentration and only the highest PSI value is widely reported (by the mass media) along with a mention of the particular pollutant that gives rise to the index value. A specification of a yearly air quality status for a particular pollutant is usually based upon an accounting of the number of days that the PSI value falls within specified ranges considered harmful. Not only may the year-to-year trend for a particular city be specified in this manner, but, since the index is used throughout the nation, the air quality of different cities may be compared.



**FIGURE 3.11** *The Relationship of the Pollution Standard Index (PSI) and the Concentration of Carbon Monoxide (Reference 44).*



**TABLE 3.3 Comparison of PSI Values with Pollutant Concentrations, Descriptor Words, Generalized Health Effects, and Cautionary Statements**

Index value	Air quality level	Pollutant Levels					Health effect descriptor	General health effects	Cautionary statements
		TSP (24 h) $\mu\text{g}/\text{m}^3$	SO <sub>2</sub> (24 h) $\mu\text{g}/\text{m}^3$	CO (8 h) $\text{mg}/\text{m}^3$	O <sub>3</sub> (1 h) $\mu\text{g}/\text{m}^3$	NO <sub>2</sub> (1 h) $\mu\text{g}/\text{m}^3$			
500	Significant harm	1000	2620	57.5	1200	3750		Premature death of ill and elderly. Healthy people will experience adverse symptoms that affect their normal activity.	All persons should remain indoors, keeping windows and doors closed. All persons should minimize physical exertion and avoid traffic.
400	Emergency	875	2100	46.0	1000	3000	HAZARDOUS	Premature onset of certain diseases in addition to significant aggravation of symptoms and decreased exercise tolerance in healthy persons.	Elderly and persons with existing diseases should stay indoors and avoid physical exertion. General population should avoid outdoor activity.
300	Warning	625	1600	34.0	800	2260	VERY UNHEALTHFUL	Significant aggravation of symptoms and decreased exercise tolerance in persons with heart or lung disease, with widespread symptoms in the healthy population.	Elderly and persons with existing heart or lung disease should stay indoors and reduce physical activity.

		↓						
200	Alert	375	800	17.0	400	1130		
							↑	Mild aggravation of symptoms in susceptible persons, with irritation symptoms in healthy population.
100	NAAQS	260	365	10.0	240	—	↓	Persons with existing heart or respiratory ailments should reduce physical exertion and outdoor activity.
							UNHEALTHFUL	
50	50% of NAAQS	75	80	5.0	120	—	MODERATE	
0		0	0	0	0	0	GOOD	

Source: Reference 34.

The cost of air pollution is, since its effects are poorly understood, difficult to estimate. In addition to health costs, air pollution may result in damage to vegetation, corrosion and deterioration of materials, and a decline in property values. A text by Ridker [45] provides a wealth of detail on the methodology of assessing economic costs, whereas that by Stern et al. [46] presents a very good general treatment of air pollution. Controlling the emission of pollutants arising from energy conversion processes and achieving the air standards of Table 3.2 is a new technological challenge.

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## PROBLEMS

1. Using the solar constant, determine the total rate at which energy is generated by the sun. Assume the sun generates energy primarily by a proton-proton reaction which on the average yields an energy of 6.675 MeV per proton. Using the mass of the sun,  $1.97 \times 10^{30}$  kg, determine the life expectancy of the sun. that is, the time necessary for the entire mass to undergo a fusion reaction.
2. The solar constant,  $S$ , is specified for the earth's distance from the sun. Since the power density is inversely proportional to distance squared, the radiation density at any point in the solar system may readily be calculated. Determine the radiation power density at a distance corresponding to the orbit of Mars. If the earth were in this orbit, what would be its temperature. assuming  $r$  and  $\epsilon$  remain the same? Determine the value of  $\epsilon$  that would restore the earth to its present temperature of 286 K (13°C) for this orbit. ( $R_{\text{Sun-Mars}} = 2.28 \times 10^{11}$  m).
3. Mercury, the planet closest to the sun ( $R_{\text{Sun-Mercury}} = 5.8 \times 10^{10}$  m), has an average surface temperature of 633 K.

- (a) Determine the sun's radiation power density at the orbit of Mercury.
  - (b) The relative emissivity,  $\epsilon$ , may readily be shown to be linearly related to a function of the reflectivity,  $1 - \tau$ . Obtain a numerical value for the proportionality constant for Mercury.
  - (c) The reflectivity of Mercury is approximately .08. What is its relative emissivity? What is the lowest possible equilibrium temperature that the planet could have for a reflectivity of .08?
4. Repeat Problem 3 for Uranus. It is  $2.87 \times 10^{12}$  m from the sun and has an average temperature of 90 K. Its reflectivity is .65. What would its relative emissivity need to be, for this reflectivity, to result in an average temperature of 250 K?
5. It has been proposed that a satellite be used to collect radiant energy from the sun (Reference 2). Photovoltaic cells would be used to convert the sun's radiation to electrical energy and a microwave system would serve to transmit the energy to an earth-based receiving antenna. A satellite in a synchronous orbit outside the earth's atmosphere would supply nearly uninterrupted power to the antenna on the earth. Assume that the photovoltaic cells are 12% efficient and that the cells occupy 75% of the overall collection surface. A 50% efficiency of transmitting the output of the photovoltaic cells to earth is expected.
- (a) Determine the maximum electrical power that can be obtained from the earth-based antenna for each square meter of collector surface. What is the yearly energy output expressed in kilowatt-hours?
  - (b) The output of the photovoltaic cells is expected to decline by 1.2%/y. Determine the total electrical energy (per square meter of collector) that will be obtained over a period of 30 years, the design life of the satellite. Assume there is no change in the performance of other components of the system.
  - (c) What is the total collection area required for a power output of 5 GW at the earth?
  - (d) Suppose the collector cost is to be no more than \$1000 per kilowatt of electrical power (at the earth). Determine the maximum allowable cost (per square meter) for the collector.
6. The area of the continental United States is approximately 3 million square miles. Assume that all the power produced by the United States ( $2.6 \times 10^{12}$  W) is dissipated in this area.
- (a) What is the power density in watts per square meter associated with fuel consumption for the United States?
  - (b) What is the value of  $\delta$ , the ratio of the above quantity to the average solar input?
  - (c) What would be the local temperature increase predicted by the solar energy balance relation?
  - (d) Suppose the total U.S. energy consumption was dissipated over only 10% of the continental area. What is the corresponding temperature increase predicted by the solar energy balance relation?

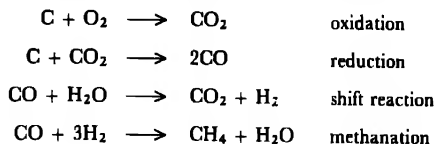
7. The solar energy balance as derived for the world was used in the above problem and in the text to estimate the temperature effect associated with energy consumption in a localized area. Comment on the validity of such calculations. What factors tend to make such a calculation suspect? What other factors related to energy consumption influence local temperature?
8. Several nations, owing to their relatively small land areas, have high average rates of energy usage per unit area (watts per square meter). Using data from the United Nation's Statistical Yearbook and a world almanac, determine the average power density, in watts per square meter, for the following nations:

The Netherlands  
Luxembourg  
Belgium  
West Germany  
United Kingdom

What is the ratio of the power densities to that of the average rate at which solar radiation is absorbed by the earth-atmosphere system? What is the power density for urban areas of the above countries in which energy usage rates (per unit area) are five times the national average?

9. An estimate of the quantity of carbon dioxide released through the combustion of petroleum products is desired. Some of the more complex hydrocarbon molecules are indicated in Figures 2.21 and 2.22. Pick a sample of ten molecules with boiling points above  $20^{\circ}\text{C}$  to "represent" petroleum products. Determine the energy of combustion per kilogram, and the quantity of carbon dioxide per unit of energy (GJ) for the combustion of each hydrocarbon selected. What is, based upon the sample chosen, the average quantity of carbon dioxide per unit of energy (kilograms per gigajoule and kilograms per watt-year)? (See the *Handbook of Physics and Chemistry* for combustion energies.)
10. The total energy consumption rate for the world was  $8.1 \times 10^{12}$  W in 1978. Coal accounted for 32% of the total, natural gas 20%, petroleum 45%, and the remainder was from nuclear and hydropower. Assume that the energy obtained from coal is primarily due to the oxidation of carbon and that from natural gas is due to the oxidation of methane. An average energy of combustion rate, per kilogram mole of carbon dioxide, of  $140 \times 10^6$  cal may be assumed for petroleum.
  - (a) What was, for the above assumptions, the quantity of carbon dioxide released to the atmosphere in 1978?
  - (b) What would have been the increase in the atmospheric carbon dioxide concentration (parts per million) if 50% of the combustion-produced carbon dioxide remained in the atmosphere? What would it have been if 100% remained?
  - (c) Assume that the total energy consumption rate increases by 20%. Determine the increase in the rate of carbon dioxide released if the energy increase is entirely due to an increase in coal consumption.
  - (d) Repeat part (c) for natural gas and for petroleum.

11. The gasification of coal depends upon the following four reactions (Chapter 2):



- (a) Obtain a single composite reaction for the coal gasification process. What is the portion of the original carbon converted to methane?
  - (b) What is the energy efficiency of this process, that is, the heating value of the methane produced relative to that of the carbon utilized?
  - (c) A quantity of synthesized methane is to be used in place of natural gas. Compare the total quantity of carbon dioxide released to the atmosphere for the synthesized methane to that released by the combustion of natural gas.
  - (d) Suppose that methane synthesized from coal is substituted for the natural gas consumed in the world (1978). Utilizing the results of Problem 10, determine the percentage increase in carbon dioxide emissions for this substitution. What would be the increase if the combustion of coal is substituted directly for natural gas?
12. The 1978 world energy consumption rate of  $8.1 \times 10^{12}$  W was due primarily to the combustion of fossil fuels. Based upon the data of Figure 3.5, the atmospheric concentration of carbon dioxide in 1978 was increasing at a yearly rate of about 2.0 ppm. Assume that the increase in carbon dioxide is due entirely to energy usage, that the 1978 rate of increase per unit of average power prevails, and that the rate of energy usage has increased, and continues to increase, at a yearly rate of 4%.
- (a) What was the total energy usage prior to 1978 for a 4%/y growth rate? What was the total carbon dioxide produced (in parts per million)? What might be considered the "natural" carbon dioxide concentration,  $q_0$ , of the atmosphere if it is assumed that all combustion-produced carbon dioxide remains in the atmosphere?
  - (b) When will, based upon the above assumptions, the atmospheric concentration of carbon dioxide reach 500 ppm?
  - (c) Suppose carbon dioxide is removed from the atmosphere by some natural mechanism and that the rate of removal is proportional to the excess carbon dioxide concentration. If the release of carbon dioxide is proportional to the rate of energy usage,  $P$ , the following differential equation for the concentration,  $q$ , results.

$$\frac{dq}{dt} = bP - \frac{1}{\tau}(q - q_0)$$

$$P = P_0 e^{at} \text{ W}$$



$b = \text{ppm/y per unit of power}$

$\tau = \text{time constant for CO}_2 \text{ removal y}$

Assuming exponential behavior for energy usage and assuming that  $\tau$  is 25 y, obtain an expression for  $q$ . What is the value of  $b$  that results in the same value of  $q_0$  obtained in part (a) ( $t \rightarrow -\infty$ )?

- (d) What is the rate of increase, in parts per million, predicted by the expression for 1978?
  - (e) Using the result of part (c), determine when the atmospheric concentration of carbon dioxide will reach 500 ppm.
13. An estimate of the total emissions of carbon monoxide is given in Figure 3.9. These emissions are almost entirely from transportation vehicles.
- (a) What were the yearly and daily per capita carbon monoxide emissions for 1977 (population of  $220 \times 10^6$ )?
  - (b) The population density of Los Angeles, an area where pollutants are frequently trapped by an atmospheric inversion layer, is about  $6000/\text{mi}^2$ . What are the daily emissions, based upon the national per capita average, per square kilometer and square meter? Suppose one day's emissions are uniformly dispersed up to a height  $h$ . Determine the value of  $h$  if the emissions result in a concentration of carbon monoxide equal to the primary standard level (8 h).
  - (c) Suppose per capita vehicle usage in Los Angeles is 50% greater than the national average and that the inversion height is 500 m. What is the average carbon monoxide concentration if one day's emissions are trapped? What is the pollution standard index for this condition? What is the ground level index if the ground level concentration is four times the average concentration?
14. The 1979 rate of energy usage for transportation in the United States was  $.66 \times 10^{12}$  W. Assume that the 1977 emissions per unit of energy, of Figure 3.10, are valid for 1979.
- (a) What was the area power density, watts per square meter, for transportation (area of  $3 \times 10^6 \text{ mi}^2$ )? What was it for an urban area with a population density 100 times the national average (energy usage also 100 times average)?
  - (b) What were the yearly and daily transportation-related emissions for the five pollutants of Figure 3.10?
  - (c) Suppose the vertical distribution of the pollutant concentration,  $x$ , had the following function of height,  $y$ .

$$x = x_0 e^{-y^2/h^2}$$

Determine, for  $h = 300$  m, the ground level pollutant concentration for one day's emissions of the urban area accumulating in the atmosphere. Consider each pollutant.

- (d) What was, for the concentrations of part (c), the ground level pollutant standard index for each of the pollutants?

15. A particular automobile has a gasoline mileage economy of 20 mpg (125,000 Btu/gal).
- (a) Using the average emission data of Figure 3.10, what are the emissions of carbon monoxide, hydrocarbons, and nitrogen dioxide expressed in gallons per mile for the automobile?
- (b) Compare the emissions with the following limits:

CO	3.4 gal/mi
HC	0.41 gal/mi
NO <sub>x</sub>	0.4 gal/mi

By what percentage would each emission have to be reduced to achieve the above limits?

- (c) Repeat parts (a) and (b) for a vehicle achieving a fuel economy of 40 mpg.

## CHAPTER 4

# *Thermodynamic Limitations*

### 1. THE FIRST AND SECOND LAWS OF THERMODYNAMICS

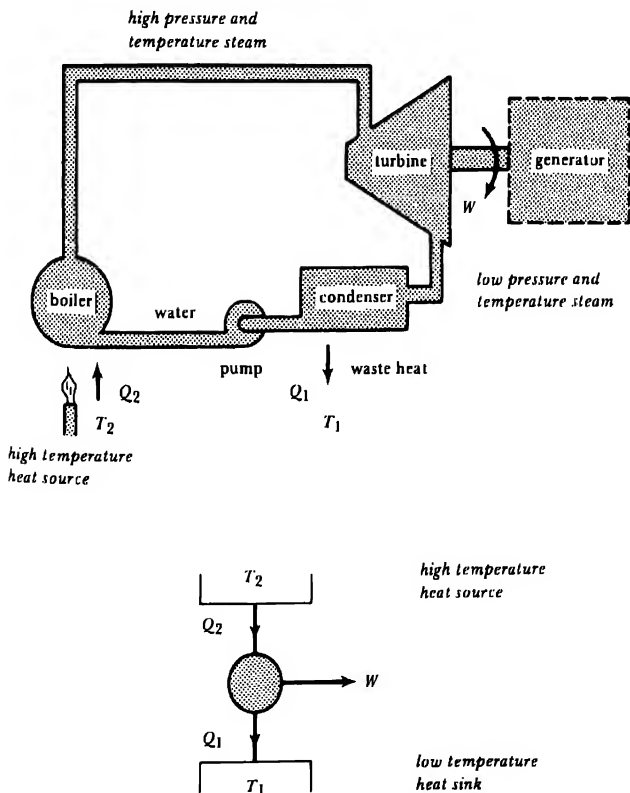
There is a slogan in George Orwell's *Animal Farm*: "All animals are equal but some animals are more equal than others" [1]. In a like manner, all energy is equal but some forms are more equal than others. Reduced to pragmatic terms, certain forms of energy are more useful than others. The energy consumption discussed in the first chapter can be classified into two broad general areas: energy used directly for heat and energy used to do work. Fuels used for residential heating and for industrial processes such as iron ore reduction fall into the first category. The gasoline engine of the automobile and the steam turbine of an electric power plant convert heat energy to mechanical work.

While energy is conserved, the conversion of heat to more useful forms (such as mechanical work) requires an expenditure of heat energy in excess of the work output. The excess, unfortunately, appears in the form of "waste." The waste heat generated by heat engines is an inevitable result of the thermodynamic process.

James Prescott Joule, a nineteenth-century English businessman, is accredited with performing the experiments that led to the principle of the conservation of heat and mechanical energy. Joule, in 1840, was able to determine the mechanical equivalent of heat, now appropriately referred to as *Joule's constant* (4.184 joules/calorie). One of the methods he used to obtain this constant was to immerse a paddle wheel in a bath of water and to rotate the paddle under the action of a falling weight. A rotation of the paddle wheel results in viscous friction which in turn heats the water. If the water bath is insulated from its surroundings, the initial

mechanical work will result in a temperature rise. The input, in this case, can be accounted for by the increase in the thermal or heat energy of the water bath. The same effect could have been achieved by a direct heat input from a source such as a Bunsen burner flame.

The First Law of Thermodynamics states that energy is conserved. For nuclear reactions in which mass is converted to energy, it can be generalized to include the equivalence of energy and mass. The first law is concerned only with an overall energy balance, that is, it simply requires inputs and outputs to be equal. It does not specify the types of processes that can occur. Purely from an energy reckoning, it would seem reasonable that a heat input to the water bath could, if one is clever



**FIGURE 4.1 A Steam Turbine Power Plant.**

enough in the design of an appropriate system, result in a work output, that is, a rotation of the paddle wheel shaft. Such an effect is not possible. The heating by mechanical work is an irreversible process. While energy is conserved, it is impossible to go back to the original condition, that is, to recover the original mechanical energy.

The Second Law of Thermodynamics, in its most elementary form, states that it is impossible to convert heat to work with no other effect. "Waste heat" necessarily results. Since heat by itself has a tendency to flow from a hot to a cold body, a heat engine requires a low-temperature sink as well as a high-temperature source. This is illustrated by the steam-turbine power plant (Figure 4.1) typical of an electrical power plant. The combustion of fuel provides the heat input to the boiler. After passing through the turbine, the high-pressure, high-temperature steam emerges as low-temperature, low-pressure steam. Heat is extracted from the steam by the condenser converting the steam to water which is, in turn, pumped back into the boiler, completing the cycle. The waste heat removed by the condenser for even the most efficient turbines is over half the original input heat. This results in a thermodynamic efficiency of less than 50%. Modern turbines will have boiler temperatures on the order of 500 to 600°C and condenser temperatures of 20 to 30°C. Thus, they operate between a high-temperature source,  $T_2$ , of 500 to 600°C and a low-temperature sink,  $T_1$ , of 20 to 30°C.

The cyclic heat engine is useful for discussing the conversion of heat to work. Since at the completion of a cycle the engine returns to its initial state, its change in internal energy (for example, energy stored in the form of a compressed gas or spring) is zero.

The Second Law of Thermodynamics, as formulated by Kelvin and Planck, states that it is impossible to construct a cyclic heat engine that wholly converts the heat energy supplied by a heat source to work. If such an engine existed, an ocean liner could propel itself by simply extracting heat from the ocean. However, a low-temperature sink that absorbs waste heat is necessary for a realizable engine.\* For such an engine, the output work,  $W$ , is equal to the difference of the input heat,  $Q_2$ , and the waste heat,  $Q_1$ .

$$W = Q_2 - Q_1$$

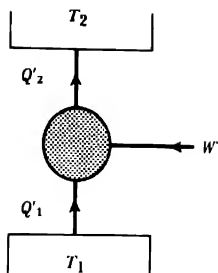
Since the heat  $Q_2$  is derived from the fuel used for the engine, efficiency is defined in terms of this quantity.

$$\eta = \frac{W}{Q_2} = \frac{Q_2 - Q_1}{Q_2} = 1 - \frac{Q_1}{Q_2}$$

Owing to the necessity of  $Q_1$  being finite (it approaches zero as the low temperature,  $T_1$ , approaches absolute zero), the efficiency must be less than one.

A cyclic heat engine, as indicated in Figure 4.2, may be used to transfer heat from a low- to high-temperature reservoir, that is, to function as a heat pump or as

\*Waste as used here is relative to the operation of the heat engine. The heat could be used for applications external to the engine. Often, however,  $Q_1$  is at a sufficiently low temperature that it has a negligible economic value.



**FIGURE 4.2** *A Heat Pump or Refrigerator.*

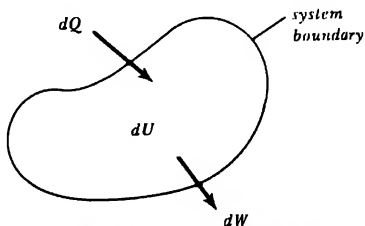
a refrigerator. Since heat would normally flow in the opposite direction, a quantity of work,  $W'$ , must be expended to accomplish the heat transfer. Conservation of energy requires that the heat supplied to the high-temperature reservoir,  $Q'_2$ , be equal to the sum of that extracted from the low-temperature source,  $Q'_1$ , plus the input work,  $W'$ .

$$Q'_2 = Q'_1 + W'$$

A heat pump is therefore able to supply a greater quantity of heat ( $Q'_2$ ) than the amount provided by a simple conversion of work to heat.

The validity of the Second Law of Thermodynamics may be established by arguments dependent upon the operation of a heat pump. Suppose a heat engine that could extract heat from a thermal reservoir, convert it to work, and produce no other effect were available. This engine would extract heat from the high-temperature reservoir of Figure 4.2 and provide the work,  $W'$ , necessary to operate the heat pump. Since the net result would be a transfer of heat from  $T_1$  to  $T_2$ , the initial supposition is invalid. The impossibility of a heat transfer from a low to a high-temperature (with no energy expenditure) is often referred to as the *Clausius statement of the second law*.

An energy balance for a thermodynamic system (the First Law of Thermo-



**FIGURE 4.3** *Energy Balance for a Thermodynamic System.*

dynamics) is indicated in Figure 4.3. The system experiences a differential change in its energy,  $dU$ , as a result of a differential heat input,  $dQ$ , and of a differential work output,  $dW$ .

$$dU = dQ - dW$$

The differential work done by a gas is related to its differential change in volume,  $dV$ .

$$dW = p \, dV$$

$$dU = dQ - p \, dV$$

By convention, heat entering the system (an input) is considered a positive quantity and work done by the system (an output) is also a positive quantity.

The energy of a gas depends upon the kinetic energy of the molecules (or atoms) comprising the gas. In addition, energies associated with molecular vibrations and/or rotation contribute to the internal energy of a gas. For pressures typical of those utilized in heat engines, the volume occupied by the molecules is negligible compared to the total volume of the gas. Total energy for such a case can then be calculated by ignoring effects resulting from the interaction of molecules of the gas. Total energy is then the sum of the energies of the individual molecules. For each degree of freedom, a molecule, on the average, has an energy of  $\frac{1}{2}kT$  (Law of Equipartition of Energy States).

$$\begin{array}{l} \text{Average molecular energy} \\ \text{for each degree of freedom} \end{array} = \frac{1}{2}kT$$

where  $T$  = temperature, K  
 $k = 1.38 \times 10^{-23}$  J/K (Boltzmann constant)

Since a molecule is free to move in three orthogonal directions, it has a minimum of three degrees of freedom. A diatomic molecule (two atoms) can rotate about three axes. Since the moment of inertia about the axis passing through the two atoms is exceedingly small, an energy associated with only two additional degrees of freedom is observed. A diatomic molecule may also exhibit a vibrational mode of energy, which adds two additional degrees of freedom. More complex molecules exhibit an internal energy associated with additional degrees of freedom.

<i>Atom/molecule</i>	<i>Degrees of freedom</i>	<i>Average energy per atom or molecule</i>
Atomic (A, He)	3	$\frac{3}{2}kT$
Diatomic (H <sub>2</sub> , N <sub>2</sub> , O <sub>2</sub> , CO)	5	$\frac{5}{2}kT$
Complex (CH <sub>4</sub> , CO <sub>2</sub> )	7	$\frac{7}{2}kT$

A kilogram-mole of molecules (or atoms) contains Avogadro's number of molecules (or atoms),  $N_0$ . The internal energy of a gas consisting of molecules with  $n$  degrees of freedom is the following.

$$U = \frac{n}{2} N_0 k T \quad \text{J/kg-mole}$$

$$= \frac{n}{2} R T$$

where  $n$  = number of degrees of freedom  
 $N_0 = 6.02 \times 10^{26}$  molecules/kg-mole (Avogadro's number)  
 $R = N_0 k = 8314.4 \quad \text{J/kg-mole}$

A differential change in energy,  $dU$ , is related to the change in temperature.

$$dU = \frac{n}{2} R dT$$

If a gas is heated at a constant volume ( $dV = 0$ ), the change in internal energy is equal to the differential quantity of heat involved.

$$dU = dQ = c_v dT \quad (\text{constant volume})$$

where  $c_v$  = specific heat for constant volume  $\text{J/K kg-mole}$

Since the specific heat for a constant volume,  $c_v$ , can be determined experimentally, a relationship between energy and temperature may be obtained for any gas or mixture of gases.

The quantity  $R$  ( $N_0 k$ ) is the universal gas constant, which relates pressure, volume, and temperature for an ideal gas.

$$pV = RT$$

where  $p$  = pressure,  $\text{N/m}^2$   
 $V$  = volume,  $\text{m}^3/\text{kg-mole}$

For a constant temperature, pressure and volume are inversely related. Therefore, on a pressure-volume diagram, lines of constant temperature form a family of hyperbolas.

## 2. THE CARNOT CYCLE AND EFFICIENCY

In 1824, when he was only 28 years old, Sadi Carnot published a short pamphlet entitled *Reflexions sur la Puissance Motrice du Feu* (Reflections on the Motive Force of Fire), dealing with the efficiency of heat engines [2]. The principle expounded is now recognized as the Second Law of Thermodynamics. While Thomas Newcomen introduced an atmospheric steam engine (an engine in which steam was condensed in the cylinder of the engine) in 1712, and James Watt patented his design for a steam engine using a separate condenser in 1769, Carnot appears to be the first to have appreciated the limitations associated with converting heat energy to work. The importance of Carnot's concepts was not fully appreciated until later in the nineteenth century. William Thomson (1824-1907), better known as Lord Kelvin,



and Rudolph Clausius (1822–1888) recognized the importance of Carnot's work and are credited with completing the theory of heat engines. It is interesting to note that Carnot's conclusions were published before the equivalence of heat and work (the First Law of Thermodynamics) was established by Joule. This undoubtedly accounts for the limited impact of Carnot's theories during the nineteenth century, even though this was the century in which the steam engine had its greatest impact on civilization. Cardwell [3] provides an excellent description of the development of the science of thermodynamics in the nineteenth century. He offers an interesting insight into the difficulties that arise when an incorrect postulate (in the case of thermodynamics, the postulate of the conservation of heat) comes to be accepted without question.

The Carnot, or maximum, efficiency for a heat engine is based upon the concept of reversibility. A reversible engine is one that results in the same heat transfers and work when operated as a heat pump as it does when operated as a heat engine, except that the signs of the quantities are changed. In general, a thermodynamic process involves interchanges of heat and work between the thermodynamic system and its surroundings. A reversible process is defined such that both the system and its surroundings may be returned to their initial condition at its conclusion. If, for example, the system does a quantity of work on the surroundings, that work could be utilized to reverse any heat transfers that occurred and completely return the system and surroundings to their initial state. However, dissipative effects, such as converting work to heat through sliding or viscous friction, are irreversible. Heat flows associated with finite temperature differentials (a practical requirement to obtain reasonable heat transfer rates) are also irreversible. If either of these effects occurs in a heat engine, the output of the engine would be insufficient to effect the return of the heat engine and its surroundings to their initial state.

The reversible thermodynamic cycle from which the Carnot efficiency is derived is, conceptually, a particularly simple heat engine. Its efficiency, which is the maximum efficiency that a heat engine can achieve, is given by an expression that depends only upon the absolute temperatures of the high-temperature source ( $T_2$ ) and the low-temperature sink ( $T_1$ ).

$$\eta_c = 1 - T_1/T_2$$

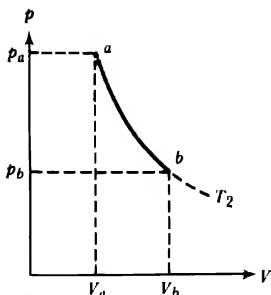
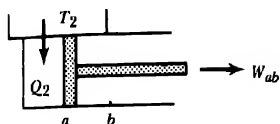
The efficiency, and hence the work output, reduces to zero if  $T_1$  equals  $T_2$ . This result, which is equivalent to eliminating the low-temperature sink, is thus consistent with the arguments of the previous section. Since a complete and rigorous derivation of the Carnot efficiency is beyond the scope of this section, the reader is referred to one of the many thermodynamics texts [4–9].

**Example** Show that a heat engine using a gas which obeys the ideal gas law can indeed have an efficiency equal to the Carnot efficiency.

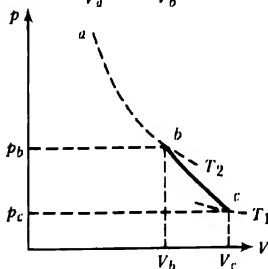
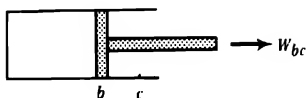
*Solution*

Since heat is extracted from a high-temperature reservoir with a temperature of  $T_2$  and is exhausted to a low-temperature sink ( $T_1$ ), isothermal and reversible heat transfers at temperatures  $T_2$  and  $T_1$ , respectively, are required. Consider

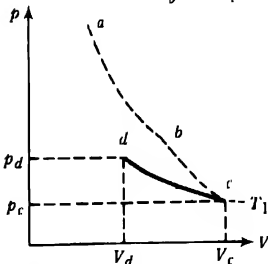
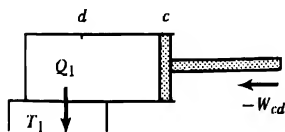
- (a) Isothermal expansion  
at temperature  $T_2$



- (b) Adiabatic expansion  
to temperature  $T_1$



- (c) Isothermal compression  
at temperature  $T_1$



- (d) Adiabatic compression  
return to temperature  $T_2$

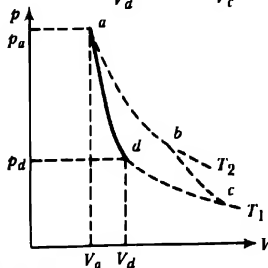
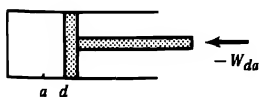


FIGURE 4.4 A Carnot Cycle.

the cylinder of Figure 4.4, which contains one kilogram-mole of a gas. With the piston at point *a*, the cylinder is brought into thermal contact with the high-temperature heat reservoir,  $T_2$ . Heat flows from the reservoir to the gas, causing the gas to expand to point *b*. The gas is then allowed to expand further, in a reversible manner with no heat transfer, to point *c* at which the temperature of the gas falls to  $T_1$ . A work output is obtained for both of these steps. The cylinder is then brought into contact with the low-temperature sink,  $T_1$ . By doing work on the gas, it is isothermally compressed to point *d*. Heat is exhausted to the low-temperature reservoir during the compression. Finally, the gas is again compressed adiabatically (zero heat transfer) until the initial point is reached in which the gas temperature is  $T_2$ . The heat input,  $Q_2$ , and the waste heat output,  $Q_1$ , may be calculated in order to determine the total work output and hence the efficiency of the cycle. The cycle consists of two isothermal (constant temperature) heat transfers (*a-b* and *c-d*) and two adiabatic paths, an expansion (*b-c*) and a compression (*d-a*). Since for a constant temperature,  $dU = 0$ , the heat transfers may be readily calculated.

$$\begin{aligned} Q_2 &= \int_a^b dQ = \int_a^b p \, dV \text{ J/kg-mole} \\ &= \int_a^b \frac{RT_2 \, dV}{V} = RT_2 \ln(V_b/V_a) \\ Q_1 &= - \int_c^d dQ = RT_1 \ln(V_c/V_d) \end{aligned}$$

For  $Q_1$ , a minus sign was introduced since  $Q_1$  is defined as being the heat exhausted by the engine.

For the adiabatic portions of the cycle, the heat transfer is zero ( $dQ = 0$ ).

$$\begin{aligned} dU &= -p \, dV \\ \frac{n}{2} R \, dT &= - \frac{RT \, dV}{V} \\ \frac{n}{2} \frac{dT}{T} + \frac{dV}{V} &= 0 \end{aligned}$$

Integrating the above equation in which the variables have been separated results in the following relation between temperature and volume.

$$T^{n/2} V = \text{a constant}$$

Using this relation, a set of equations involving the end points of both adiabatic paths is obtained.

$$\begin{aligned} T_2^{n/2} V_b &= T_1^{n/2} V_c \\ T_2^{n/2} V_a &= T_1^{n/2} V_d \end{aligned}$$

This results (by division) in the following volume relationship.

$$V_b/V_a = V_c/V_d$$

The heat transfers may be now used to obtain an expression for efficiency.

$$\begin{aligned}\eta &= 1 - \frac{Q_1}{Q_2} = 1 - \frac{T_1 \ln(V_c/V_d)}{T_2 \ln(V_b/V_a)} \\ &= 1 - \frac{T_1}{T_2}\end{aligned}$$

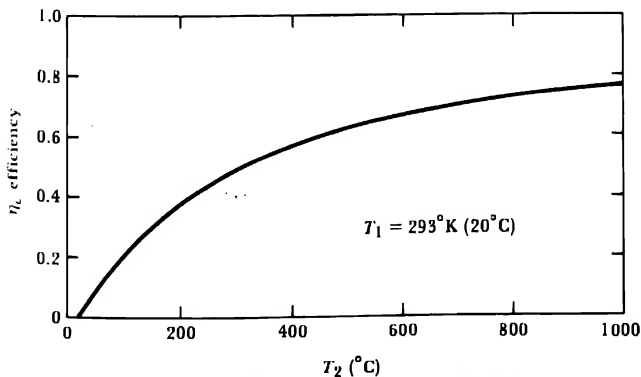
This is thus the Carnot efficiency,  $\eta_c$ .

The low temperature sink of a heat engine depends upon the method by which heat is removed from the engine. Since this temperature must necessarily be greater than the ambient temperature, an efficiency for a low temperature of 293 K (20°C) represents an upper limit for the maximum theoretical efficiency that might be expected for most applications. As indicated in Figure 4.5, the efficiency is also limited by the upper temperature,  $T_2$ . The upper temperature for a modern steam turbine, for example, is on the order of 540°C (~ 1000°F). The efficiency of a Carnot engine operating between this temperature and a low temperature sink of 20°C is 65%. The overall efficiency of a modern electric generating plant is approximately 40%. Even if all theoretically avoidable losses due to heat losses and irreversibilities could be eliminated, the efficiency of a plant would not exceed 65% unless the turbine temperature could be increased.

A Carnot cycle (reversible heat engine), when used as a heat pump, produces the greatest heat transfer for a given amount of work. The measure of the performance of a heat pump is its refrigeration coefficient, the ratio of the heat removed from a low temperature reservoir to the amount of work required.

$$\text{Refrigeration coefficient} = Q'_1/W'$$

For a Carnot cycle, this may be readily shown to be dependent on the temperature of the reservoirs.



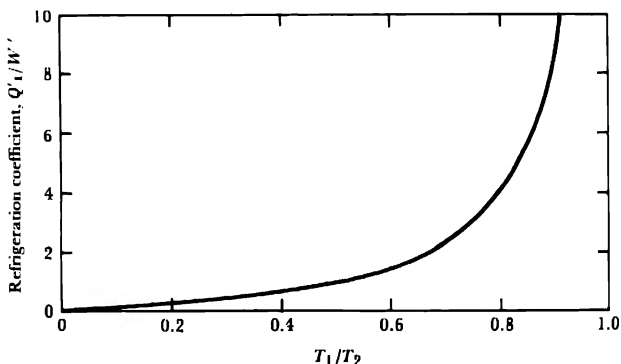
**FIGURE 4.5** Carnot Efficiency for  $T_1 = 293\text{ K}$  (20°C).

$$Q'_1/W' = \frac{T_1}{T_2 - T_1}$$

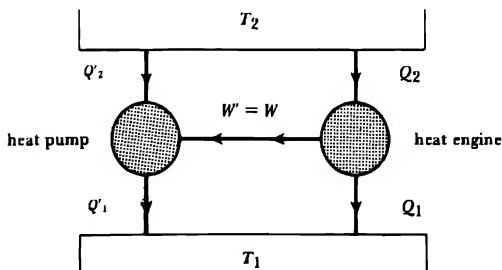
Figure 4.6 is a plot of this coefficient.

Through the use of a heat pump, it may be established that the Carnot efficiency is indeed the greatest efficiency that can be obtained for a heat engine. Consider the heat engine and heat pump of Figure 4.7, which operate between the same thermal reservoirs. If both the heat pump and engine operate on a Carnot cycle, and the quantities of work are equal ( $W = W'$ ), the heat transfers are equal and in the opposite directions.

$$\begin{aligned} Q'_2 &= Q_2 \\ Q'_1 &= Q_1 \end{aligned} \quad \text{Carnot cycles}$$



**FIGURE 4.6** Refrigeration Coefficient for a Carnot Cycle.



**FIGURE 4.7** A Heat Pump and a Heat Engine.

The heat transfers due to the two thermodynamic systems thus cancel. If the heat engine has an efficiency less than the Carnot efficiency, its output work will be less and the heat pump will be unable to produce a heat transfer which will cancel that of the heat engine. Similarly, if the heat engine is ideal and the refrigerator is not, a balancing heat flow will not be achieved.

Suppose, however, that a heat engine with an efficiency greater than that of a Carnot engine were available. This engine could be used to drive the Carnot heat pump. For this case an inequality in heat transfers results.

$$\begin{array}{ll} Q_2 < Q'_2 \\ Q_1 < Q'_1 \end{array} \quad \text{Hypothetical heat engine } (\eta > \eta_c)$$

The net result is a transfer of heat from  $T_1$  to  $T_2$ , an obviously impossible situation. Therefore the Carnot efficiency is indeed the maximum heat engine efficiency.

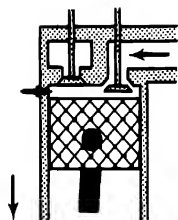
### 3. THE OTTO CYCLE AND THE INTERNAL-COMBUSTION ENGINE

The Otto cycle is an idealized version of the cycle of a gasoline internal-combustion engine. The idealized cycle for a diesel engine differs only slightly from this cycle. Within the United States, the internal-combustion engine accounts for over 95% of the horsepower of all heat engines, although, owing to the intermittent use of these engines, their yearly energy consumption accounts for only one-quarter of the total energy consumed. Internal-combustion engines are used nearly exclusively for transportation and for applications requiring small prime movers.

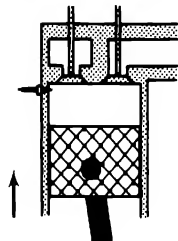
Nicolaus August Otto, a German salesman, is credited with the development of the internal-combustion engine. Although he and his partner, Eugen Langen, experimented with an engine before 1876, their 1876 model is probably the earliest recognizable predecessor of the modern engine. Rather than using gasoline, they used illuminating gas, then widely distributed for lighting. It was a four-cycle engine with an intake, compression, power, and exhaust stroke similar to that of today's engines. Two additional inventions, however, were necessary before the engine's widespread use in the twentieth century. The first was that of the carburetor, a device to regulate and control the mixture of air and fuel. Wilhelm Mayback is credited with this 1893 development. The second major contribution was that of a dependable ignition system to replace the external flame used for early engines. During the 1890s, Karl Benz experimented with a low-voltage ignition system that used mechanically movable points located within the cylinder of the engine. In 1902, Robert Bosch developed a high-voltage magneto that, when used in conjunction with a spark plug, resulted in a reliable ignition system. A detailed and interesting history of the internal-combustion engine may be found in the two volumes of *Technology in Western Civilization* [10, 11].

The four distinct processes of a four-cycle engine are indicated in Figure 4.8. During intake, a mixture of fuel and air is drawn into the cylinder, the intake valve being open. When the piston is fully withdrawn, the cylinder volume is  $V_1$  and the pressure,  $p_a$ , is approximately atmospheric. During the next stroke, in which the

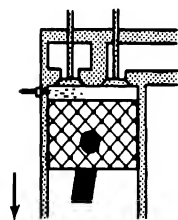
Intake



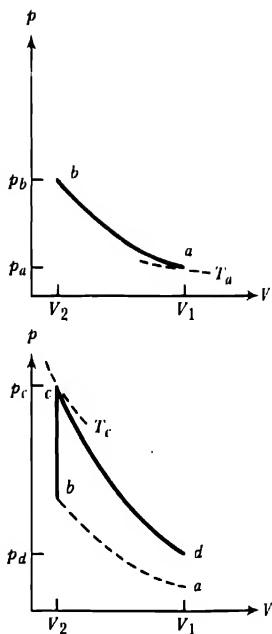
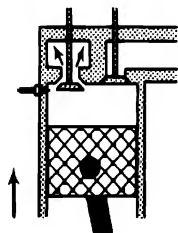
Compression



Power



Exhaust



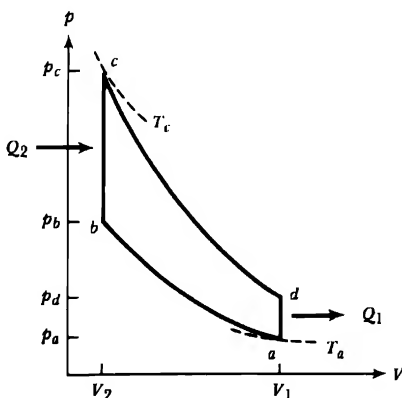
**FIGURE 4.8** Internal-Combustion Engine.

intake valve is closed, the fuel-air mixture is compressed. The compression corresponds to a nearly adiabatic process in which the final volume (top of stroke) is  $V_2$  and the pressure is  $p_b$ . Near the peak of the compression stroke, the mixture is ignited by a high-voltage spark. Combustion results in a rapid increase in pressure, while the volume remains nearly constant. The expansion of the hot gases within the cylinder results in the power stroke, at the completion of which the volume returns to  $V_1$ . A return of the piston, now with the exhaust valve open, completes the four strokes of what is commonly referred to as a *four-cycle engine*.

Four up and down movements (assuming the cylinder is vertical), requiring two complete rotations of the crank shaft, are necessary to obtain one power stroke in the four-cycle engine. However, two-cycle engines, in which a power stroke is obtained for each rotation of the crank shaft, are also built. These engines tend to be less reliable and hence find use only in applications which require extremely small and lightweight engines. Two-cycle engines, having power ratings comparable to that of a modern automobile, are seldom used.

The  $p$ - $V$  diagram of Figure 4.8 is not closed. At the end of the power stroke, point  $d$ , the gas is exhausted. A new mixture of fuel and air is introduced at the beginning (point  $a$ ) of the compression stroke that follows the exhaust and intake strokes. The combustion products which normally carry heat away could, in principle, be cooled at a constant volume to reduce the pressure to  $p_a$  (atmospheric pressure). The heat extracted in cooling the exhaust gases would thus be equal to that which would normally be carried off. An idealized closed cycle (with  $Q_1$  representing the extracted heat) is shown in Figure 4.9.

Combustion results in a heat input of  $Q_2$ . For this idealized cycle, the chemical change associated with combustion is ignored. Heat, it will be noted, is not supplied and extracted at a constant temperature (as for a Carnot cycle) but at a constant



**FIGURE 4.9** An Idealized Otto Cycle.



volume. Since  $T_c$  is the highest temperature and  $T_a$  the lowest, the resultant efficiency would be expected to be less than that of a Carnot cycle operating between these temperature extremes.

The volume,  $V$ , used in the ideal gas relation was the volume (in cubic meters) of one kilogram-mole of gas. As indicated on the previous figures, the volumes  $V_1$  and  $V_2$  correspond to cylinder volumes in which  $V_1$  is the maximum volume that occurs with the piston fully extended (bottom of stroke as shown in Figure 4.7) while  $V_2$  is the minimum volume. At the beginning of the intake stroke, the pressure is nearly atmospheric. Since atmospheric pressure supports a .76 meter column of mercury, the pressure in newtons per square meter may readily be found to be  $1.01 \times 10^5 \text{ n/m}^2$ . For a temperature of  $20^\circ\text{C}$ , the volume of one kilogram-mole is  $24.1 \text{ m}^3$ . The volume is considerably larger than the cylinder volume of even a very large internal-combustion engine. The ideal gas relation, however, can be modified to relate pressure, volume, and temperature for a fraction,  $n_m$ , of a mole.

$$p n_m V = n_m R T$$

$$p V' = n_m R T$$

where  $n_m$  = number (or fraction) of kilogram-moles  
 $V'$  = volume ( $\text{m}^3$ )

The new quantity,  $V'$ , is thus the volume of  $n_m$  kilogram-moles of gas. For example, the volume  $V_1$  for a four-cylinder 1600 cubic centimeter displacement engine is  $400 \text{ cm}^3$  or  $4 \times 10^{-4} \text{ m}^3$ . The fraction of mole involved, assuming the above pressure and temperature, is this volume divided by  $24.1 \text{ m}^3$  or  $1.67 \times 10^{-5} \text{ kg-moles}$ . An exceedingly small molar volume is thus involved for this engine. A calculation of efficiency, however, does not depend upon the number of moles involved. Very often a hypothetically large cylinder containing one mole of gas is assumed to simplify calculations.

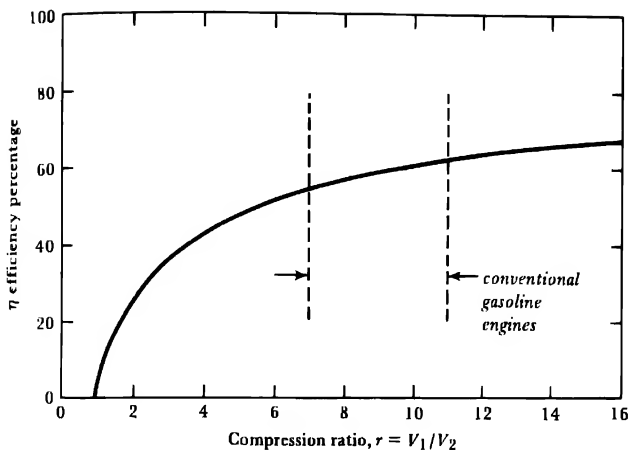
For convenience, the volume ratio,  $V_1/V_2$ , is defined as the compression ratio  $r$ . A particularly simple expression for the theoretical efficiency of an idealized Otto cycle is then obtained.

$$\eta = 1 - \left( \frac{V_2}{V_1} \right)^{2/n} = 1 - \frac{1}{r^{2/n}}$$

The theoretical efficiency is thus dependent only upon the compression ratio. For air, which tends to behave as an ideal gas with five degrees of freedom, the efficiency expression reduces to the following:

$$\eta = 1 - \frac{1}{r^{.4}}$$

Since compression ratios between 7 and 11 are typical for gasoline engines, theoretical efficiencies of 54% to 62% are predicted. In practice, however, efficiencies greater than 20% are seldom realized.



**FIGURE 4.10** Otto Cycle Efficiency as a Function of Compression Ratio.

**Example** Show that a gas obeying the ideal gas law has a theoretical Otto cycle efficiency given by the above relationship (see Figure 4.10).

*Solution*

Assume a heat engine utilizes one kilogram-mole of gas and operates with the idealized cycle indicated in Figure 4.9. Since the heat transfers,  $Q_2$  and  $Q_1$ , occur at a constant volume ( $dV = 0$ ), the differential heat ( $dQ$ ) is equal to the differential change in energy ( $dU$ ).

$$Q_2 = \int_b^c dU = \int_b^c \frac{n}{2} R dT = \frac{n}{2} R(T_c - T_b)$$

$$Q_1 = - \int_d^a dU = \frac{n}{2} R(T_d - T_a)$$

The sign of the waste heat term,  $Q_1$ , is chosen to be positive for heat leaving the engine.

$$\eta = 1 - \frac{Q_1}{Q_2} = 1 - \frac{T_d - T_a}{T_c - T_b}$$

In the problem for the Carnot engine, a temperature-volume relationship was established for the adiabatic portion of the cycle.

$$T^{n/2} V = \text{constant}$$

$$TV^{2/n} = \text{a new constant}$$

Using this relationship, an expression for the temperature differences occurring in the efficiency expression may be obtained.

$$T_c V_2^{2/n} = T_d V_1^{2/n}$$

$$T_b V_2^{2/n} = T_a V_1^{2/n}$$

$$(T_c - T_b) V_2^{2/n} = (T_d - T_a) V_1^{2/n}$$

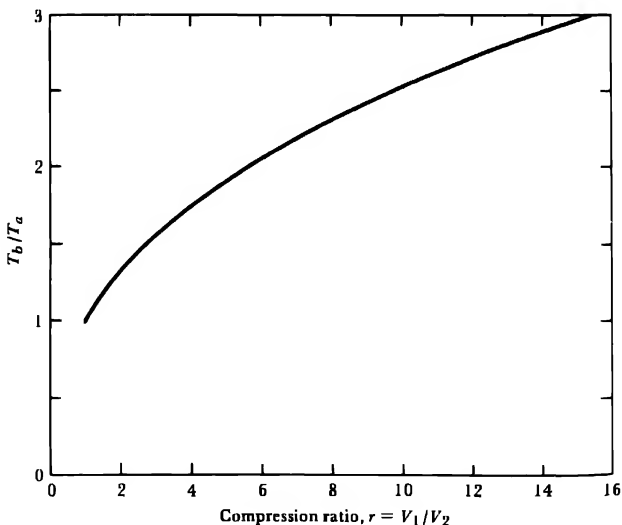
$$\frac{T_d - T_a}{T_c - T_b} = \left( \frac{V_2}{V_1} \right)^{2/n} = \frac{1}{r^{2/n}}$$

The compression ratio  $r$ , which is equal to  $V_1/V_2$ , was introduced in the last expression. This yields the following expression for efficiency.

$$\eta = 1 - \frac{1}{r^{2/n}}$$

The temperatures before and after combustion,  $T_b$  and  $T_c$  respectively, are important. The initial temperature,  $T_b$ , is the result of the adiabatic compression. It is related to  $T_a$ , essentially the ambient temperature, by the following relation.

$$\frac{T_b}{T_a} = r^{2/n}$$



**FIGURE 4.11** *Precombustion Temperature as a Function of Compression Ratio.*

A plot of  $T_b/T_a$  for air as a function of the compression ratio is given in Figure 4.11. For a compression ratio of 10,  $T_b/T_a = 2.5$  and hence  $T_b = 736$  K ( $463^\circ\text{C}$ ) for  $T_a = 293$  K ( $20^\circ\text{C}$ ). The higher the compression ratio, the higher  $T_b$ .

While increasing the compression ratio increases the thermodynamic efficiency, it also increases  $T_b$ , increasing the likelihood of preignition. To minimize preignition, a high octane fuel is necessary. A common means of obtaining a high octane rating is the addition of tetraethyl lead to gasoline. Lead, unfortunately, results in a particularly obnoxious pollutant, which, in addition to being a biological hazard, impairs the operation of exhaust pollution controls.

The temperature at the completion of combustion,  $T_c$ , depends upon the heat due to combustion,  $Q_2$ . The larger the heat input for a given size cylinder and compression ratio, the higher the combustion temperature. For a specified shaft speed (which determines the number of power strokes per second), compression ratio, and output power, the upper temperature is inversely dependent upon the displacement of an engine (the number of cylinders times the volume  $V_1$ ). The smaller the displacement for a given power, the higher the temperature involved.

A high value of upper temperature,  $T_c$ , tends to increase the oxidation of nitrogen within the cylinder forming nitric oxide and nitrogen dioxide. When a mixture of nitrogen oxides ( $\text{NO}_x$ ) and hydrocarbons are exposed to intense sunlight, a photochemical reaction occurs that results in smog. Reducing  $T_c$  tends to reduce the formation of nitrogen oxides. Therefore, a large displacement power ratio is desired to minimize the formation of nitrogen oxides. Reducing the temperature, however, tends to increase the emission of unburnt hydrocarbons and carbon monoxide. These can be reduced by using a lean fuel to air ratio and by exhaust after-burners or catalytic reactors. Converters to reduce nitrogen oxides are considerably less effective than those for hydrocarbons. An excellent discussion of pollutants can be found in an article by Heywood [12]. For a treatment dealing with the quantities involved for a typical engine (VW 1600 cc), Problem 5 at the end of this chapter is recommended.

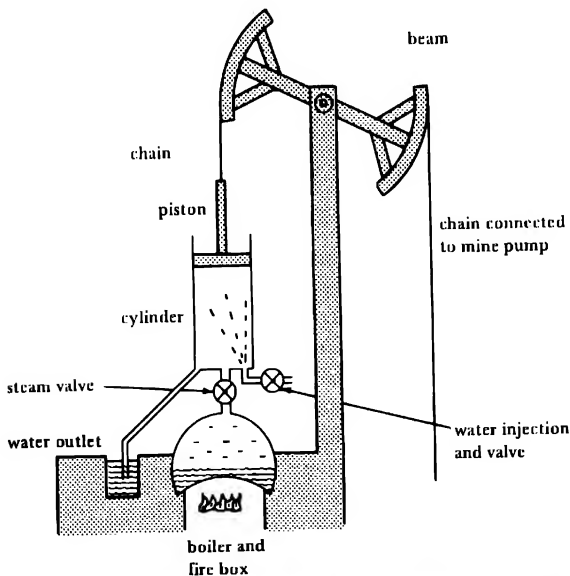
A diesel engine uses a slightly modified version of the Otto cycle. Since the fuel is injected near the peak of the compression stroke, preignition is no problem and hence large compression ratios ( $r > 10$ ) are normally used. No ignition system is necessary since the temperature at the time of injection is sufficient to ignite the fuel. Not only can high thermal efficiencies associated with the high compression ratio be achieved, but low-grade fuel can be used.

An interesting modification of the piston-type gasoline engine is the rotary engine invented by Felix Wankel in 1926. While thermodynamically it uses a conventional Otto cycle, the rotary engine eliminates the oscillatory motion of the piston of a conventional internal-combustion engine. Basically this engine uses a triangular rotor which revolves eccentrically in a specially designed housing. Only in the past few years have seals been available to produce a sufficiently reliable engine. Wide-scale use of this engine for automobiles may come about in the future. A discussion of the Wankel engine may be found in an article by Cole [13]. Since thermodynamically this engine differs little from a conventional piston-type engine, the main advantage is derived from its mechanical design, which depends entirely upon rotary motion.

Automobiles, since shortly after the introduction of the mass-produced Model T Ford, have been wholly dependent upon the internal-combustion engine. Even U.S. railroad transportation is now tied to the diesel locomotive (electric propulsion is common in other countries). Widespread usage of the internal-combustion engine may not necessarily be due to its inherent superiority but to its being the first mass-produced engine. Other heat engines (the steam engine), having lower pollution emissions may be superior to the internal-combustion engine [14].

#### 4. THE STEAM ENGINE AND THE RANKINE CYCLE

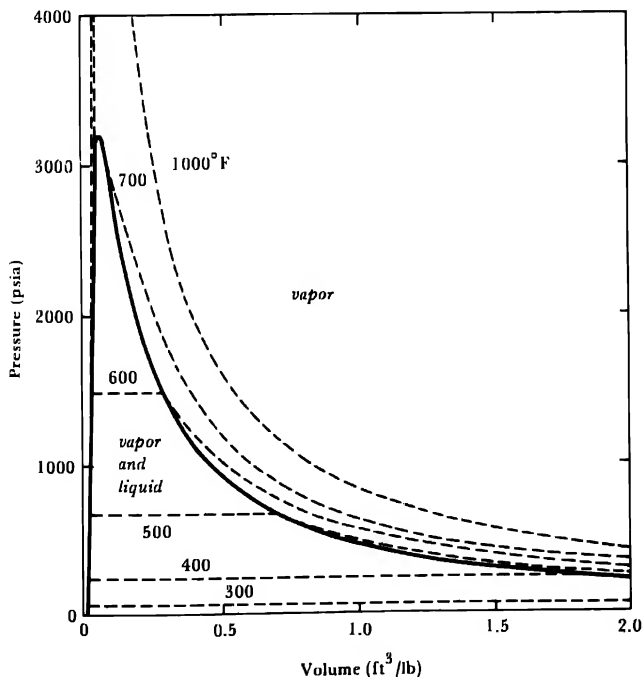
The history of the steam engine dates back to the first century A.D. when Hero of Alexandria constructed an "aeolipile" (a "wind" machine). The rotating wheel and shaft of this engine was made to turn by steam jets located on its periphery. A reaction of the jet nozzle produced the torque which overcame the inherent friction of the wheel. Modern jet-aircraft engines utilize this same reaction principle. The aeolipile, however, was never developed beyond the stage of a mere, albeit interesting, toy. In 1629 Giovanni Branca proposed driving a horizontal waterwheel



**FIGURE 4.12** *Simplified Diagram of an Early Newcomen Steam Engine.*

with a jet of steam. While it is not known whether this engine was actually constructed, its principle of operation would have been the same as that of a modern impulse turbine.

The first steam engine that did perform useful work was invented by Thomas Newcomen in 1712. (See Figure 4.12.) This engine, designed primarily for pumping water from mine shafts, was used throughout the eighteenth century. At the time of its invention, high-pressure boilers were not available. Rather than rely upon high-pressure steam to “push” a piston, steam at atmospheric pressure was admitted into the cylinder of the engine when the piston was fully withdrawn. The steam was then condensed within the cylinder by a jet of cooling water. This reduced the pressure to less than that of the atmosphere. Atmospheric pressure pushing on the other side of the piston outside the cylinder produced the working force of the engine. Consequently, this and similar engines were referred to as *atmospheric engines*. The engine was designed such that the combined weight of the chain extending to



**FIGURE 4.13** *Pressure-Volume Diagram for Water (English Units).*

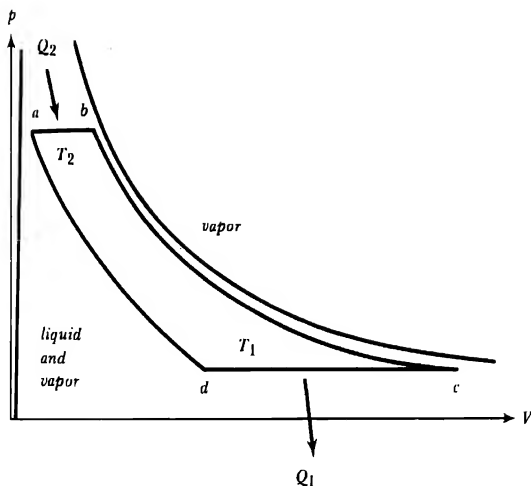
the bottom of the mine shaft and the pump mechanism was sufficient to return the piston to the top of its stroke. Since condensing steam was a slow process, 12 strokes per minute were typical for the early engines. Engines eventually developed outputs of several horsepower.

Despite the widespread use of the Newcomen engine prior to the date when James Watt patented his engine (1769) and attained his first commercial success (1775), Watt is usually credited with "inventing" the steam engine. Watt's engine differed from Newcomen's in that the steam was condensed outside the cylinder of the engine. Since the cylinder of Watt's engine did not need to be cooled, a significant improvement in efficiency was achieved. Early engines were sold on a royalty basis, the royalty being determined by the coal saved over that of a Newcomen-type engine. As for the internal-combustion engine, Kranzberg and Pursell provide an interesting historical description of it in the early days [15]. For additional reading, an extremely delightful and detailed history of the steam engine by Storer [16] is recommended.

Steam engines differ from the heat engines discussed in previous sections in that a change in phase of the working fluid is utilized. A pressure-volume diagram for water (with commonly used English units) is given in Figure 4.13. Since mixtures of vapor and liquid are usually involved in the design of steam engines, steam tables are normally needed for numerical solutions [17, 18]. The narrow left-hand portion is the liquid region; the right-hand region is that of vapor. The region under the heavy curve consists of a mixture of liquid and vapor. Several isotherms are also shown on the diagram. The 1000°F line is entirely removed from the liquid-vapor region and is thus always vapor. Its hyperbolic behavior is similar to the isotherms for an ideal gas. The peak of the vapor-liquid region occurs at a temperature of 705.4°F. At temperatures lower than this, isotherms traverse the mixture region coinciding with constant pressure (isobaric) lines. Since the fractional parts of liquid and vapor change along an isotherm of the mixture, a corresponding volume change also occurs. The vapor end (right side) represents saturated vapor (no liquid), whereas the liquid end (left side) represents saturated liquid (no vapor).

Boiling water at atmospheric pressure results in the well-known liquid-vapor change of state for a temperature of 212°F or 100°C. When heated, water first reaches the liquid edge of the mixture region. Upon further heating, vapor or steam is formed which remains in equilibrium with the water. As more water is changed to steam, the right-hand edge of the mixture region is eventually reached. Further heating, if the pressure remains constant, results in an increase in temperature. Forming steam at this pressure results in a volume increase by a factor of 1600. Conversely, condensing steam as in a Newcomen engine, results in a similar volume decrease.

A Carnot engine, similar to that discussed for an ideal gas, may also use a liquid-vapor cycle, as shown in Figure 4.14. The isothermal heat transfers coincide with constant pressure (horizontal) lines for the vapor-liquid mixture. Portion  $a-b$  is an isothermal expansion. The heat input,  $Q_2$ , results in a vaporization of liquid and, hence, a volume increase. Portion  $b-c$  is an adiabatic expansion resulting in a new liquid-vapor mixture at point  $c$ . Heat,  $Q_1$ , is then extracted from the mixture by condensing some of the vapor, arriving at the new mixture corresponding to point  $d$ . Path  $d-a$  is an adiabatic compression. Since the heat transfer occurs entirely at



**FIGURE 4.14 A Carnot Cycle Using a Liquid-Vapor Cycle.**

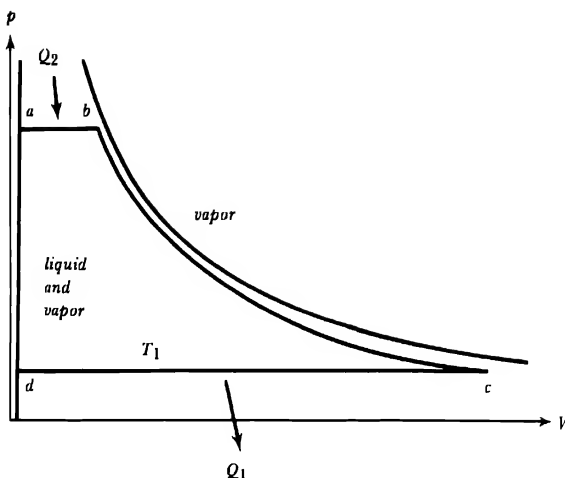
temperatures  $T_2$  or  $T_1$ , this engine has an efficiency equal to that of a Carnot cycle.

$$\eta = \eta_c = 1 - \frac{T_1}{T_2}$$

A steam engine operating on a Carnot cycle is impractical. After the expansion ( $b \rightarrow c$ ) in the piston or turbine, a precise quantity of heat  $Q_1$  must be extracted from the liquid-vapor mixture to reach point  $d$ . Both temperature and pressure remain constant for this process. From point  $d$ , an adiabatic compression of a liquid-vapor mixture is required. Conventional closed-cycle steam engines condense the liquid-vapor mixture of point  $c$ , at a constant pressure, to the liquid region. Sufficient heat is removed, as indicated in Figure 4.15, so that only fluid emerges at point  $d$ . The fluid (water) is then adiabatically compressed by the feedwater pump. In Figure 4.1 the boiler is portion  $a-b$  of the cycle; the turbine,  $b-c$ ; the condenser,  $c-d$ ; and the pump,  $d-a$ . Since water is nearly incompressible ( $p \, dV \simeq 0$ ), very little energy is required to operate the pump.

Condensing the liquid-vapor mixture of point  $c$  results in a greater waste heat output and, hence, a lower efficiency for this type of engine. The decrease in efficiency is partially offset by the convenience of the compression portion of the cycle that requires only a minimal energy expenditure. Additional heat also must be supplied at point  $a$  to bring the water (essentially at the condenser temperature of  $T_1$ ) up to the boiler temperature of  $T_2$ . This results in a further decrease in thermal efficiency. This type of steam cycle is named after William Rankine (1820–1872), a professor of engineering at Glasgow University. While the engine naturally evolved from those of Watt, Rankine is credited with developing a manual, published in 1859,





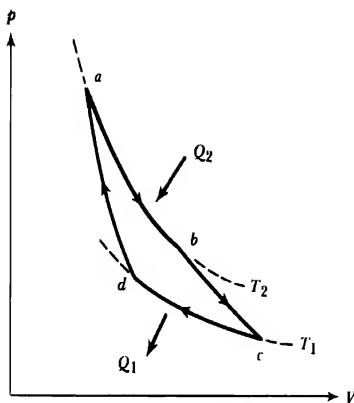
**FIGURE 4.15 A Rankine Cycle Engine.**

dealing with the design of steam engines. Therefore, it is appropriate that the steam cycle be referred to as the *Rankine cycle*.

## 5. ENTROPY

While energy is conserved and hence cannot be “used up” or exhausted, the usefulness of a quantity of energy does decrease through usage. Energy is degraded through usage, eventually to the point of zero usefulness. Units of energy such as the joule or Btu do not include or even imply a measure of usefulness. Very often energy is referred to in the context of the energy content of resources with a high potential. The energy of low-utility sources, such as the exhaust heat of a thermal power plant, is seldom confused with a high-utility source such as petroleum. Common usage of energy terms often tends to include a distinction associated with usefulness.

Entropy is a negative measure associated with utility. An increase in entropy corresponds to a decrease in utility; hence, entropy is a measure of disutility. The concept of entropy was developed by the nineteenth-century thermodynamicists that followed Carnot, particularly Fourier, Joule, Rankine, and Kelvin, along with Clausius, who is credited with the presently used definition of entropy (1865). Clausius recognized that the entropy of a closed system tends to increase to a maximum value. Even though energy is conserved in a closed system, the system tends toward an energy state corresponding to that of minimum usefulness. Clausius’s insights concerning the entropy principle extended beyond simple heat engines, taking on



**FIGURE 4.16** Pressure-Volume Diagram for a Carnot Cycle.

a cosmological significance. If a closed universe is assumed (an obviously unverifiable assumption), the energy of the universe is constant. Entropy, however, tends to increase to a maximum. With a continual increase in entropy (a continual decrease in energy potential), Clausius was led to speculate on an ultimate "heat death" for the universe. Entropy is discussed in all texts on thermodynamics, such as those referred to already. Additional insights are provided by other references dealing more exclusively with entropy [19-21].

In the development of the entropy concept, a cyclic reversible heat engine will be considered. This approach, based upon the operation of heat engines, tends to parallel the historical development of the concept by Clausius. A Carnot cycle (Figure 4.16) has been shown to have the greatest efficiency,  $\eta_c$ , of any heat engine operating between the temperatures  $T_2$  and  $T_1$ .

$$\eta_c = W/Q_2 = \frac{Q_2 - Q_1}{Q_2} = 1 - \frac{Q_1}{Q_2} = 1 - \frac{T_1}{T_2}$$

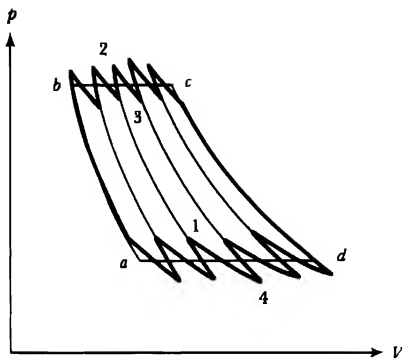
Using the above equation, a relationship between the heat transfers,  $Q_1$  and  $Q_2$ , and the temperature at which they occur may be obtained.

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = 0$$

Both  $Q_2$  and  $Q_1$  are the result of an integration of differential heat transfers.

$$Q_2 = \int_a^b dQ \quad Q_1 = - \int_c^d dQ$$

For the reversible adiabatic portions of a Carnot cycle, that is  $b-c$  and  $d-a$ , the heat



**FIGURE 4.17** *A Reversible Thermodynamic Cycle.*

transfers are zero. If the temperature is placed within the integral, the following is obtained for the cyclic integral of  $dQ/T$ .

$$\oint \frac{dQ}{T} = 0 \quad (\text{Carnot cycle})$$

The efficiency of a cyclic heat engine other than a Carnot engine ( $\eta$ ), will be no greater than that for a Carnot engine ( $\eta_c$ ) operating between the same temperatures.

$$\eta \leq \eta_c$$

This results in the following inequality for the heat transfers.

$$\frac{Q_2}{T_2} - \frac{Q_1}{T_1} \leq 0$$

A similar inequality is obtained for the cyclic integral of  $dQ/T$ .

$$\oint \frac{dQ}{T} \leq 0 \quad (\text{any cyclic heat engine})$$

This, the inequality of Clausius, may be shown to be valid for a cyclic heat pump. In general, the deviation of the integral from zero is a measure of how close the behavior of a particular cyclic engine approaches that of a Carnot engine.

The cyclic integral of  $dQ/T$  vanishes not only for a Carnot cycle but for any closed reversible thermodynamic cycle. Consider the reversible cycle ( $a-b-c-d-a$ ) of Figure 4.17 on which a series of approximating Carnot cycles are superimposed. The cyclic integral  $dQ/T$  for a particular Carnot cycle, for example 1-2-3-4-1, is zero. Furthermore, since the differential heat transfer,  $dQ$ , is zero along the common adiabatic portions of adjacent Carnot cycles, the integral along the outer portions of the Carnot cycles is zero. The outer portion of the approximating Carnot cycles,

however, may be made to approach the actual cycle by increasing the number of the individual cycles. In the limiting case, the cyclic integral of  $dQ/T$  of the reversible path approaches that of the Carnot cycles and hence vanishes.

$$\oint \frac{dQ}{T} = 0 \quad (\text{for any closed reversible cycle})$$

The above condition is therefore valid for any closed reversible cycle.

The zero value of the cyclic integral of  $dQ/T$  implies that the value of the integral between any two points is independent of the path. Consider two arbitrary reversible paths,  $A$  and  $B$ , connecting points 1 and 2 of Figure 4.18. Since these reversible paths form a closed cycle, the following is valid.

$$\oint \frac{dQ}{T} = \int_1^2 \frac{dQ}{T} + \int_2^1 \frac{dQ}{T} = 0$$

Rearranging the integrals establishes the desired equality.

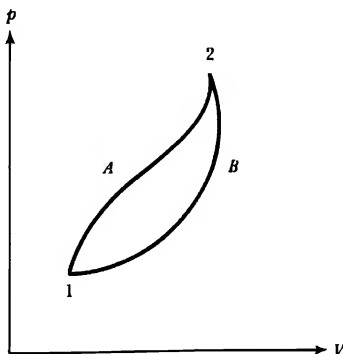
$$\int_1^2 \frac{dQ}{T} = \int_1^2 \frac{dQ}{T}$$

Since paths  $A$  and  $B$  were arbitrary, the value of the integral must be independent of the path, that is, it must depend only upon the endpoints. This implies the existence of a state variable.

$$dS = dQ/T$$

$$\int_1^2 \frac{dQ}{T} = \int_1^2 dS = S_2 - S_1 \text{ J/K kg-mole}$$

The change in entropy which depends only upon the endpoints,  $S_2 - S_1$ , is expressed



**FIGURE 4.18 Two Arbitrary Reversible Paths.**

in units of energy per Kelvin. If one kilogram-mole of gas is assumed, the change in  $S$  is expressed as the change per kilogram-mole. The above relationship by which  $S$  is defined does not yield an absolute value for entropy but only the difference of entropy between two points. For convenience, an arbitrary condition is often chosen which corresponds to zero entropy. For example, steam tables set the entropy (as well as the internal energy) equal to zero for saturated water at  $0^\circ\text{C}$ . If only changes in entropy are considered, the zero point is of no importance.

Entropy is not the only thermodynamic state variable. Pressure, volume, and temperature are obviously state variables. Internal energy depends upon the state of a system being dependent only upon temperature for an ideal gas. As for entropy, the integral of  $dU$  between two points depends only upon the endpoints.

An expression for the change in entropy for one kilogram-mole of an ideal gas may readily be obtained. For a reversible path, the following results.

$$\begin{aligned}dQ &= dU + dW = dU + p dV \\ \frac{dQ}{T} &= \frac{dU}{T} + \frac{p dV}{T} \\ \int_1^2 \frac{dQ}{T} &= \int_1^2 \frac{dU}{T} + \int_1^2 \frac{p dV}{T}\end{aligned}$$

If the internal energy is a function of the temperature only, the first integral on the right-hand side of the above relation may be integrated. For an ideal gas,  $dU = (n/2)R dT$  in which  $n$  is the number of degrees of freedom. Using the ideal gas law, an integrable expression results for the second integrand.

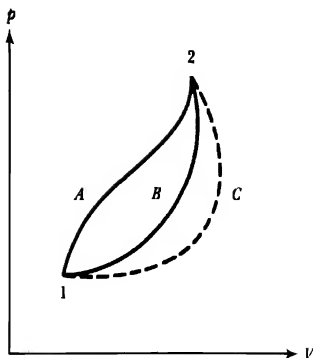
$$\begin{aligned}p &= RT/V \\ S_2 - S_1 &= \frac{n}{2}R \int_1^2 \frac{dT}{T} + R \int_1^2 \frac{dV}{V} \\ &= \frac{n}{2}R \ln(T_2/T_1) + R \ln(V_2/V_1) \quad \text{J/K kg-mole}\end{aligned}$$

Expressions involving only pressure and temperature or pressure and volume may also be obtained.

The change in entropy between two points is equal to the integral of  $dQ/T$  between these two points if and only if the path is reversible. For an irreversible path, the integral of  $dQ/T$  is no longer equal to the change in entropy. In Figure 4.19 an irreversible path,  $C$ , is added to the reversible paths,  $A$  and  $B$ , which connects points 1 and 2. If the reversible paths  $A$  and  $B$  are used for the cyclic integral of  $dQ/T$ , a zero result is obtained.

$$^A \int_1^2 \frac{dQ}{T} + ^B \int_2^1 \frac{dQ}{T} = 0$$

The integration over paths  $C$  and  $B$  can also be considered. Since  $C$  is an irreversible path, the inequality of Clausius applies for a cyclic integral involving paths  $C$  and  $B$ .



**FIGURE 4.19** An Irreversible Path.

$$\int_1^2 \frac{dQ}{T} + \int_2^1 \frac{dQ}{T} < 0$$

Eliminating the integral over path *B* of the above two relations yields the following inequality.

$$\int_1^2 \frac{dQ}{T} < \int_1^2 \frac{dQ}{T}$$

As *A* is a reversible path, the value of the integral is equal to the change in entropy,  $S_2 - S_1$ .

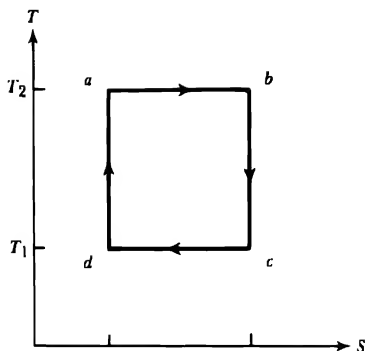
$$\int_1^2 \frac{dQ}{T} < S_2 - S_1$$

The above is valid for any irreversible path. The general result is that the integral of  $dQ/T$  is less than the change in entropy for an irreversible path and equal to the change for a reversible path.

$$\int_1^2 \frac{dQ}{T} \leq S_2 - S_1$$

Irreversible processes always result in the inequality condition.

The state of a thermodynamic system may be described in terms of its state variables. While pressure and volume have been extensively used (*p-V* diagrams), other pairs can be equally valid and may, if appropriately chosen, yield additional insight into the processes being discussed. Temperature and entropy comprise such a pair of variables. Processes are often indicated on temperature-entropy diagrams (often referred to as *Mollier diagrams* after George Mollier), such as that of Figure 4.20. Adiabatic, reversible processes imply constant entropy and result in vertical



**FIGURE 4.20** A Temperature-Entropy Diagram for a Carnot Cycle.

lines while isothermal processes form a family of horizontal lines. The Carnot cycle of Figure 4.20 is thus represented by a rectangle,  $a-b-c-d-a$ . Since  $dS = dQ/T$  for a reversible process, a differential area element,  $T dS$ , is equal to the corresponding differential heat transfer.

$$dQ = T dS \quad (\text{reversible process})$$

Therefore, the area under a particular path, for example  $a-b$ , is equal to the heat transfer for that path,  $Q_2$ .

$$Q_2 = \int_a^b dQ = \int_a^b T dS$$

The area under the path  $c-d$  is the heat rejected,  $Q_1$ .

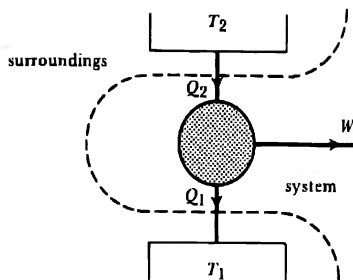
$$Q_1 = - \int_c^d dQ = - \int_c^d T dS$$

Since the value of the integral is negative, a minus sign has been introduced for  $Q_1$ . The area enclosed by the curve is thus equal to  $Q_2 - Q_1$  or the output work.

The concept of entropy change may be applied to the surroundings as well as to the thermodynamic system. For a transfer of heat, such as from a thermal reservoir, a change of entropy of the reservoir is given by the integral of  $dQ/T$  in which  $dQ$  and the temperature apply to the reservoir. The total change in entropy for a given process is the sum of the changes in entropy for the surroundings and the thermodynamic system.

$$(S_2 - S_1)_{\text{total}} = (S_2 - S_1)_{\text{surr}} + (S_2 - S_1)_{\text{system}}$$

This result may be applied to any thermodynamic system including a cyclic heat engine such as that of Figure 4.21. Both the high-temperature heat source and the low-temperature heat sink, with temperatures of  $T_2$  and  $T_1$ , respectively, are included



**FIGURE 4.21** A Cyclic Heat Engine and Surroundings.

as part of the surroundings. A complete cycle of a cyclic engine returns the working fluid to its original state and therefore the entropy change of the system is zero. Based upon the inequality of Clausius, the value of the cyclic integral of  $dQ/T$  is less than or equal to zero.

$$\oint \frac{dQ}{T} \leq 0$$

During the transfer of a differential quantity of heat from  $T_2$ , the temperature of the thermodynamic fluid,  $T$ , is less than or at most equal to  $T_2$ .

$$\frac{dQ}{T_2} \leq \frac{dQ}{T} \quad (\text{high temperature input})$$

A similar inequality results for the low-temperature heat rejection in which the temperature of the fluid is greater than or equal to  $T_1$  during the heat transfer. The differential heat transfer for the fluid is negative over this portion of the cycle.

$$\frac{dQ}{T_1} \leq \frac{dQ}{T}$$

Hence the inequality is also valid if finite temperature differentials occur during the heat transfers.

$$\frac{Q_2}{T_2} - \frac{Q_1}{T_1} \leq 0$$

Both the high- and low-temperature reservoirs must be considered in calculating the change of entropy for the surroundings.

$$(S_2 - S_1)_{\text{surr}} = -\frac{Q_2}{T_2} + \frac{Q_1}{T_1}$$

A minus sign is required for  $Q_2$ , since it is a measure of the heat leaving the surroundings, while a plus sign is required for  $Q_1$ , since it represents heat received. Utilizing the inequality for the heat engine, the following is obtained.



$$(S_2 - S_1)_{\text{total}} = (S_2 - S_1)_{\text{sur}} \geq 0$$

Associated with the operation of an irreversible cyclic heat engine, as well as any other heat engine (such as one depending upon a steady flow process), is an inevitable increase in entropy. A less than ideal refrigerator may also be shown to result in an overall entropy increase.

An additional increase in entropy occurs when the output work of the heat engine is eventually converted to heat through frictional losses. If the low-temperature reservoir ( $T_1$ ) accepts the heat, an additional entropy increase of  $W/T_1$  results for the surroundings. The total entropy increase is therefore the following.

$$\begin{aligned}(S_2 - S_1)_{\text{total}} &= -\frac{Q_2}{T_2} + \frac{Q_1 + W}{T_1} \\ &= Q_2 \left( -\frac{1}{T_2} + \frac{1}{T_1} \right)\end{aligned}$$

Conservation of energy ( $Q_1 + W = Q_2$ ) was used to obtain the second expression. If the high temperature ( $T_2$ ) is large compared to the temperature at which heat is rejected (usually the ambient temperature), an approximate expression for the change in entropy results.

$$(S_2 - S_1)_{\text{total}} \simeq Q_2/T_1 \quad \text{J/K kg-mole}$$

While  $Q_2$  corresponds to the heat transfer associated with a cyclic engine containing one kilogram-mole of fluid, a similar result occurs for any energy usage,  $E$ , in which the output is eventually dissipated in heat.

$$\text{Total change in entropy} \simeq E/T_1 \quad \text{J/K}$$

For this case, the same total entropy increase occurs regardless of the irreversibilities of the thermodynamic system. The systems differ, however, in the amount of mechanical work that was available before it was eventually converted to heat.

The combustion of fossil fuels and the fissioning of nuclear fuels result in an ultimate entropy increase. Approximately half of the total energy consumed is used for direct heat and hence in the process is degraded to heat with an average temperature that is approximately the temperature of the earth's surface ( $13^\circ\text{C}$ ). The other half is used to produce mechanical work, used primarily to drive electrical generators and for transportation. The mechanical work is eventually dissipated as heat. Therefore, for each unit of energy utilized, an entropy increase of approximately the reciprocal of the average temperature of the earth (286 K) results.

$$\begin{aligned}\text{Total change in entropy per unit of energy} &\simeq \frac{1}{286} \\ &\simeq .0035\end{aligned}$$

An increase in entropy implies a decrease in low-entropy energy sources, that is, sources of energy with a high utility, and occurs even though energy is conserved. Total entropy thus provides the measure of utility lacking for units of energy.

## 6. AVAILABLE WORK

Even though energy is always conserved, something is generally lost when it is used to accomplish desired tasks (when, according to common usage, energy is "consumed"). The entropy increase that occurs because of irreversibilities is a measure of this loss.

The same energy measure is generally used for both work, that is, ordered motion, and for thermal energy, a random motion of molecules and atoms. Based upon the Second Law of Thermodynamics, only a fraction of a given quantity of thermal energy, even when using an ideal thermodynamic system, may be converted to work. This fraction, referred to as the *Carnot efficiency*, depends upon the temperature of the thermal energy source (and the low-temperature sink); the higher its temperature, the greater the quantity of work that may be obtained. This suggests the desirability of an alternative energy specification based upon the amount of work, for a specified ambient temperature, that can be obtained from an energy source with a reversible thermodynamic system. This is known as *available work* or *availability*.<sup>\*</sup> For a quantity of thermal energy,  $Q$ , with a temperature,  $T$ , the available work,  $B$ , depends upon the Carnot efficiency.

$$B = \eta_c Q = (1 - T_A/T)Q$$

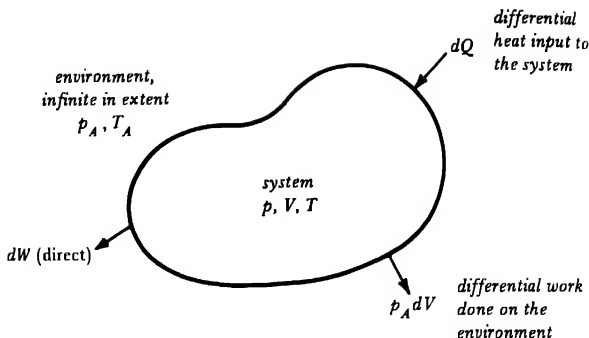
As mentioned, work is also dependent upon the ambient temperature,  $T_A$ .

While specified for thermal energy, the concept of available work may be expanded to all forms of energy: chemical, gravitational, kinetic, nuclear, etc. It is the maximum amount of work that can be obtained from an energy source with an ideal reversible system. Since work, mechanical energy, is the basis of this measure, it is treated as having the highest value or quality. This is correct for most applications, since even with irreversibilities to accomplish a particular task less energy in the form of work is required than if any other form of energy were used.

The available work of a system, for example, a kilogram-mole of a gas, depends upon the state of the system as well as the condition of the surrounding environment. By definition, available work is the maximum amount of useful work that can be obtained from the system in the process of bringing the system into equilibrium with the environment (Figure 4.22). The environment is assumed to be infinite in extent and hence it is not affected by a change of the system. Since the maximum useful work is to be obtained, the change of the system must be reversible. Transfers of

<sup>\*</sup>The American Physical Society's study, *Efficient Use of Energy*, recommends that the concept of available work be stressed [22]. Following the notations of the APS study, the symbol  $B$  (*bereitstehender brauchbarer Arbeitsbestand*) will be used for available work. While this is not a new concept, it tends to receive little attention in the more frequently used thermodynamic treatments. A 1941 text by Keenan [23], now available in paperback from the M.I.T. Press, provides an excellent treatment of this topic.

Available work is closely related to the concept of "lost work," a measure also neglected in recently published thermodynamic texts.



**FIGURE 4.22** *A Thermodynamic System Surrounded by an Infinite Environment.*

only heat and work between the system and the environment will be considered.

In bringing the system into equilibrium with the environment, useful work,  $dW(\text{direct})$ , may be obtained directly from the system. In addition, a heat transfer,  $dQ$ , provides an opportunity for obtaining work owing to the temperature difference between the system and the environment. According to the notation of Figure 4.22,  $dQ$  is positive if there is a heat transfer into the system. For this case, the system temperature,  $T$ , is less than the temperature of the environment.\* A heat engine could use the heat transfer from the environment to produce a useful quantity of work,  $dW_Q$ . The heat input for such a heat engine would be  $dQ + dW_Q$ .

$$\text{Differential heat input} = dQ + dW_Q$$

$$dW_Q = \eta_c(dQ + dW_Q)$$

$$= \frac{\eta_c dQ}{1 - \eta_c} = \frac{T_A - T}{T} dQ$$

If  $dQ$  is negative (heat is transferred from the system to the environment),  $T_A$  is less than  $T$ . The role of the temperatures in the Carnot efficiency is reversed and the resultant expression for  $dW_Q$  is the same as that above. A positive quantity of useful work is obtained regardless of the direction of the heat transfer.

The differential work done by the system may be considered as the sum of two quantities, that obtained directly,  $dW(\text{direct})$ , and that done on the environment.

$$dW = p dV = dW(\text{direct}) + p_A dV$$

The work done on the environment,  $p_A dV$ , depends upon the pressure of the environment, not that of the system. Using the first-law energy balance for the system, the following is obtained.

---

\* For this condition,  $\eta_c = 1 - T/T_A$ , since  $T$  is less than  $T_A$ .

$$\begin{aligned}
 dW &= dQ - dU \\
 dW(\text{direct}) &= dW - p_A dV \\
 &= dQ - dU - p_A dV
 \end{aligned}$$

The total useful work that can be obtained for the differential change of the system includes  $dW_Q$ .

$$\begin{aligned}
 \text{Total differential quantity of useful work} &= dW(\text{direct}) + dW_Q \\
 &= dQ - dU - p_A dV + \frac{T_A - T}{T} dQ \\
 &= T_A \frac{dQ}{T} - dU - p_A dV
 \end{aligned}$$

If the available work of the system is  $B$ , the differential work obtained in changing the system is the differential decrease,  $-dB$ , in the available work. Since the change is reversible, the differential change,  $dQ/T$ , is the change in entropy of the system.

$$dB = dU - T_A dS + p_A dV$$

The right-hand side of the above expression involves only differentials of state variables ( $T_A$  and  $p_A$  are constants). Therefore, available work,  $B$ , is a function only of the state of the system.

$$B = (U - U_A) - T_A(S - S_A) + p_A(V - V_A)$$

The subscripted quantities,  $U_A$ ,  $S_A$ , and  $V_A$ , are values of these variables corresponding to the system's being in equilibrium with the surrounding environment. If in bringing the system to equilibrium, its volume decreases ( $V_A < V$ ), work will be done by the environment on the system. Available work, a measure of the energy potential of a system, reflects this energy gain. If the volume of the system increases ( $V_A > V$ ), work done on the environment decreases the system's work-producing potential.

**Example** The available work,  $B$ , of one kilogram-mole of a gas at a temperature of  $T$  is desired. The gas is at the same pressure as the environment,  $p = p_A$ , and behaves as an ideal gas with  $n$  degrees of freedom. Determine the ratio of the available work to the thermal energy that could be obtained from the gas.

*Solution*

Expressions for the change in internal energy, volume, and entropy of the gas must be determined. Internal energy depends directly upon temperature.

$$U - U_A = \frac{n}{2} R(T - T_A)$$

An expression for the change in entropy for one kilogram-mole of an ideal gas with  $n$  degrees of freedom was obtained in the previous section.

$$S - S_A = \frac{n}{2} R \ln T/T_A + R \ln V/V_A$$

Since the pressure is constant, the change in entropy may be expressed as a function of only the temperature.

$$V/V_A = T/T_A$$

$$S - S_A = \frac{n+2}{2} R \ln T/T_A$$

The effect of the volume change may also be expressed in terms of temperature.

$$P_A(V - V_A) = R(T - T_A)$$

Finally, an expression for available work may be written.

$$\begin{aligned} B &= (U - U_A) - T_A(S - S_A) + P_A(V - V_A) \\ &= \frac{n+2}{2} R(T - T_A) - \frac{n+2}{2} R T_A \ln T/T_A \end{aligned}$$

The available work,  $B$ , it will be noted, is positive regardless of the sign of the temperature difference,  $T - T_A$ .

The first term on the right-hand side of the expression for available work will be recognized as the thermal energy that is, in effect, stored by the system.

$$p \, dV = R \, dT \quad \text{constant pressure}$$

$$dQ = dU + p \, dV$$

$$= \frac{n}{2} R \, dT + R \, dT = \frac{n+2}{2} R \, dT$$

$$Q = \frac{n+2}{2} R(T - T_A)$$

The coefficient of the temperature difference in the above expression is the specific heat of an ideal gas at constant pressure.

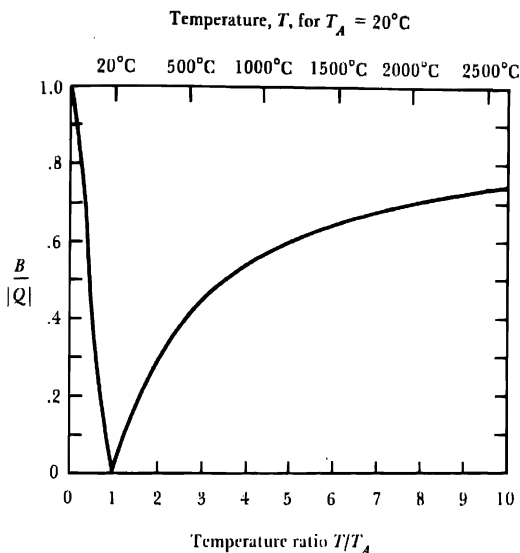
$$C_p = \frac{n+2}{2} R$$

Dividing by  $Q$ , the desired result is obtained.

$$B/Q = 1 - \frac{T_A}{T - T_A} \ln T/T_A$$

While available work,  $B$ , is positive regardless of the sign of the temperature difference, the thermal energy,  $Q$ , depends upon the sign of the temperature difference. The magnitude of  $Q$  is the quantity of heat that could be transferred from the higher to lower temperature. The ratio of  $B/|Q|$ , a quantity that is always positive, is indicated in Figure 4.23.

Fossil fuels are the energy input of many energy conversion systems. The energy value for a fuel is usually specified as its heating value determined with a calorimeter. Since water is a combustion product of hydrocarbon fuels, heating values for these fuels depend upon the final state of the water. A fuel's "high heating value"



**FIGURE 4.23** *Availability for a Finite Volume of an Ideal Gas ( $p = p_A$ ).*

corresponds to the water being condensed to its liquid state. An alternative specification, "low heating value," corresponds to the water being exhausted as a vapor. These two heating values differ by the latent heat of condensation of the combustion-produced water. While the final state for the water has a significant effect on the heating value of a fuel (10% for methane), it has little effect on the available work of the fuel. At atmospheric pressure, the condensation process occurs

**TABLE 4.1** *Available Work of Fossil Fuels*

Fuel	Heat of combustion <sup>a</sup> MJ/kg-mole	Available work <sup>b</sup> MJ/kg-mole	Percent difference
Carbon, C	394	395	+ .2
Methane, CH <sub>4</sub>	802	809	+ .8
Ethane, C <sub>2</sub> H <sub>6</sub>	1428	1455	+ 1.9
Propane, C <sub>3</sub> H <sub>8</sub>	2044	2093	+ 2.4
Ethylene, C <sub>2</sub> H <sub>4</sub>	1323	1325	+ .1
Liquid octane, C <sub>8</sub> H <sub>18</sub>	5075	5268	+ 3.8

<sup>a</sup>Low heating values (water vapor).

<sup>b</sup>Without diffusion. Adapted from data of the APS study *Efficient Use of Energy*.

at a temperature of  $100^{\circ}\text{C}$ , a temperature at which only a very small quantity of work may be obtained.

The available work of a fuel may be determined through a generalized interpretation of the expression already developed for a closed system. The combustion system initially consists of a mixture of fuel and air or oxygen. A small entropy increase occurs with the initial mixing of the fuel and air or oxygen. In addition, the combustion products are eventually released to the environment resulting in a further entropy increase. But when all effects are considered, the available work of commonly utilized fossil fuels is found to be close to the "low heating value" of the fuel. Table 4.1 summarizes data prepared by the American Physical Society.

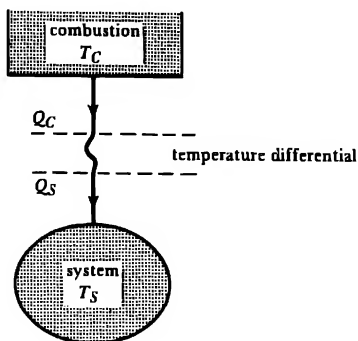
## 7. SECOND-LAW EFFICIENCIES

To illustrate the role of second-law efficiencies, a conventional space heating system (Figure 4.24) will be considered. In a furnace, a high-temperature combustion process is used and the combustion-produced thermal energy is transformed to the heating fluid, air or water, with a heat exchanger. The system temperature,  $T_s$ , is rather low,  $40$  to  $50^{\circ}\text{C}$  for warm air heating and less than  $100^{\circ}\text{C}$  for water systems. The combustion temperature is much higher; in principle it could be about  $2000^{\circ}\text{C}$  for commonly used fossil fuels. The available work of the combustion-produced thermal energy and of the system thermal energy,  $B_C$  and  $B_S$  respectively, may readily be obtained.

$$B_C = (1 - T_A/T_C)Q_C$$

$$B_S = (1 - T_A/T_S)Q_S$$

If the heat exchanger has no heat losses, the thermal energies,  $Q_C$  and  $Q_S$ , will be



**FIGURE 4.24** A Conventional Heating System.

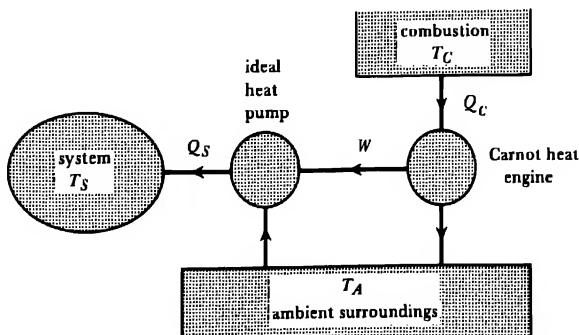
equal and, since the system temperature is less than the combustion temperature, available work is lost ( $B_S < B_C$ ). An overall entropy increase occurs.

$$\begin{aligned}(S_2 - S_1)_{\text{total}} &= (S_2 - S_1)_{\text{system}} + (S_2 - S_1)_{\text{surr}} \\ &= \frac{Q_S}{T_S} - \frac{Q_C}{T_C} \\ &= Q_C \left( \frac{1}{T_S} - \frac{1}{T_C} \right) \quad \text{for } Q_S = Q_C\end{aligned}$$

Because of the temperature difference of the heat exchanger, the irreversible heat flow results in an entropy increase, that is, a loss in thermodynamic potential. The loss in available work,  $B_C - B_S$ , is another measure of the loss of potential. A heat engine, substituted for the heat exchanger, could utilize the temperature difference to convert part of the thermal energy to work. Alternatively, the combustion-produced thermal energy could be used for an energy input of an ideal reversible heat engine and heat pump system (Figure 4.25).

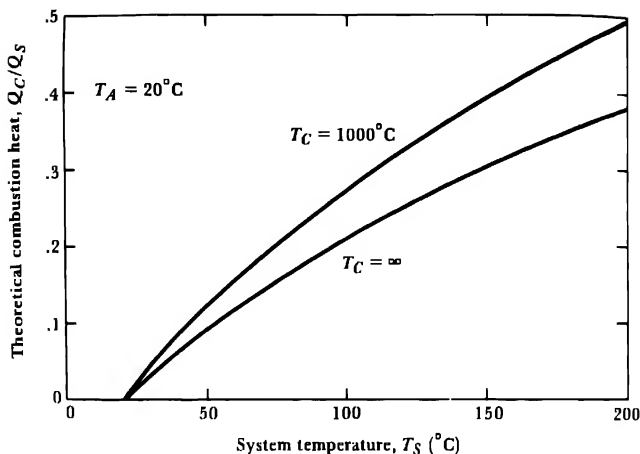
$$\begin{aligned}W &= \eta_C Q_C = (1 - T_A/T_C) Q_C \\ Q_S &= \frac{W}{1 - T_A/T_S} = \frac{1 - T_A/T_C}{1 - T_A/T_S} Q_C \\ \frac{Q_C}{Q_S} &= \frac{1 - T_A/T_S}{1 - T_A/T_C}\end{aligned}$$

As indicated in Figure 4.26, the combustion-produced heat that is theoretically required for low-temperature applications is only a small fraction of that normally used. Compared to an ideal heat engine and heat pump system, a conventional furnace, even ignoring heat losses, is very inefficient. Since there is no entropy increase for the ideal system, the available work of the low-temperature thermal energy used



**FIGURE 4.25** An Ideal Heating System.





**FIGURE 4.26** *Combustion-Produced Heat Theoretically Required for a Heating System.*

for heating,  $Q_S$ , is the same as that of the combustion-produced thermal energy,  $Q_C$ , albeit that the quantities of energy are unequal ( $Q_S > Q_C$ ).

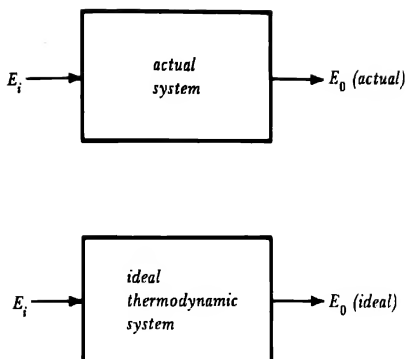
$$B_S = (1 - T_A/T_S)Q_S = (1 - T_A/T_C)Q_C$$

$$= B_C$$

The concept of efficiency depends upon a standard of comparison. As normally interpreted, the standard of comparison is an ideal, reversible system. This comparison is illustrated in Figure 4.27 in which the energy inputs of the two systems are identical. An efficiency measure based upon this comparison is the quotient of the energy outputs.

$$\varepsilon = \frac{E_o(\text{actual})}{E_o(\text{ideal})}$$

The inputs for this comparison are truly equal in both quantity and quality (equal temperatures, for example, if thermal energy). Epsilon ( $\varepsilon$ ) has been used to distinguish this measure from the input-output efficiency measure,  $\eta$ , that has previously been used. Since a thermodynamically ideal system, constrained by the Second Law of Thermodynamics has been used for a comparison, the above efficiency,  $\varepsilon$ , is referred to as the *second-law efficiency*. The input-output measure,  $\eta$ , limited only by the First Law of Thermodynamics (output is equal to input for an "ideal" system), is referred to as a *first-law efficiency*.



**FIGURE 4.27** *A Comparison of an Actual and Ideal Thermodynamic System for Determining a Second-Law Efficiency.*

The first-law efficiency measure used in the previous sections for heat engines was defined as the output work divided by the input heat.

$$\eta = W/Q$$

The energy quantities,  $W$  and  $Q$ , are not the same: one is work and the other is thermal energy. The Carnot efficiency, again a first-law efficiency measure, is the efficiency of an ideal, reversible heat engine, and for a finite high-temperature source,  $T_1$ , is necessarily less than unity.

$$\eta_c = 1 - T_1/T_2$$

The efficiency,  $\eta_c$ , tends to suggest that something has been lost even though the ideal engine is reversible. However, no loss has occurred (there has been no increase in entropy) since the original conditions can be restored with a reversible heat pump. This efficiency measure tends to imply that an engine could be devised which would convert all input thermal energy to work in violation of the Second Law of Thermodynamics.\*

Based upon the definition of the second-law efficiency, the efficiency of a reversible heat engine,  $\epsilon_c$ , regardless of the temperature of the input heat source, is always 1.

$$\epsilon_c = 1$$

The second-law efficiency of an irreversible engine, operating between thermal sources

\* It should be emphasized that this efficiency,  $\eta = W/Q$ , is never introduced in such a manner as to suggest this conclusion. Rather, it is the conventional use of the concept of efficiency that gives this connotation.

of the same temperature, is less than one. An expression for its second-law efficiency, in terms of its first-law efficiency, may readily be obtained.

$$W(\text{actual}) = \eta Q$$

$$W(\text{ideal}) = \eta_c Q$$

$$\varepsilon = \frac{W(\text{actual})}{W(\text{ideal})} = \eta/\eta_c$$

$$\varepsilon = \frac{\eta}{1 - T_1/T_2}$$

If  $T_1$  is equal to the ambient temperature,  $T_A$ , the efficiency may be expressed in terms of available work.

$$\varepsilon = \frac{\eta Q}{(1 - T_A/T_2)Q} = \frac{W(\text{actual})}{B}$$

The output of the heat engine would be  $B$ , the available work, if the heat engine were ideal.

Comparing the input thermal energies for the heating system of Figure 4.24 with that of an ideal system, Figure 4.25, the following second-law efficiency is obtained.

$$\varepsilon = \frac{Q_S(\text{ideal})}{Q_S(\text{actual})} = \frac{1 - T_A/T_S}{1 - T_A/T_C}$$

This expression, assuming that no heat escapes from the heat exchanger (a first-law efficiency of one), is rather small for normal heating temperatures. It will be noted that this ratio is equal to the ratio of the available work of the system's thermal energy to that of the combustion-produced heat of Figure 4.24 ( $Q_S = Q_C$ ).

$$B_S = (1 - T_A/T_S)Q_S$$

$$B_C = (1 - T_A/T_C)Q_C$$

$$\varepsilon = B_S/B_C \quad \text{for } Q_S = Q_C$$

The second-law efficiency for a conventional heating system is reduced to output quantity divided by input quantity with the quantities being available work.

The second-law efficiency of many energy conversion systems may be determined by comparing the performance of the actual system to that of an ideal system. A unique ideal system, however, cannot be devised when the actual system has multiple inputs and outputs of energy. It is then necessary to use available work to determine the second-law efficiency. In this case, the system's efficiency depends upon the minimum available work,  $B(\text{min})$ , that would provide the same energy outputs from an ideal reversible system. The reference quantity for the efficiency measure is the actual available work,  $B(\text{actual})$ , associated with the various inputs.

$$\varepsilon = \frac{B(\min)}{B(\text{actual})}$$

For many systems,  $B(\min)$  is equal to the available work of the system's energy outputs.\* The second-law efficiency is then the ratio of the output to input available work, a measure similar to that used for first-law efficiencies except that available work rather than energy is used for the comparison. Available work, unlike energy, is used up by irreversible processes.

Since the available work of a fossil fuel is approximately equal to its "lower heating value" (Table 4.1), the second-law efficiency of a conventional heat engine, based upon its fuel input, is approximately equal to its output work divided by the heat of combustion of the fuel.

$$\varepsilon = \frac{W(\text{out})}{E(\text{input fuel})}$$

Its second-law efficiency and its first-law efficiency are essentially equal. This was not the case when the input for the heat engine was thermal energy. Its second-law efficiency depended upon the available work of the thermal energy source and was greater than its first-law efficiency ( $\varepsilon \geq \eta$ ). Often, however, the input thermal energy is derived from the combustion of a fossil fuel. If flue and other thermal losses can be ignored, the thermal input is equal to the heat of combustion of the fuel.

$$Q = E(\text{input fuel})$$

$$\eta = W(\text{out})/Q = W(\text{out})/E(\text{input fuel})$$

$$= \varepsilon$$

The second-law efficiency, based upon the energy value of the fuel input, is thus equal to the first-law efficiency based upon the thermal energy input. The first-law thermal efficiency,  $W/Q$ , normally determined for heat engines is thus important.

A first-law efficiency may provide a valid measure for the performance of a heat engine, but it tends to be misleading in judging the performance of other systems. This occurred for the heating system previously considered. When the input of a system is work, such as for an electrical resistance heater, a first-law efficiency rating is inappropriate. From a first-law perspective, a resistance heater is 100% efficient if no energy is lost. Resistance heaters, however, utilize an irreversible process in which electricity, a high-quality form of energy equivalent to work, is converted to low-temperature thermal energy. Resistance heaters are essentially equivalent to generating thermal energy through the use of a frictional brake placed on the output shaft of a heat engine (the only difference is convenience of distributing electrical energy). An ideal heating system would use an electrically driven heat pump. If the electrical input energy is  $W$ , the following is obtained.

\* This is the case if no work is done on (or by) the surrounding environment.

$$Q_S(\text{resistance}) = W$$

$$Q_S(\text{ideal}) = \frac{T_S}{T_S - T_A} W$$

$$\varepsilon = \frac{Q_S(\text{resistance})}{Q_S(\text{ideal})} = 1 - \frac{T_A}{T_S}$$

The notation is the same as that previously used:  $T_S$  is the system temperature and  $T_A$  the ambient temperature. A resistance heater with  $T_S = 50^\circ\text{C}$  and  $T_A = 20^\circ\text{C}$  has a second-law efficiency of only 9.3%.

Electrical resistance heaters are used not only for space heating but also for water heating. For this task, a quantity of water, initially at the ambient temperature, is warmed to a desired temperature,  $T_W$ . Not all the thermal energy needs to be supplied at the final temperature,  $T_W$ . The minimum quantity of work necessary to warm the water is that required by the ideal heat pump of Figure 4.28. A differential quantity of heat,  $dQ_W$ , at a temperature of  $T$ , requires a differential input quantity of work,  $dB$ .

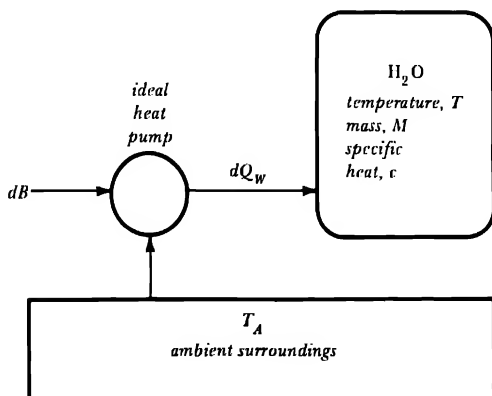
$$dB = (1 - T_A/T) dQ_W$$

The thermal input to the water system (mass of  $M$  and specific heat per unit of mass,  $c$ ) results in a differential change of temperature,  $dT$ .

$$dQ_W = cM dT$$

$$dB = cM(1 - T_A/T) dT$$

The minimum work required is that necessary to increase the temperature of the water from  $T_A$  to  $T_W$ .



**FIGURE 4.28** *An Ideal Water Heater.*

$$B(\min) = \int_{T_A}^{T_W} dB = cM \int_{T_A}^{T_W} (1 - T_A/T) dT$$

$$= cM(T_W - T_A) - cMT_A \ln T_W/T_A$$

For a resistance heater with no thermal losses (a first-law efficiency of 100%), the actual work used (equal to the electrical energy) is equal to the thermal energy of the warmed water.

$$B(\text{actual}) = cM(T_W - T_A)$$

This results in the following second-law efficiency for a conventional electric water heater.

$$\varepsilon = \frac{B(\min)}{B(\text{actual})}$$

$$= 1 - \frac{T_A}{T_W - T_A} \ln T_W/T_A$$

For a required water temperature of 50°C and an ambient temperature of 20°C, the second-law efficiency is 4.8%. Owing to thermal losses, the second-law efficiency of a heater fueled with a fossil fuel is even less. Since the available work of a fuel is approximately equal to its heating value, the following is obtained.

$$\varepsilon = \eta \left( 1 - \frac{T_A}{T_W - T_A} \ln T_W/T_A \right)$$

The first-law efficiency,  $\eta$ , accounts for the thermal losses.

A loss of available work can be associated with the combustion of a fuel. Combustion-produced thermal energy would have to be at an infinitely high temperature for its available work to be equal to that of the fuel. The combustion of fossil fuels, in air, results in a maximum flame temperature of approximately 2000°C, a temperature only slightly dependent upon the type of fuel. The thermal energy of the combustion products is equal to the heat of combustion of the fuel.

$$Q_C = c_p(T_C - T_A) \quad (\text{one kilogram-mole})$$

The coefficient,  $c_p$ , is the specific heat for constant pressure and  $T_C$  is the combustion temperature. If the combustion products behave as an ideal gas, then the combustion temperature and heat of combustion may readily be related.

$$Q_C = \frac{n+2}{2} R(T_C - T_A)$$

For an input of thermal energy with a temperature of 2000°C, the efficiency of a Carnot heat engine is 87.1% ( $T_A = 20^\circ\text{C}$ ). The finite combustion temperature corresponds to a loss of 12.9% of the fuel's available work. However, not all the combustion-produced thermal energy may be converted to work at this efficiency.

The combustion products at an elevated temperature and at atmospheric pressure may be considered a thermodynamic system. The available work of this

system is that obtained when the products are cooled, reversibly, to the ambient temperature. As the products are cooled, the efficiency of an ideal heat engine which converts the thermal energy to work decreases. To the extent that the combustion products behave as an ideal gas, the results of the example problem in Section 6 apply (Figure 4.23). A temperature of 2000°C for the system yields a quantity of available work that is approximately 70% of the thermal energy. Since for combustion the system's thermal energy is approximately equal to available work of the fuel, approximately 30% of the fuel's available work is lost in the combustion process. Therefore, the overall efficiency of a heat engine (both first and second law) dependent upon the combustion of a fossil fuel in air will never exceed 70%. Owing to other irreversibilities, the overall efficiency will be considerably less.

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## PROBLEMS

1. Determine the molecular density (in molecules per cubic meter and molecules per cubic centimeter) for air at a pressure of one atmosphere and a temperature of  $20^{\circ}\text{C}$ . Air may be assumed to behave as an ideal gas.
2. What is the velocity of an average oxygen molecule,  $\text{O}_2$  (average kinetic energy  $= \frac{3}{2}kT$ ) for  $T = 300\text{ K}$ . An oxygen molecule has two additional degrees of freedom associated with vibrational energy, that is a total energy of  $\frac{5}{2}kT$ . Determine the total energy (in joules) of a kilogram-mole of oxygen at a temperature of  $300\text{ K}$ . What is this expressed in Btu?
3. Consider a Carnot cycle for which the upper temperature  $T_2$  is  $600\text{ K}$  and the lower temperature  $T_1$  is  $300\text{ K}$ . The efficiency is obviously .5 or 50%. For the high-temperature isothermal expansion the expansion ratio,  $V_b/V_a$ , is equal to five. The working fluid is air which behaves as an ideal gas with five degrees of freedom.
  - (a) Using the notation of the text, calculate the ratios  $V_c/V_a$  and  $V_d/V_a$ . What are the corresponding pressure ratios,  $p_b/p_a$ ,  $p_c/p_a$ ,  $p_d/p_a$ ?
  - (b) Sketch, to scale, the corresponding  $p$ - $V$  diagram. Assume pressure and volume are measured in units of  $V_a$  and  $p_a$ , that is, these quantities may be considered to have a value of one.
4. A Carnot engine of Problem 3 contains one kilogram-mole of air.
  - (a) Determine, for the path  $a$ - $b$ - $c$ , the mechanical work output, that is,  $\int p dV$  for the path.
  - (b) What is the work necessary for the compression  $c$ - $d$ - $a$ ?
  - (c) What is the net work output,  $W$ ? Is this indeed 50% of  $Q_2$ ? Consider a more practical size engine of  $10^{-4}\text{ kg-mole}$  of air in which point  $c$  is at atmospheric pressure and a temperature of  $20^{\circ}\text{C}$ .
  - (d) What is  $V'_c$ , the largest volume, for this engine?
  - (e) Suppose the engine goes through 60 complete cycles each second. What is its power output in kilowatts?



5. Consider a gasoline engine with an idealized Otto cycle (Figure 4.9). The compression ratio,  $r = V_1/V_2$ , is equal to 7.5.
- For  $T_a$  equal to an ambient temperature of  $20^\circ\text{C}$ , determine  $T_b$ .
  - Determine the energy,  $Q_2$ , necessary to result in an upper temperature,  $T_c$ , of  $2000^\circ\text{C}$ . Assume one kilogram-mole of air.
  - Assume the volume of the cylinder,  $V^\eta$ , is  $400\text{ cm}^3$  (a 1600-cc VW engine) and that  $p_a$  is equal to atmospheric pressure. Determine the fraction of a kilogram-mole present.
  - The heat  $Q'_2$  for one power stroke is therefore this fraction times that calculated for part (b). Find  $Q'_2$ .
  - Assume the engine is operating at a shaft speed of 3600 rpm. For a four-cylinder, four-cycle engine, this will result in 120 power strokes per second. Determine the rate of heat input (joules per second).
  - Using the Otto efficiency, what is the power output expressed in watts and horsepower?
6. Suppose the compression ratio,  $r$ , of the gasoline engine of Problem 5 is increased to 10.
- What would its output power be if  $Q'_2$  remains unchanged? What is the temperature  $T_c$  for this condition?
  - What is its output power if  $T_c$  remains unchanged, that is,  $2000^\circ\text{C}$ ?
7. Experimentally, it is often convenient to determine the specific heat of a gas at constant pressure,  $c_p$ , and at constant volume,  $c_v$ .

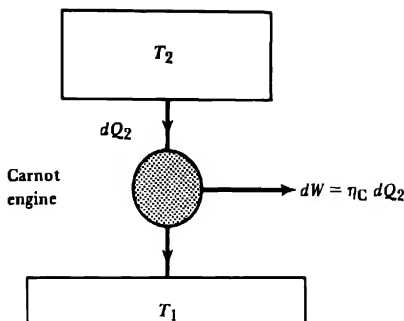
$$\begin{aligned} dQ &= c_p dT && \text{constant pressure} \\ dQ &= c_v dT && \text{constant volume} \end{aligned}$$

- Determine the specific heats for an ideal gas.
- Show that the ratio of  $c_p/c_v$  is equal to  $(n + 2)/n$ .
- Consider the following gases for which the ratios of their specific heats are specified.

		$\gamma = c_p/c_v$	
Air	1.40	Acetylene, $\text{C}_2\text{H}_2$	1.23
Hydrogen, $\text{H}_2$	1.41	Ethane, $\text{C}_2\text{H}_6$	1.18
Helium, He	1.66	Argon, Ar	1.67
Water vapor, $\text{H}_2\text{O}$	1.33	Carbon dioxide, $\text{CO}_2$	1.30
Isobutane, $\text{C}_4\text{H}_{10}$	1.09	Propane, $\text{C}_3\text{H}_8$	1.12

Determine the number degrees of freedom that can be associated with each gas.

8. A heat reservoir is to be used to store energy. Its initial temperature is  $T_{20}$ . A Carnot engine is to operate between this reservoir and an infinite low-temperature sink at a temperature of  $T_1$ . (See Figure P4.8.)



**FIGURE P4.8**

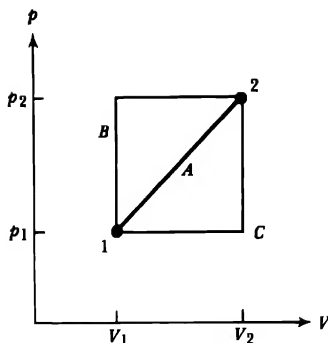
Assume the energy of the reservoir is linearly proportional to its temperature. Therefore its differential change in temperature is proportional to  $dQ_2$ .

$$K dT_2 = -dQ_2$$

- (a) Determine the total work output that may be obtained by lowering the reservoir to the sink temperature.
- (b) The initial energy of the reservoir,  $Q_{\text{stored}}$ , relative to  $T_1$  is simply  $K(T_{20} - T_1)$ . Show that the following relationship is valid for the output work.

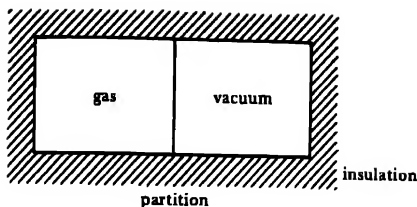
$$W_{\text{out}} = Q_{\text{stored}} \left( 1 - \frac{T_1 \ln T_{20}/T_1}{T_{20} - T_1} \right)$$

- (c) Determine the ratio of  $T_{20}/T_1$  necessary for an efficiency of one-half ( $W_{\text{out}} = \frac{1}{2} Q_{\text{stored}}$ ).
  - (d) Suppose water is used for the energy storage in which  $T_{20} = 100^\circ\text{C}$  and  $T_1 = 20^\circ\text{C}$ . Determine the efficiency expected.
  - (e) Compare the above efficiency to that of a Carnot heat engine operating between a constant temperature source at  $100^\circ\text{C}$  and a low temperature sink of  $20^\circ\text{C}$ .
9. Three reversible paths,  $A$ ,  $B$ , and  $C$ , connect an initial point ( $p_1, V_1$ ) and a final point ( $p_2, V_2$ ) on the  $p$ - $V$  diagram of Figure P4.9. Assume that the gas is ideal and behaves as a gas with  $n$  degrees of freedom. The closed system contains one kilogram-mole of gas.
- (a) Calculate the change in internal energy,  $U_2 - U_1$ , for the three paths.
  - (b) Calculate the integral of  $dW$  and  $dQ$  for the three paths. Does  $W$  or  $Q$  behave as a state variable?
  - (c) Show that the integral of  $dQ/T$  is the same for these three paths.

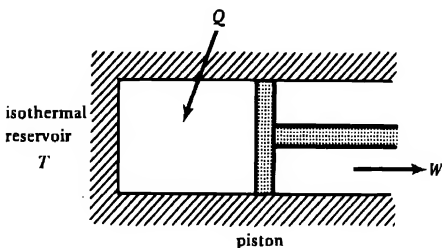


**FIGURE P4.9**

10. Assume that an electric power plant has a combustion temperature of  $1000^{\circ}\text{C}$  and exhausts its waste heat at an ambient temperature of  $20^{\circ}\text{C}$ .
  - (a) What would the overall efficiency of the power plant need to be to result in a zero increase in total entropy? Assume the power plant has an actual overall efficiency of 40%.
  - (b) Calculate, assuming cyclic operation of the power plant, the input and waste heat for an output of one kilowatt-hour of electrical energy. What is the entropy increase (J/K) associated with the 1 kWh of electrical energy output?
  - (c) Assume that resistive power line losses result in 10% of the output electrical energy being converted to heat (at the ambient temperature). What is the entropy increase for the 1 kWh of electrical energy associated with this loss?
  - (d) Assume that the remaining electrical energy (0.9 kWh) is converted to heat and degraded to the ambient temperature by the consumer. What is the entropy increase caused by the consumer?
  - (e) What is the sum of the entropy increases for parts (b), (c), and (d)? Is this equal to that which would be expected for a simple temperature degradation of the combustion produced heat to heat at the ambient temperature?
11. An example of an irreversible thermodynamic process is the sudden expansion of a gas. Assume that one kilogram-mole of an ideal gas is initially contained in the left-hand portion of the container of Figure P4.11a. If the partition separating the portions is removed, the gas will expand from its initial volume of  $V_1$  to a final volume of  $V_2$ . For a container insulated from its surroundings, no heat is transferred to the gas and furthermore, since no work is performed by the gas, its internal energy remains unchanged. As a consequence, the temperature of the gas after the expansion is equal to its initial temperature,  $T$ .
  - (a) In order to calculate the entropy increase for a gas, a reversible path for the expansion must be used. The expansion could have been accomplished in a reversible manner by allowing the gas to slowly expand against a piston



**FIGURE P4.11a**



**FIGURE P4.11b**

- (Fig. P4.11b). Calculate the heat transferred to the gas for this expansion. What is the corresponding work output?
- (b) What is the entropy increase for the gas for the reversible expansion? What is the entropy change for the surroundings and the total entropy change associated with the reversible expansion?
- (c) What is the entropy increase for the gas for the sudden irreversible expansion? What is the total entropy change for this case?
12. What is the quantity of available work that can be associated with a vacuum? Assume that the volume corresponds to that of one kilogram-mole of air at atmospheric pressure and a temperature of  $20^\circ\text{C}$ . Express the result in joules, British thermal units, and kilowatt-hours.
13. A thermodynamic system consists of one kilogram-mole of air at atmospheric pressure and temperature ( $p_A = 1.01 \times 10^5 \text{ n/m}^2$ ,  $T_A = 20^\circ\text{C}$ ). Assume that air behaves as an ideal gas with five degrees of freedom.
- (a) A quantity of thermal energy,  $10^6 \text{ J}$ , is used to heat the system. What is the available work of the system if its volume remains constant?
- (b) Obtain an expression for the available work of the system,  $B$ , in terms of the thermal energy supplied,  $Q$ , if the volume remains constant. Why does the ratio,  $B/Q$ , increase as  $Q$  increases?

- 14.** Compressed air can be used for energy storage. Assume that one kilogram-mole of air (five degrees of freedom) is compressed adiabatically and reversibly to a pressure ten times that of atmospheric pressure. The ambient temperature,  $T_A$ , is  $20^\circ\text{C}$ .
- What are the temperature and pressure of the compressed air?
  - What is the work done in compressing the air,  $\int p \, dV$ ? What is the available work of the compressed air? Why do these quantities differ? What is the mechanical work that needs to be supplied to the compressor when the effect of the atmosphere is taken into account,  $\int (p - p_A) dV$ ? The compressed air is stored in a noninsulated container and eventually comes to thermal equilibrium with the environment.
  - What is the resultant pressure of air?
  - Determine the overall entropy increase associated with the heat transfer.
  - What is the available work of the compressed air after thermal equilibrium is achieved?
  - What is the overall efficiency of the compressed air system?  
Suppose, for the above problem, only a portion of the thermal energy is lost in the storage of the compressed air.
  - What is the overall efficiency if only 10% of the thermal energy is lost?
  - What is the overall efficiency for a loss of 50% of the thermal energy?
- 15.** For a particular space heating system, warm air with a temperature of  $40^\circ\text{C}$  is sufficient. A conventional natural gas furnace with a first-law efficiency of 70% is used.
- What is the second-law efficiency of the furnace for an outdoor (ambient) temperature of  $0^\circ\text{C}$ ?
  - What is the second-law efficiency of the furnace for an outdoor temperature of  $-20^\circ\text{C}$ ?
  - For the above two cases, what is the second-law efficiency of the furnace in providing warmed air at a temperature of  $20^\circ\text{C}$ ?
- 16.** An electrically driven heat pump used for space heating has a coefficient of performance (COP) of 3. The heat supplied is three times the electrical energy input. The ambient outdoor temperature is  $0^\circ\text{C}$ .
- What is the second-law efficiency of the heat pump if it supplies thermal energy at a temperature of  $40^\circ\text{C}$ ?
  - What would its coefficient of performance need to be if its second-law efficiency in supplying heat at  $40^\circ\text{C}$  were 50%?
- 17.** A water heater provides a household with 200 liters of water at a temperature of  $60^\circ\text{C}$ . The entering water is at an ambient temperature of  $20^\circ\text{C}$ .
- What is the thermal energy, expressed in joules and kilowatt-hours, necessary to heat the water?
  - What is the minimum quantity of available work that is needed to heat the water? Assume that the entering water is heated in a reversible fashion, that is, thermal energy is supplied at the water's temperature.

- (c) What is the second-law efficiency of an electrical resistance heater used to heat the water if it has a first-law efficiency of 100%?
18. Suppose the entering water for Problem 17 is  $10^{\circ}\text{C}$  and the ambient temperature remains  $20^{\circ}\text{C}$ .
- What is the available work of the entering water (200 liters)?
  - What is the minimum available work necessary to heat the water to  $60^{\circ}\text{C}$ ?
  - What is the second-law efficiency of an electrical resistance heater used to heat the water from 10 to  $60^{\circ}\text{C}$ ?
19. An air conditioner requires an input of 1.5 kWh of electrical energy to result in a cooling of 12,000 Btu. (This corresponds to an energy efficiency rating (EER) of  $12/1.5 = 8$ .) Assume that the ambient outdoor temperature is  $40^{\circ}\text{C}$  and the indoor air is cooled to a temperature of  $10^{\circ}\text{C}$ .
- What is the coefficient of performance,  $Q(\text{cooling})/E(\text{input})$ , for the air conditioner?
  - What is the minimum work (and hence electrical energy) necessary to provide 12,000 Btu of cooling at a temperature of  $10^{\circ}\text{C}$  for an ambient temperature of  $40^{\circ}\text{C}$ ? What is the second-law efficiency of the air conditioner relative to this input?
  - Suppose the indoor air is cooled reversibly from an indoor room temperature of  $25^{\circ}\text{C}$  to a temperature of  $10^{\circ}\text{C}$ . What is the minimum work now needed? What, relative to this quantity, is the second-law efficiency of the air conditioner?
20. Consider a block of ice with a mass of one metric ton and a temperature of  $0^{\circ}\text{C}$ . The ambient temperature is  $20^{\circ}\text{C}$ . The specific heat of water is approximately  $1 \text{ cal/g}^{\circ}\text{C}$  and the heat necessary for melting the ice, latent heat of fusion, is  $80 \text{ cal/g}$ .
- Determine the quantity of thermal energy, expressed in joules, necessary to melt the ice.
  - Determine the available work of the block of ice by calculating the work obtained in melting the ice with a reversible heat engine which uses the ice (or water) for exhausting its waste heat.
21. Consider the task of heating one kilogram of water at an ambient temperature of  $20^{\circ}\text{C}$  to its boiling point of  $100^{\circ}\text{C}$ . Its specific heat is  $1 \text{ cal/g}^{\circ}\text{C}$ .
- What, in kilowatt-hours, is the thermal energy required?
  - What is the minimum work, in kilowatt-hours, necessary to heat the water?
  - What is the second-law efficiency for heating the water with an electric range in which half the thermal energy is lost to the environment?
  - Determine the additional quantity of thermal energy and work necessary to boil the water and convert it entirely to steam at  $100^{\circ}\text{C}$ . The heat vaporization for water at atmospheric pressure is  $580 \text{ cal/g}$ .

## CHAPTER 5

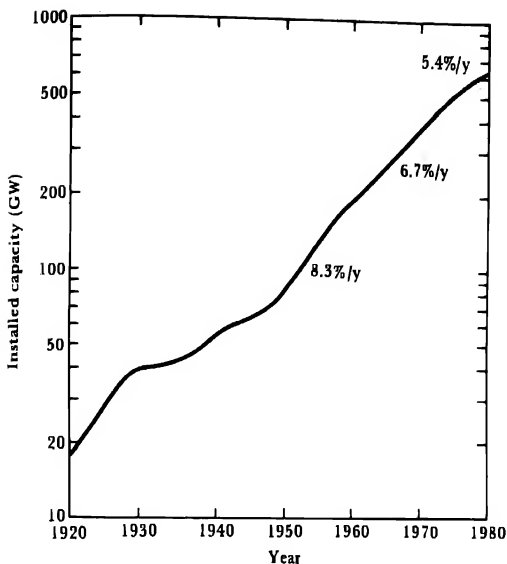
# *Electrical Energy from Fossil Fuels*

### 1. A PERSPECTIVE

Prior to 1973, the consumption of electrical energy in the United States had been steadily increasing at an average annual rate of 7%. Over the last five years of the 1970s, however, the average increase, based upon utility sales, was only 3.2%/y [1]. Because of rapidly increasing fuel and generating equipment costs, a decline in the usage of electrical energy is likely. Over the 1980 to 1982 period consumption remained essentially unchanged.

At the end of 1979, the generating capacity of electric utilities in the United States was 598 GW. Figure 5.1 shows the increase in generating capacity that has occurred since 1920 [1-2]. During the 1950s the average growth rate was 5.4%/y, while during the 1960s it was 6.7%/y. The average growth rate for the 1970s was 5.4%/y. As a result of reduced growth rates for electrical energy consumption, increases in generating capacity over the next decades are expected to be much smaller than the 6 to 7%/y average of the first 70 years of the century.

Based upon the 1979 U.S. population of approximately 220 million, the installed electric generating capacity was 2.8 kW per capita. Dividing by the 1979 energy consumption of  $2.34 \times 10^{12}$  kWh by 8760 h per year, an average electric power of 267 GW is obtained. This is 43% of the installed capacity. On a per capita basis, the average U.S. electrical energy consumption rate was 1.2 kW—approximately six times the world's average. An installed capacity greater than twice the average power was necessary to provide for both daily and seasonal fluctuations in demand (and for repairs and maintenance of equipment). Based upon a fuel conversion efficiency of about one-third, the 1.2 kW per capita U.S. consumption of electrical

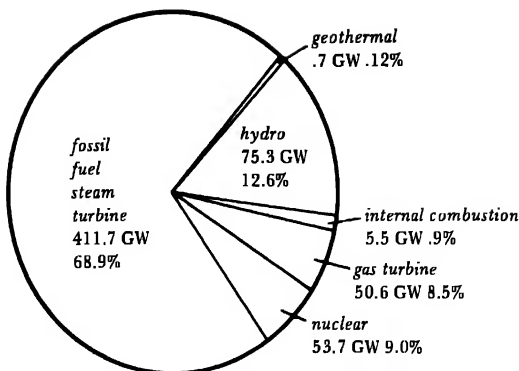


**FIGURE 5.1** *United States Electric Generating Capacity.*

power corresponds to a thermal power of 3.6 kW (31% of the 1979 total energy consumption rate of 11.9 kW per capita). On a world basis, energy used for the production of electricity is approximately one-fourth of total consumption. The average world per capita total energy consumption rate is approximately 2 kW; 1/2 kW of thermal power was used to produce 1/6 kW of electricity.

Electric utilities in the United States accounted for 97% of the electrical energy for 1979. The industrial capacity was for specialized needs and to provide power for high peak industrial requirements. Fossil-fueled steam turbines accounted for nearly 69% of the installed capacity with hydropower and nuclear power being the next most significant energy sources (Figure 5.2). Other energy sources were very small. Fossil-fueled power plants will undoubtedly provide over half the electric generating capacity until the year 2000, if not later. Steam turbine boilers utilize all three fossil fuels: oil, gas, and coal. While boilers may be converted from one fuel to the other, depending upon fuel availability, a few modern boilers can simultaneously utilize a mixture of fuels. The gas turbine is used by utilities to provide for intermittent peak electric loads. Gas turbines have lower installation costs per kilowatt capacity than do steam turbines, but their lower efficiency results in higher fuel costs.

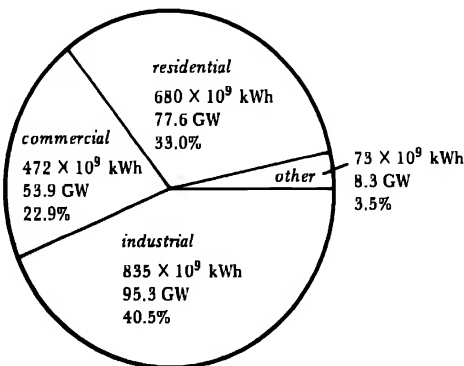




**FIGURE 5.2 1979 Installed Capacity of Utilities.**

Electrical energy usage in the United States (based upon utility sales) is presented in Figure 5.3. Total energy sold, it may be noted, was approximately 8% less than the amount generated, the difference being attributable to transmission and distribution losses.

The average electrical energy cost in 1979 for a residential consumer was 4.5¢/kWh; for a commercial consumer it was 5.7¢/kWh and for an industrial user it was 4.7¢/kWh. Since the overall cost of supplying electrical energy depends upon demand (the peak power required) as well as upon the quantity of energy, these prices are the national averages that would be charged to users having a specified



**FIGURE 5.3 Electrical Energy Usage for 1979.**

requirement.\* While the cost of electrical energy increased considerably over the 1970 to 1980 period, the increase was only slightly more than that which would have occurred due to inflation. (In 1970 residential costs averaged 2¢/kWh, commercial 2.7¢/kWh, and industrial 1.7¢/kWh.)

On an average, approximately three units of thermal energy (fuel) are required to generate one unit of electrical energy. Each kilowatt-hour of electrical energy therefore requires a fuel input of approximately 3 kWh or 10,240 Btu. A fuel cost of 30¢/10<sup>6</sup> Btu, typical of that paid by utilities prior to the oil embargo of 1973-1974, resulted in a cost of only .3¢/kWh of electricity. If fuel oil priced at \$5.20/10<sup>6</sup> Btu) is used to generate electricity, the fuel cost is 5.3¢/kWh. The cost of electrical energy also depends upon the cost of generating equipment, that is, the power plant. A large coal or nuclear facility presently requires an investment on the order of \$1000/kW of generating capacity (\$1 billion for a 1-GW capacity). For a 75% load factor, each kilowatt of generating capacity would produce, on a yearly basis, 6570 kWh of electrical energy. If the utility is allowed an 18% return on its investment, the yearly return for each kilowatt of capacity (\$1000/kW) would be \$180. Prorated over the 6570 kWh of electrical energy, this is a return of 2.7¢/kW.† Both increasing fuel prices and high interest rates hence increased the cost of electrical energy.

## 2. STEAM TURBINES

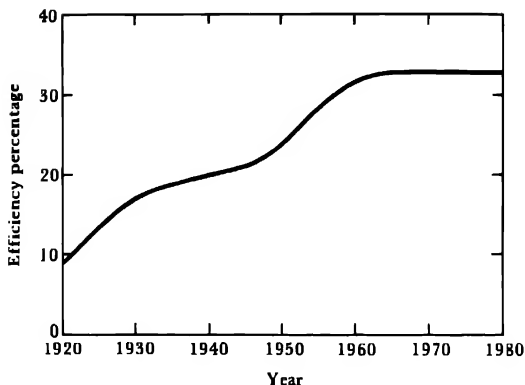
The development of prime movers has been marked by a steady increase in the efficiency of converting fuels to mechanical work. This has been due to both mechanical design improvements and the advances in the science of thermodynamics during the nineteenth century. Recent improvements have been highly dependent upon thermodynamics and metallurgical developments that have made high-temperature boilers and turbines possible.

Based upon the data of Storer [3], a 50-hp engine of the early eighteenth century consumed coal at an hourly rate of approximately 1600 lb (727 kg). This corresponded to an efficiency of less than 1%. Watt's engine with an external condenser (1776) had an efficiency just slightly greater than 2%. An efficiency of 10% was not achieved until the latter part of the nineteenth century. Based upon the average coal rate for 1902, one kilowatt-hour of electricity required the combustion of 6.85 lb (3.4 kg) of coal [4], that is, the efficiency of generation was only 3.9%.

In 1920 the Federal Power Commission (FPC) was established by the Congressional passage of the Federal Power Act. Since the establishment of the commission, data pertaining to all aspects of electricity generation are readily available.

\* The residential cost is based upon a monthly consumption of 500 to 750 kWh. The commercial rate is based upon a consumption of 6000 kWh with a peak demand of 30 kW whereas the industrial rate is based upon  $2 \times 10^3$  kWh with a 1-MW peak demand [1].

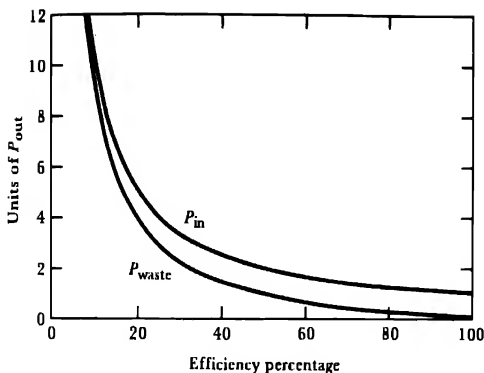
† If the money to build the plant were borrowed at an 18%/y interest rate, this would be the interest cost to be charged against each kilowatt-hour of electrical energy.



**FIGURE 5.4** *U.S. Average Efficiency for Base-Load Steam-Electric Utility Plants.*

Early reports tabulated an average coal rate (pounds of coal per kilowatt-hour of electricity), while subsequent reports presented heat rates (Btu/kWh). Recent reports have converted heat rates to efficiencies. The efficiency data of Figure 5.4 were obtained from FPC statistics. Present efficiencies are approximately eight times those of 1900, that is, only one-eighth as much fuel is now required per unit of electrical energy. If electricity were still generated with the 1920 efficiency of 9.1% (as opposed to an efficiency of 33%), over half the total U.S. energy consumption, assuming the same electrical usage would be required for the generation of electricity, and total consumption would be 65% greater than it is at present. At present, less than one pound (.84 lb or 38 kg) of coal is necessary, on the average, to produce one kilowatt-hour of electrical energy. Efficiencies as high as 40% (a coal rate of .68 lb or .31 kg) have been achieved by a few modern steam turbines. However, the average efficiency has remained nearly constant at approximately 33% for the past two decades. Higher average efficiencies will very likely require new techniques of generation rather than mere improvements in current techniques.

Since electrical energy is equivalent to work, the first- and second-law thermodynamic efficiencies of generating electricity with fossil fuels are equal. A portion of the available work of the fuel is lost as a result of the irreversible nature of combustion and the limitations of practical heat engines. The loss might be considered the energy cost, arising from practical limitations, of converting fossil fuels to electrical energy. Based upon the concept of available work, a unit of electrical energy is equal to a unit of fossil fuel energy: the electrical energy is generally more valuable. From a practical perspective, less electrical energy tends to be required to accomplish a particular task than if a fuel were to be used. Electric motors, for example, may have efficiencies of 90% or greater. Also, electrical lighting is much more efficient than fuel-powered lights.



**FIGURE 5.5** *Input and Waste Power in Terms of  $P_{out}$  vs. Efficiency.*

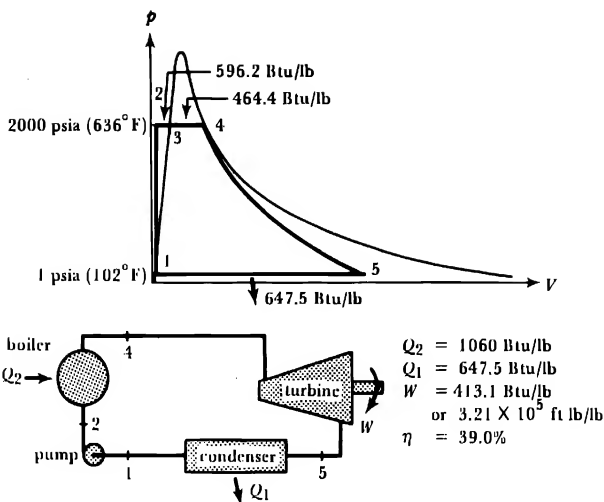
Small changes in the overall efficiency of electric power plants can have a significant impact on fuel and cooling requirements. This may most readily be seen by expressing the input heat rate,  $P_{in}$ , and the waste heat rate,  $P_{waste}$ , in terms of the output power,  $P_{out}$ , and the efficiency,  $\eta$ .<sup>\*</sup> See Figure 5.5.

$$\begin{aligned}\eta &= P_{out}/P_{in} \\ P_{in} &= 1/\eta P_{out} \\ P_{waste} &= P_{in} - P_{out} = (1/\eta - 1)P_{out}\end{aligned}$$

In contrast to the reciprocating-piston-steam engines of the eighteenth and nineteenth centuries, modern electric steam plants utilize more efficient and less bulky turbines. The first practical reaction turbine was invented in England by Charles Parsons in 1884, while in France Carl Gustaf de Laval developed the first impulse turbine in 1889. For both turbines, high-temperature and high-pressure steam is expanded through a nozzle. In the impulse turbine, the kinetic energy of the low-pressure steam leaving the nozzle is converted to mechanical energy. For a reaction turbine, high-pressure steam is allowed to expand to a lower pressure within the turbine. A modern turbine consists of several stages that may utilize both impulse and reaction sections. The expansion within the turbine and its nozzle is, owing to the very small heat loss, adiabatic and closely approximates a reversible (isentropic) process.

The steam cycles which have led to the high efficiencies of modern steam turbines, may most readily be understood by considering modifications of the basic Rankine cycle discussed in the previous chapter. It will, however, be necessary to

<sup>\*</sup> "Waste heat" is probably a poor choice of terms since it tends to imply zero economic value. This heat can be used for residential space heating, industrial process heating, or, if the plant is suitably located, for desalinization of sea water. It is waste only when a plant is limited to the exclusive generation of electricity.



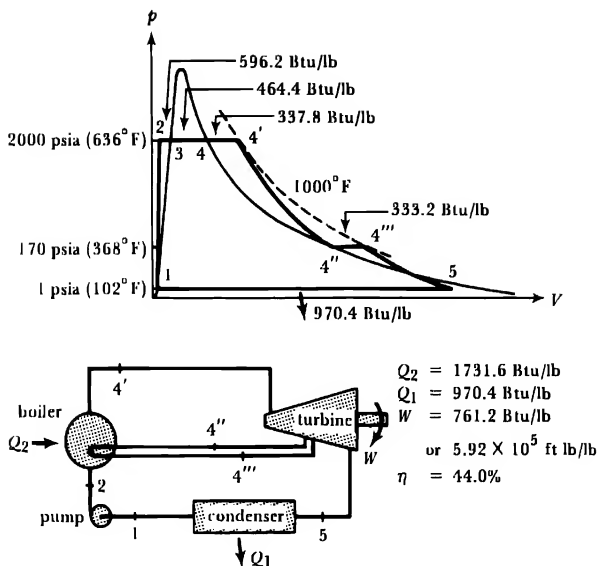
**FIGURE 5.6 A Basic Rankine Cycle.**

consider numerical results derived from the experimentally measured properties of steam [5].\* These calculations are extensively treated in most engineering thermodynamics texts. The simplest Rankine cycle is that of Figure 5.6. A pressure of 2000 psia (136 times the atmospheric pressure) corresponds to the upper pressure limit set by structural considerations for a relatively modern power plant. The temperature of the fluid-vapor portion (3 to 4) of the cycle is 636°F (336°C), whereas the low pressure, 1 psia, corresponds to a temperature of 102°F (39°C). The piping and condenser necessary for the very large volume corresponding to point 5, as well as the available cooling, determine the lower temperature limit. While the efficiency of a Carnot cycle is 49% for these temperatures, that of the Rankine cycle in which saturated vapor (4) leaves the boiler, is only 39%. For this cycle, the water emerging from the condenser (1) is compressed by the pump and enters the boiler (2). Since water is nearly incompressible, the water entering the boiler is essentially at the condenser temperature. Over half the input heat is used to heat the feedwater to the upper temperature, that is, to move from 2 to 3 on the  $p$ - $V$  diagram. Except for point 3, the water is at a temperature considerably less than the maximum. The

\* Following the convention used in the design of U.S. steam power plants, English units will be used for heat, pressure, and temperature. The heat energies given are in British thermal units per pound mass of steam or water. Since one Btu is equivalent to 1054 joules, one Btu per lb is equal to 2321 joules per kilogram. A pressure of one psia (pounds per square inch absolute) is equal to 6894.6 newtons per square meter. A frequently used pressure unit, the bar, is equivalent to  $10^5$  newtons per square meter. A pressure of one bar is nearly equal to a pressure of one atmosphere (.986923 atmospheres).

The flame temperature within the boiler, however, is considerably greater than the upper temperature of the cycle of Figure 5.6. Metallurgical considerations limit the upper steam temperature. If an upper temperature of 1000°F (538°C) is utilized, as indicated in Figure 5.7, a further improvement in efficiency is obtained. From 4 to 4' the vapor is superheated, thus increasing the average upper temperature and hence the efficiency. A further increase in efficiency would be obtained if structural limits permitted increasing the boiler pressure, since this would increase the temperature of the isotherm from 3 to 4. Since the output work per pound of steam has also increased, the steam or water circulation rate for a given output has decreased. A concurrent operational benefit occurs with superheat. Point 5, the mixture of water and vapor as it emerges from the turbine, has been shifted to the right, that is, the ratio of vapor to fluid has been increased. Superheat for this example results in a vapor ratio of 77% (.77 lb vapor and .23 lb fluid) as compared to 62.5% obtained without superheat.

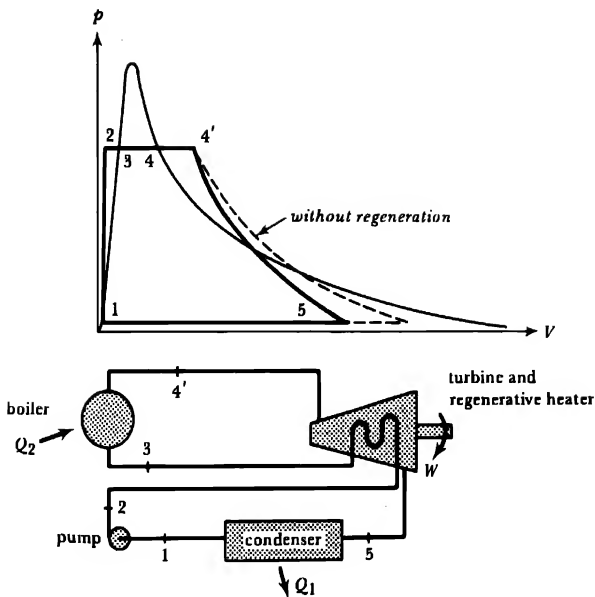




**FIGURE 5.8 A Steam Cycle with Both Superheat and Reheat.**

In addition to the increased efficiency obtained with superheat, the corresponding increase in exit vapor percentage or quality results in reduced blade wear. A high fluid content tends to result in rapid pitting of blades and hence can be very costly. As a consequence, a reheat cycle, as shown in Figure 5.8, is usually employed to further increase the vapor content of the turbine exit steam. In this example, the steam expands in the high pressure turbine to point 4'', the boundary of the vapor region. The steam at this point (170 psia, 368°F) enters the boiler in a separate heat exchanger and is heated at constant pressure to the upper temperature of the ensuing example of 1000°F. It is then further expanded in a low-pressure, high-volume turbine that may or may not be connected to the same shaft as the high-pressure turbine. The steam quality of 93.7% vapor obtained for point 5 is sufficiently high for an extended turbine life. A slight increase in efficiency as well as substantial increase in work output per pound of working fluid also result.

As has already been noted, a large input is necessary to heat the feedwater (2 to 3 of the previous figures). Since the water's temperature is considerably less than the upper permissible limit, the thermal efficiency is considerably less than that which would result if all heating was at or near the maximum temperature. A significant improvement may be achieved by means of a regenerative feedwater heater. During expansion in the turbine, the temperature of the steam decreases until it reaches the

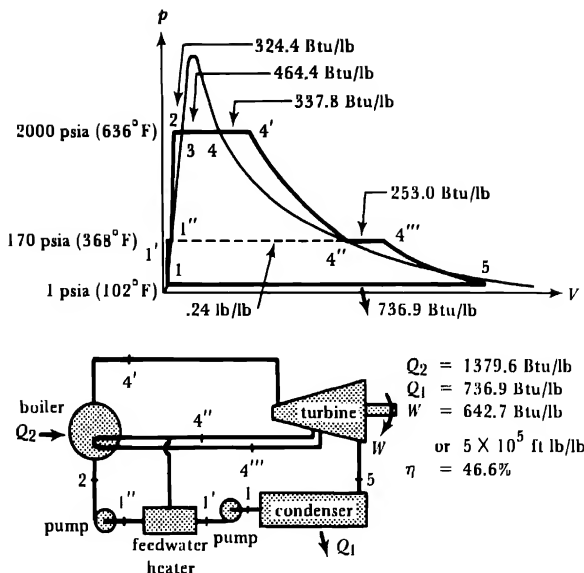


**FIGURE 5.9** *An Ideal Regenerative Feedwater Heater.*

condenser temperature. In a regenerative system, a turbine heat exchanger is utilized to heat the feedwater (2 to 3) as shown in Figure 5.9. The feedwater flow in the regenerative heater is in the opposite direction of turbine steam flow. Ideally, only a very small temperature difference exists between the feedwater heater and the expanding steam within the turbine at any given point. The heat input from the boiler is only from 3 to 4' and the cooling is from 5 to 1. Point 5, it should be noted, has shifted to a lower specific volume since the turbine expansion, 4 to 5, is no longer adiabatic. In the cycle of Figure 5.9, the average temperature of the input heat has been significantly increased. For an ideal turbine heat exchanger (negligible temperature difference), the efficiency of the steam cycle could be substantially increased.

Owing to practical considerations, the decrease in the quality of steam at point 5 is undesirable. Also, combining an efficient heat exchanger with a turbine is much easier to conceptualize than to realize. Practical regenerative cycles usually extract some of the steam at various points in the turbine and use it to either directly or indirectly heat the feedwater. Consider the cycle of Figure 5.10, a modification of the cycle of Figure 5.8, in which a small quantity of steam at 4" is mixed with the





**FIGURE 5.10** A Regenerative Cycle with an Open Water Feedwater Heater.

feedwater at point 1'. Two feedwater pumps are used for this cycle. The first one pressurizes the water to 170 psia, the pressure corresponding to the reheat pressure of 4''. Before reheat, a quantity of steam (.24 lb/lb) just sufficient to heat the feedwater at point 1' to the temperature of saturated liquid at 1' is mixed with the feedwater in an open heater. The second pump pressurizes the water to the boiler pressure. Since the feedwater is partially preheated, the heat input for the portion 2 to 3 is substantially reduced. In addition, since the temperature of point 2 is now approximately that of point 1' (368°F or 187°C), the average boiler temperature is significantly increased. The work output of the low-pressure turbine and the waste heat (per pound of fluid heated by the boiler) is reduced, since some of the working fluid which originally traversed this portion of the cycle (Figure 5.8) is now used for the feedwater heater. Another increase in efficiency, albeit small, has been achieved.

A further increase in efficiency may be achieved by using several feedwater heaters in which steam is extracted from various segments of the turbine(s). However, each feedwater heater requires an additional pump, since the pressure of the heater must correspond to that of the extracted steam. Additional increases in efficiency necessitate an increase in either or both the boiler temperature and pressure. Increasing the pressure increases the temperature of the path 3 to 4' and hence the

average temperature at which heat is absorbed. A higher upper boiler temperature ( $t_1$ ) also increases the average temperature.

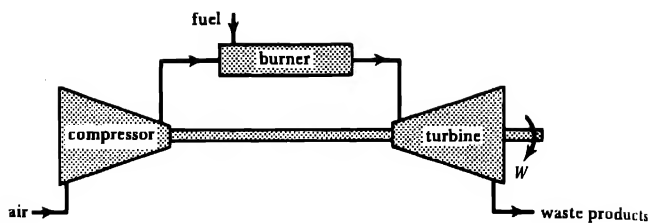
The most modern steam turbine electric power plants have overall efficiencies approaching 40%. This includes all losses, the overall efficiency being the product of the efficiency due to combustion, the turbine, and the generator. Combustion is on the order of 90% efficient, that is, 10% of the heat typically is lost through the flue. Large electric generators have conversion efficiencies of 98% or better. An overall efficiency of 40% therefore requires a steam turbine efficiency of at least 45%. In order to account for thermodynamic losses, an ideal cycle efficiency of at least 50% is thus required. Since this is on the order of the efficiencies that were achieved with the 2000 psia and 1000°F boiler of the previous examples, higher temperatures and pressures would be needed to achieve still higher overall efficiencies. Due to the cost of increasing the working temperatures and pressures, only small further increases in efficiency seem likely for the conventional steam cycle.

### 3. GAS TURBINE AND COMBINED CYCLES

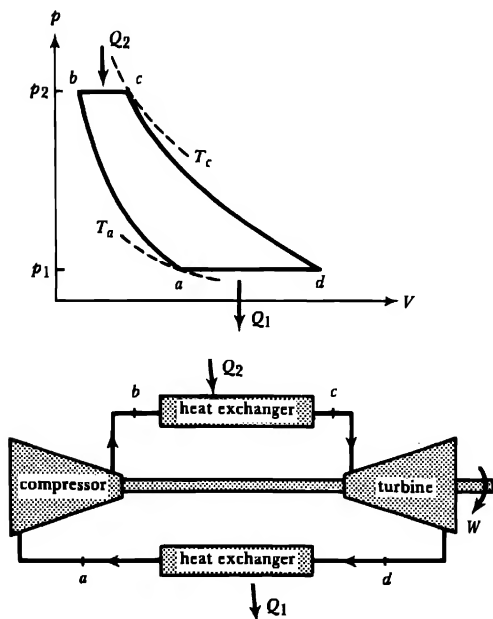
The gas turbine, an internal-combustion engine, is similar in many respects to the Otto- and diesel-cycle reciprocating engines. In contrast to the mechanically inefficient piston and crankshaft of a reciprocating engine, the power transfer of the gas turbine cycle is provided by a more efficient rotary compressor and turbine. High-temperature, fairly efficient gas turbines are the result of aircraft jet engine development and research. While the upper turbine temperatures presently used for industrial power plants are on the order of 1000°C, temperatures as high as 1600°C are anticipated by 1990 [6, 7].

Even though 1000°C is considerably higher than the temperatures of the more efficient steam turbine boilers (1000°F or 538°C), present gas turbines have thermal conversion efficiencies of only 20 to 30%. The low capital cost of gas turbines, that is, the cost per kilowatt of power, makes them particularly attractive, despite the higher fuel cost, for intermittent peaking power. Anticipated improvements in the thermal efficiency of the gas turbine and the combining of gas- and steam-turbine cycles are expected to result in substantial overall efficiency increases. The upper temperature of 1000°C, it should be noted, corresponds, for a low temperature of 20°C, to a Carnot efficiency of 77%. An 84% efficiency could be achieved for 1600°C. Unlike the steam-turbine cycle, extremely high pressures are unnecessary for the gas-turbine cycle.

An open-cycle internal-combustion gas turbine is shown schematically in Figure 5.11. Air at atmospheric pressure and temperature is compressed before entering the burner or combustion chamber. Combustion of the fuel-air mixture produces a high-temperature gas that expands within the turbine and provides the work output. The turbine and compressor are frequently on the same shaft, separated by the combustion chamber. For industrial applications, a substantial portion of the turbine power, 30 to 50%, is required to drive the compressor. For an aircraft jet engine, however, the turbine is merely large enough to provide the mechanical power



**FIGURE 5.11** An Open-Cycle Internal-Combustion Gas Turbine.



**FIGURE 5.12** A Closed Brayton Cycle Gas Turbine.

necessary for the compressor. The hot exhaust gases expanded by a nozzle produce the high-velocity exit gases which in turn result in the engine's thrust.

The gas turbine may most readily be analyzed by considering the ideal closed-cycle air-standard Brayton cycle of Figure 5.12. For this cycle (as for the Otto cycle), the exhaust gas is recycled after being cooled in the low-temperature heat exchanger. The combustion process is replaced by a high-temperature heat exchanger. Heat transfers for the ideal cycle occur at constant pressures:  $p_1$  for the waste heat transfer,  $Q_1$  and  $p_2$  for the heat input,  $Q_2$ . The heat transfer is not, as opposed to the Carnot cycle, at a constant temperature. The overall efficiency for the gas-turbine cycle is therefore considerably less than the Carnot efficiency corresponding to  $T_c$  and  $T_a$ , the upper and lower respective temperatures. To simulate an open cycle, point  $a$  corresponds to ambient conditions. The compression from  $a$  to  $b$  and the turbine expansion from  $c$  to  $d$  are, ideally, both adiabatic and reversible.

The theoretical analysis of the cycle, assuming air behaves as an ideal gas, is straightforward. First consider the change in internal energy,  $dU$ , for the adiabatic portions in which  $dQ = 0$ .

$$dU = dQ - p dV = -p dV$$

Expressing the internal energy differential in terms of a temperature differential and using the ideal gas law, an equation which may readily be solved is obtained.

$$dU = \frac{n}{2} R dT$$

$$\frac{n}{2} R dT = -p dV = -(RT/V) dV$$

$$\frac{n}{2} \frac{dT}{T} + \frac{dV}{V} = 0$$

$$\frac{n}{2} \ln T + \ln V = C' \quad (\text{a constant})$$

$$T^{n/2} V = \ln C'$$

Substituting for the volume,  $V$ , an expression involving the pressure and temperature is obtained.

$$V = RT/p$$

$$\frac{T^{(n+2)/2}}{p} = C' \quad (\text{a new constant})$$

The following is therefore obtained for the endpoints of path  $a$  to  $b$ .

$$\frac{T_b^{(n+2)/2}}{p_b} = \frac{T_a^{(n+2)/2}}{p_a}$$

$$\frac{T_b}{T_a} = \left( \frac{p_b}{p_a} \right)^{2/(n+2)} = \left( \frac{p_2}{p_1} \right)^{2/(n+2)}$$

A similar expression is obtained for the turbine expansion of path  $c$  to  $d$ .

$$\frac{T_c}{T_d} = \left( \frac{p_c}{p_d} \right)^{2/(n+2)} = \left( \frac{p_2}{p_1} \right)^{2/(n+2)}$$

The temperature ratios for the endpoints are thus equal.

$$T_b/T_a = T_c/T_d$$

The input heat transfer,  $Q_2$ , occurs over the path  $b$  to  $c$ .

$$Q_2 = \int_b^c dQ = \int_b^c (dU + p dV)$$

Since the integral for  $Q_2$  corresponds to a constant pressure ( $dp = 0$ ), a simplification of the integrand is possible.

$$d(pV) = p dV + V dp$$

$$= p dV \quad (\text{since } dp = 0)$$

$$p dV = d(pV) = R dT$$

$$\begin{aligned} Q_2 &= \int_b^c \left( \frac{n}{2} R dT + R dT \right) = \frac{n+2}{2} R \int_b^c dT \\ &= \frac{n+2}{2} R (T_c - T_b) \end{aligned}$$

A similar result is obtained for the waste heat,  $Q_1$ .

$$Q_1 = - \int_d^a dQ = - \frac{n+2}{2} R (T_d - T_a)$$

Since the net work output,  $W$ , is  $Q_2 - Q_1$ , an expression for the efficiency follows.

$$\begin{aligned} \eta &= \frac{W}{Q_2} = \frac{Q_2 - Q_1}{Q_2} = 1 - \frac{Q_1}{Q_2} \\ &= 1 - \frac{T_d - T_a}{T_c - T_b} = 1 - \left( \frac{T_a}{T_b} \right) \left( \frac{T_d/T_a - 1}{T_c/T_b - 1} \right) \end{aligned}$$

This expression may be simplified by using the temperature relations obtained for the adiabatic paths.

$$\frac{T_a}{T_b} = \left( \frac{p_1}{p_2} \right)^{2/(n+2)}$$

$$T_b/T_a = T_c/T_d \quad T_d/T_a = T_c/T_b$$

$$\eta = 1 - \left( \frac{p_1}{p_2} \right)^{2/(n+2)}$$

Since air behaves as an ideal gas with five degrees of freedom ( $n = 5$ ), the following result is obtained for the efficiency.

$$\eta = 1 - (p_1/p_2)^{2/7}$$

Figure 5.13 is a plot of efficiency as a function of pressure ratio.

For an ideal Brayton cycle, the efficiency depends only upon the pressure ratio. For a practical gas-turbine cycle in which turbine and compression losses are considered, the upper temperature,  $T_c$ , is important since the heat input,  $Q_2$ , depends upon it.

$$Q_2 = \frac{n+2}{2}R(T_c - T_b)$$

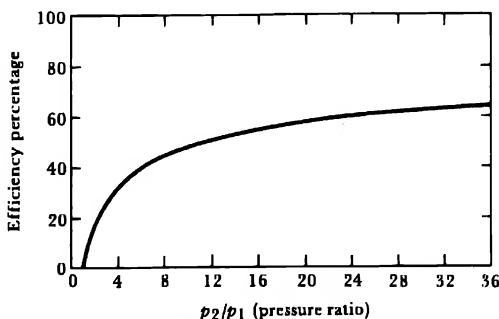
For a given pressure ratio and hence  $T_b$ , the higher  $T_c$  is, the greater the heat input per kilogram-mole of gas. Since the efficiency remains unchanged, the work output per kilogram-mole also increases. The work necessary to drive the compressor, however, remains unchanged. Reducing the quantity of gas that needs to be circulated for a specified output tends to increase the efficiency when losses occur.

The highest present turbine and compressor efficiencies are no greater than 90% and only a few percentage points more are anticipated for future improvements. If  $W_T$  is the work output for an ideal turbine, the turbine efficiency,  $\eta_T$ , is defined such that the actual work output,  $W'_T$ , is given by the following expression.

$$W'_T = \eta_T W_T$$

For the compressor with an efficiency of  $\eta_c$ , the actual work input,  $W'_c$ , is greater than that for an ideal compressor,  $W_c$ .

$$W'_c = W_c/\eta_c$$



**FIGURE 5.13** Efficiency as a Function of Pressure Ratio for a Brayton Cycle Gas Turbine.

The net work output for an ideal cycle is the difference of the turbine and compressor work.

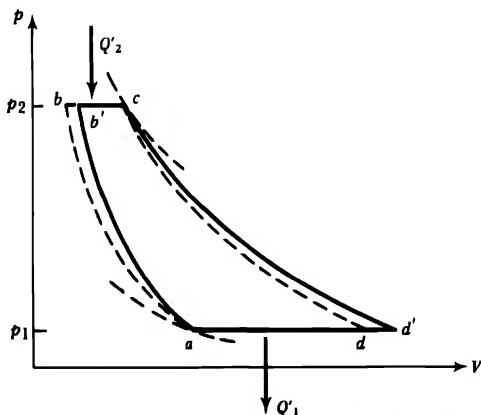
$$W = W_T - W_C$$

For a nonideal cycle, the actual net work depends upon the inefficiencies of the compressor and the turbine.

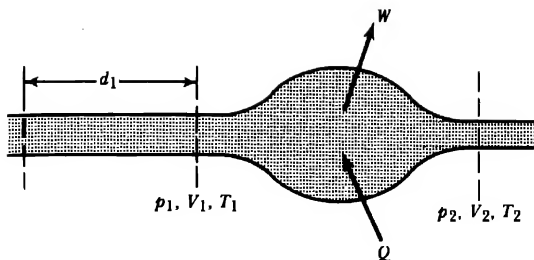
$$W' = W'_T - W'_C = \eta_T W_T - W_C / \eta_C$$

Since  $W'$  is equal to the difference of the two quantities, even for turbine and compressor efficiencies as high as .9 (90%), it will be considerably less than  $W$ .

The nonideal compression and expansion, even if adiabatic, will modify the cycle of Figure 5.12. Frictional heat resulting from turbulent flow in both the compressor and turbine tends to increase the respective exit temperatures, as shown by Figure 5.14, in which  $b'$  and  $d'$  are the new endpoints ( $T'_{b'} > T_b$  and  $T'_{d'} > T_d$ ). Points  $a$  and  $c$  have been assumed to remain unchanged. In order to determine the new conditions for points  $b'$  and  $d'$ , the energy balance for a steady-state, steady-flow process will need to be developed. This is necessary since, unlike the Otto and Carnot cycles considered in the previous chapter, the various processes (compression, combustion, expansion) occur in distinct physical regions. Energy associated with the mass flow of the gas from one region to another cannot be ignored. Consider the apparatus of Figure 5.15, which represents the flow of a gas in an open system. A work output of  $W$ , a heat input of  $Q$  per kilogram-mole of flow, and a steady-state



**FIGURE 5.14** Effect of Compressor and Turbine Inefficiencies on a Gas-Turbine Cycle.



**FIGURE 5.15 A Steady-Flow System.**

condition in which the net rate of mass flow into the volume is equal to that leaving the volume will be assumed. An important aspect of the steady-flow process is the work done by the gas in entering or leaving the apparatus. If at the entrance, point 1, the pressure is  $p_1$  and the area is  $A_1$ , the total force on the entering gas is  $p_1 A_1$ . Furthermore, if one kilogram-mole occupies a volume corresponding to the length  $d_1$ , the work necessary to transfer that mole into the apparatus will be the product of the force and the distance.

$$\begin{aligned} \text{Work necessary for one mole} &= p_1 A_1 d_1 = p_1 V_1 \\ \text{to enter apparatus} \end{aligned}$$

The quantity  $A_1 d_1$  is the volume of one mole,  $V_1$ . By a similar argument, the work done by the gas in leaving the volume is  $p_2 V_2$ . Adding the internal energy to the flow-work terms, and taking into account  $W$  and  $Q$ , a steady-flow energy balance is obtained.

$$p_1 V_1 + U_1 + Q = p_2 V_2 + U_2 + W$$

If the kinetic energy of the gas is significant, a term  $\frac{1}{2} M v^2$ , in which  $M$  is the mass of one mole, would need to be included. Also, a change in potential energy associated with elevation can, if necessary, be accounted for. The variables  $p_1 V_1 + U_1$  and  $p_2 V_2 + U_2$  are state variables, that is, they depend only upon the state of the gas and not upon the process by which the gas reached that state. For convenience,  $pV + U$  is defined as the enthalpy,  $h$ .

$$h = pV + U$$

Enthalpy has the same dimensions as the internal energy, namely, joules per kilogram-mole. For an ideal gas, enthalpy depends only upon temperature.

$$\begin{aligned} h &= RT + \frac{n}{2} RT \\ &= \frac{n + 2}{2} RT \end{aligned}$$



Rewriting the steady-flow energy balance, the following is obtained.

$$h_1 + Q = h_2 + W$$

Consider the compressor of the ideal gas-turbine cycle in which work is done on the gas, that is,  $W = -W_c$ , and the heat transfer is zero,  $Q = 0$ . For an ideal cycle, the following inlet and outlet temperatures ( $T_a$  and  $T_b$ , respectively) result.

$$h_a = h_b - W_c$$

$$\frac{n+2}{2}RT_a = \frac{n+2}{2}RT_b - W_c$$

$$\frac{n+2}{2}R(T_b - T_a) = W_c$$

For a nonideal compressor there is a new outlet temperature,  $T'_b$ .

$$\frac{n+2}{2}R(T'_b - T_a) = W'_c$$

The inlet temperature,  $T_a$ , has been assumed to remain unchanged. Since  $W'_c$  is greater than  $W_c$ ,  $T'_b$  is necessarily greater than  $T_b$ . Taking the ratio of the above relations, the following is obtained.

$$\frac{T'_b - T_a}{T_b - T_a} = \frac{W'_c}{W_c} = \frac{1}{\eta_c}$$

The numerator is the difference of the actual temperatures and the denominator is the difference of the temperatures for an ideal cycle. A similar expression is obtained for the turbine in which the upper temperature,  $T_c$ , is assumed unchanged.

$$\frac{T_c - T'_d}{T_c - T_d} = \eta_T$$

Turbine and compressor inefficiencies modify both  $Q_2$  and  $Q_1$ .

$$Q'_2 = \frac{n+2}{2}R(T_c - T'_b)$$

$$Q'_1 = \frac{n+2}{2}R(T'_d - T_a)$$

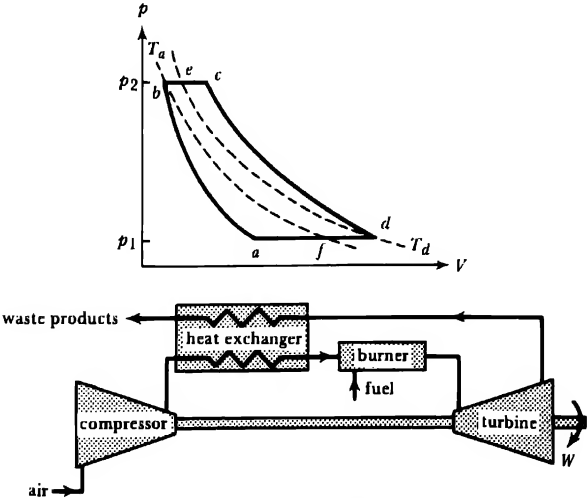
The effect of turbine and compressor inefficiencies may best be illustrated with a numerical example. Consider the cycle of Figure 5.14 in which the pressure ratio,  $p_2/p_1$ , is equal to 10, the upper temperature  $T_c$  is equal to 1000°C, and  $T_a$  corresponds to an ambient temperature of 20°C. The ideal efficiency is 48.2%. The work of the turbine and compressor, as well as the net work output per kilogram-mole of air, is presented in Table 5.1. For the ideal case, the compressor work is 44.5% of the turbine output. As a consequence, the turbine output power must be 80% greater than the net output power. Included in the table are the effects of finite

turbine and compressor efficiencies (90% and 80%). A small decrease in the turbine work and a concurrent increase in compressor work have a substantial effect on the net output. Even though  $Q_2$  decreases slightly due to the increase in  $T_b$ , the net efficiency is drastically reduced.

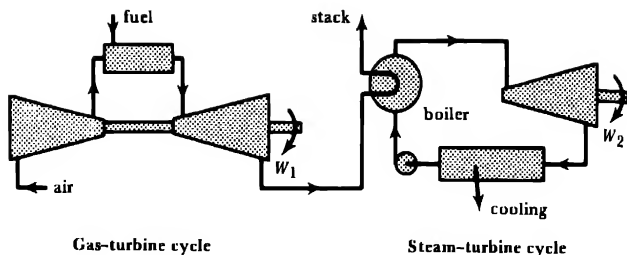
**TABLE 5.1    Effect of Turbine and Compressor Inefficiency on Overall Efficiency for a Gas-Turbine Cycle**

Turbine and compressor	Work and heat ( $10^6$ J/kg-mole)				Resultant efficiency
	$W_T$	$W_C$	$W$	$Q_2$	
100%	17.86	7.94	9.92	20.58	48.2%
90%	16.08	8.82	7.26	19.70	36.9%
80%	14.29	9.92	4.37	18.60	23.5%

One means of increasing the efficiency of gas turbines is through the use of a regenerator. Normally the turbine exit temperature of  $T_d$  or  $T'_d$  of Figure 5.14 is considerably greater than the compressor exit temperature of  $T_b$  or  $T'_b$ . For the previous example,  $T_d = 659$  K (386°C) and  $T_b = 566$  K (293°C). A portion of the input heat,  $b$  to  $e$  of Figure 5.15, may, in principle, be recovered from the turbine exhaust gas. The input heat,  $Q_2$  is reduced to that of  $e$  to  $c$  only. Since



**FIGURE 5.16    A Gas-Turbine Cycle with a Regenerator.**



**FIGURE 5.17 A Combined Gas and Steam Turbine Power Plant.**

the output is unchanged, a net increase in efficiency is achieved. As the pressure ratio is increased, however, the compressor exhaust temperature,  $T_b$ , increases, and the regenerator effectiveness decreases.

Further modifications of the basic gas-turbine cycle are possible. Multiple compressors with intercooling and multiple turbines with successive combustion chambers can result in increased efficiency. A further improvement can be achieved by using the gas-turbine "waste" heat to either partially or totally power a conventional steam-turbine power plant. Since the boiler temperature of the basic Rankine cycle of Figure 5.16 was only 636°F or 336°C, the exhaust gases of the gas-turbine example with a temperature of 386°C could be used for the boiler heat input. A gas-turbine topping cycle is illustrated in Figure 5.17. In general, modifications of both the gas- and steam-turbine cycles could also be included. Since the net work output is equal to the sum of the two outputs and the heat input is that of the gas-turbine combustion chamber alone, a substantial efficiency increase is possible.

Figure 5.18 illustrates the operation of a combined-cycle heat engine in which the waste heat of the high-temperature engine,  $Q_{1A}$ , is the heat input of the low-temperature engine,  $Q_{2B}$ . The overall thermodynamic efficiency of the combined-cycle engine is the total work divided by the heat input,  $Q_{2A}$ .

$$\eta = \frac{W}{Q_{2A}} = \frac{W_A + W_B}{Q_{2A}}$$

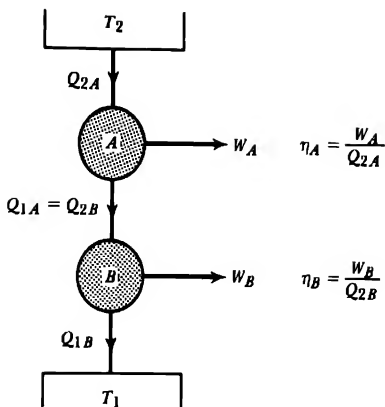
$$W_A = \eta_A Q_{2A}$$

$$W_B = \eta_B Q_{2B} = \eta_B Q_{1A} = \eta_B (Q_{2A} - W_A) = \eta_B (1 - \eta_A) Q_{2A}$$

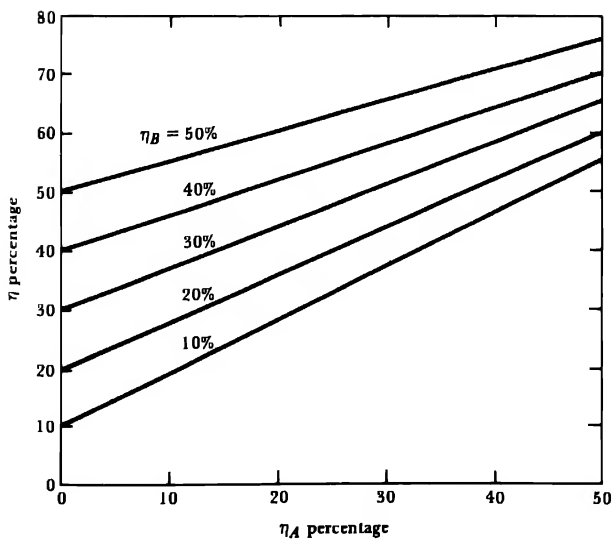
Substituting for the work outputs, the following efficiency expression is obtained.

$$\begin{aligned} \eta &= \eta_A + \eta_B (1 - \eta_A) \\ &= \eta_A + \eta_B - \eta_A \eta_B \end{aligned}$$

The net efficiency therefore may be substantially greater than either efficiency, as shown in Figure 5.19. Combined cycle power plants are not only being proposed



**FIGURE 5.18** A Combined Cycle Heat Engine.



**FIGURE 5.19** Overall Efficiency for a Combined Cycle.

and constructed for gas turbines but also for magnetohydrodynamic (MHD) electric generators, the subject of Section 5.

#### 4. COGENERATION AND DISTRICT HEATING

Combined-cycle heat engines of the type discussed in the previous section can reduce the fuel required for producing work or electrical energy. In a similar fashion, the joint production of thermal and electrical energy can also result in significant energy savings [8-10]. Large quantities of process steam, saturated steam at temperatures generally less than 200°C (392°F), are used for numerous industrial processes. The second-law efficiencies of conventional furnaces used for raising process steam and for providing thermal energy for space and water heating are rather low, no more than 30% for process steam boilers and less than 10% for space and water heaters. The thermodynamic potential of the high-temperature combustion process is not utilized for these applications, and hence available work of the fuel is lost. If instead of using a heat exchanger to reduce the temperature of the thermal energy (a highly irreversible system), a heat engine were to be used, some of the available work that would be lost could be recovered. This is the underlying principle of cogeneration. High-temperature combustion products are first used to drive a heat engine (usually connected to an electric generator) and the heat rejected by the engine is used for the thermal need. As a result, the second-law efficiency of a system providing both thermal and electrical energy may be considerably greater than when the tasks are accomplished individually.

In many systems, raising process steam, for example, a steady-flow process is involved. Hence, an expression for the available work of the steady flow is desired. An expression for the available work of a closed system was obtained in Chapter 4.

$$B = (U - U_A) - T_A(S - S_A) + p_A(V - V_A)$$

The subscripted quantities refer to the state of the system when it is in equilibrium with the surrounding environment or ambient condition. The energy balance for a steady-flow system developed in the previous section entailed an accounting of the work associated with the gas entering and leaving the system, that is, the pressure-volume product of the respective flows. Although the work associated with a steady flow is  $pV$ , not all this work can be recovered in bringing the moving gas to rest. Only the pressure in excess of the ambient pressure,  $p_A$ , contributes to a force from which work can be obtained. While a steady flow might be used to push a piston (its pressure being  $p$ ), the other side of the piston experiences a force due to atmospheric pressure,  $p_A$ . Since the work done on the atmosphere is not recoverable, the following is obtained.

$$\begin{aligned} \text{available work} \\ \text{due to a steady flow} &= (p - p_A)V \end{aligned}$$

Adding this term to the available work of a closed system yields the following.

$$\begin{aligned} B &= (U - U_A) + (p - p_A)V - T_A(S - S_A) + p_A(V - V_A) \\ &= (U + pV) - (U_A + p_A V_A) - T_A(S - S_A) \end{aligned}$$

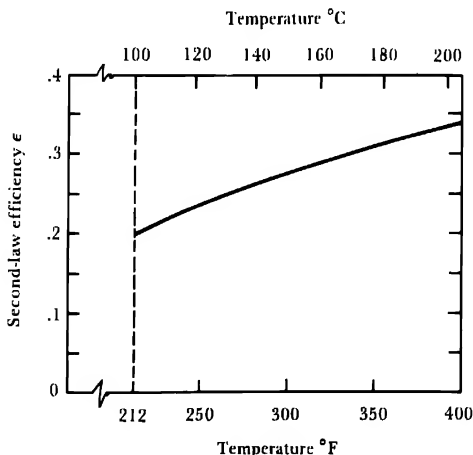
The first term on the right-hand side may be recognized as the enthalpy of the stream,  $h$ , and the second is the enthalpy of the gas when it is brought into equilibrium with the environment,  $h_A$ .

$$B = h - h_A - T_A(S - S_A)$$

The enthalpy difference is the energy of the steady flow (relative to the environment) whereas the term involving the entropy difference is that energy which cannot be converted to work.\*

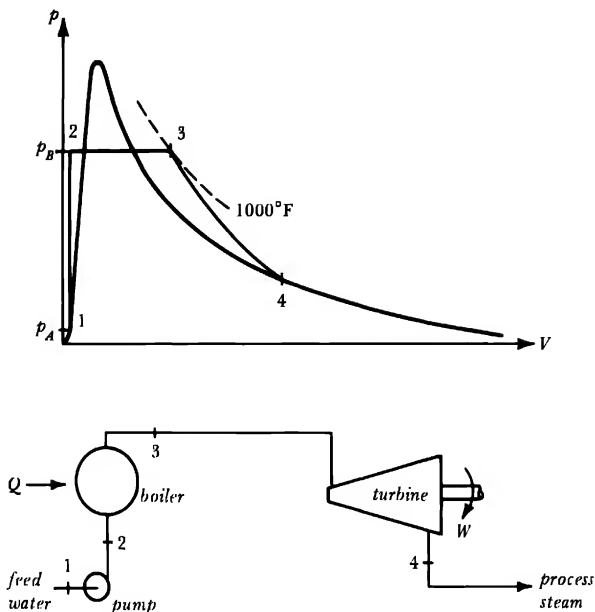
Through the use of published steam tables, the available work of saturated steam (100% vapor) may be determined. Dividing by  $h - h_A$ , a quantity essentially equal to the thermal energy required for producing the steam, the result indicated in Figure 5.20 is obtained.

$$\frac{B}{h - h_A} = 1 - T_A \left( \frac{S - S_A}{h - h_A} \right)$$



**FIGURE 5.20 Second-Law Efficiency of Raising Process Steam.**

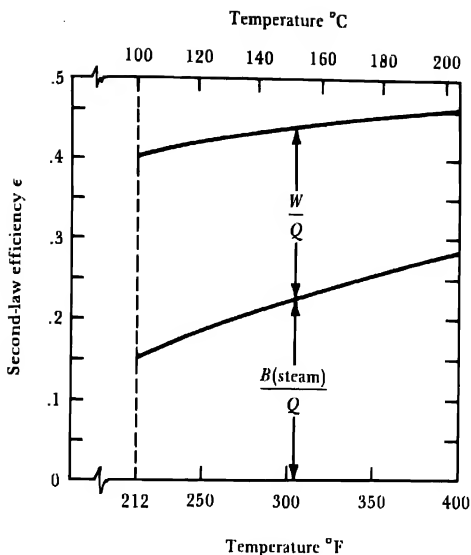
\* The term *exergy* has also been used for the available work of a steady-flow system [11]. "Potential work" is probably a more descriptive term.



**FIGURE 5.21** A Steam-Turbine Cogeneration System for Raising Process Steam.

If the very small quantity of mechanical work needed to pressurize the feedwater is ignored, this result corresponds to the second-law efficiency of raising the steam in a boiler with no thermal losses. Thermal losses, including those of the flue, can be accounted for by multiplying the efficiency of Figure 5.20 by the conventional first-law efficiency,  $\eta$ , of the boiler.

An elementary cogeneration system for producing process steam is indicated in Figure 5.21. The boiler pressure,  $p_B$ , is chosen so that when the steam expands reversibly in the turbine, saturated steam at the desired temperature (point 4 on the diagram) is obtained. The data of Figure 5.22 was obtained assuming an upper temperature limitation of 1000°F (538°C) and the entering water at a temperature of 68°F (20°C). The upper curve is the second-law efficiency (assuming no thermal losses). Indicated in the figure is the fraction of the thermal energy that is converted to work by the turbine and the fraction corresponding to the available work of the process steam. The elementary cogeneration system of Figure 5.21 is considerably more efficient than a conventional boiler which produces only process steam. As in the case of the steam-turbine cycles discussed in an earlier section, the overall



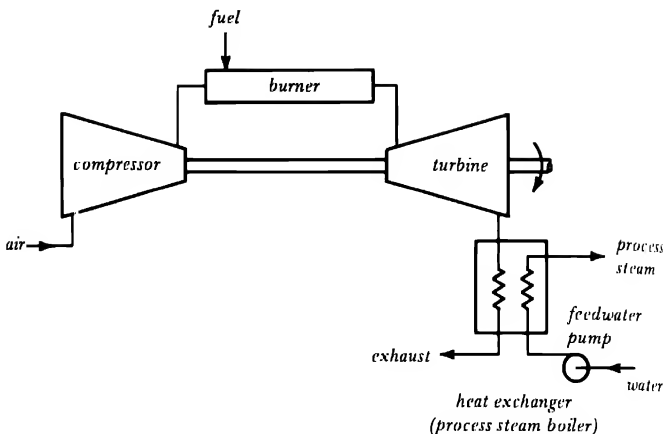
**FIGURE 5.22** *Second-Law Efficiency for a Cogeneration System.*

second-law efficiency of the cogeneration system can be increased through the judicious use of regenerative feedwater heaters.

More fuel per unit of process steam is required for cogeneration than for raising process steam alone. This added fuel energy, however, is entirely converted to electrical energy (with a reversible turbine and lossless boiler). This suggests an incremental rating system for specifying the electrical generating performance of an actual cogeneration system. For an ideal cogeneration system, one kilowatt-hour of added fuel energy is required for one kilowatt-hour of electrical energy, a fuel rate of 3412 Btu/kWh. Actual cogeneration systems can achieve a fuel rate of 4600 Btu/kWh [12], which is only about half the fuel required by a conventional power plant producing only electrical energy.

Other heat engines, such as the gas turbine indicated in Figure 5.23, can also be used in a cogeneration system. For the same conditions used for the gas turbine of the previous section, an upper temperature of 1000 $^{\circ}\text{C}$ ,  $T_c$ , and a pressure ratio of 10, the temperature of the turbine exhaust, 386 $^{\circ}\text{C}$ ,  $T_d$ , is considerably higher than that needed for raising process steam. The output of the turbine-compressor system, however, is unaffected by the heat exchanger. Hence, the process steam requires no additional fuel; it is generated from the thermal energy that would otherwise have been rejected to the environment. If all the exhaust heat were used for raising



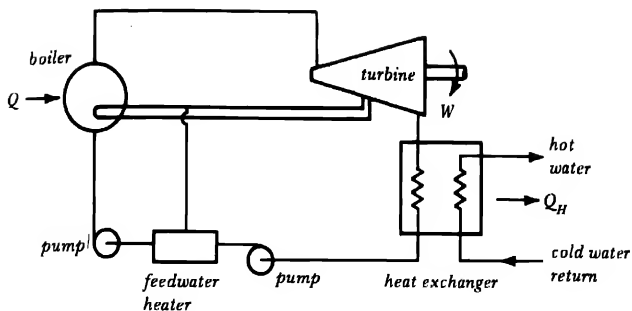


**FIGURE 5.23** A Gas-Turbine Cogeneration System.

saturated steam at a temperature of 400°F (204°C), 17.6% of the fuel's input energy would be converted to available work of the steam. The overall second-law efficiency of the system with a reversible compressor and turbine would be 65.8%! This is achieved despite the irreversibility of the heat exchanger—the temperature of the turbine exhaust is considerably higher than that of the process steam.

While the second-law efficiency of a gas turbine cogeneration system is reduced by turbine and compressor losses, the reduction is less than when no process steam is produced. Since a nonideal gas turbine system rejects a greater quantity of waste heat for a given input, a greater quantity of process steam can be generated. For a 90% efficient turbine and compressor (Table 5.1), 21.5% of the input is converted to available work of the process steam (400°F), resulting in an overall second-law efficiency of 58.4%. The 80% efficient turbine and compressor reduce the second-law efficiency to 49.5%. If a lower steam temperature is desired, the second-law efficiency of the gas turbine cogeneration system would be less, owing to the greater temperature differential in the heat exchanger. But the efficiency could be increased if the heat rejected by the gas turbine were first used to raise steam suitable for a steam turbine (as in Figure 5.17). The steam-turbine portion would be suitably chosen to exhaust saturated steam at the desired temperature.

Closely related to the production of process steam for industrial uses is the cogeneration of electrical energy and hot water suitable for a district heating system. District heating systems typically use a nonpressurized hot water distribution system. Individual users, such as a private residence, tap a small quantity of the hot water which, by means of a heat exchanger, warms water for space heating and provides hot water for domestic usage. In the summer, the heat can also be used to power



**FIGURE 5.24** A Cogeneration System for District Heating.

an absorption-type air conditioning system (Section 6 of Chapter 8). District heating systems, common in Europe, could also be widely used in the United States, thus reducing energy needs [12].

Saturated steam at a temperature of 212°F (100°C) could be used to provide the hot water. The system of Figure 5.21 would have an overall second-law efficiency of 40%, assuming no thermal losses or irreversibilities. Modifications of the steam cycle, such as shown in Figure 5.24, improve the overall efficiency of the system. This cycle is similar to that of Figure 5.10, except the condenser temperature for the turbine has been increased to 212°F (100°C). The steam from the turbine is condensed in the heat exchanger, transferring a thermal energy of  $Q_H$  to the district heating system. For this system,  $W/Q_2 = 36.2\%$ , a considerable reduction from the 46.6% efficiency of Figure 5.10. The district heating system, however, is supplied with 855.6 Btu of thermal energy per pound of steam that flows through the boiler. The ratio of  $B_H$  (the available work of the thermal energy) to  $Q_2$  is .147 and hence the second-law efficiency of the overall system is 50%. Since the district heat would substitute for heat supplied by conventional furnaces that are less than 10% efficient, significant energy savings could be realized even when losses of the cogeneration district heating system are taken into account.

## 5. MAGNETOHYDRODYNAMICS

While the twentieth century has been marked by major improvements in both prime movers and electric generators, electricity is still produced in much the same manner as in 1900. Basically, an engine that converts heat to mechanical work is connected to an electric generator by a rotating shaft. Though it is possible to convert heat and chemical energy directly to electrical energy, large-scale application of direct conversion techniques has been lagging.

The importance of high temperatures in achieving high efficiencies has been discussed. Even though the upper temperature for modern gas turbines exceeds that of steam-turbine boilers, the temperature is still considerably less than achievable combustion temperatures of 2000°C or higher, resulting in a significant "temperature loss." Magnetohydrodynamics, MHD for short, is a process by which high-combustion temperatures can more fully be utilized. It is also a direct conversion process eliminating the mechanical link between the heat engine and the electric generator. There are presently proposals to use an MHD generator as a topping cycle for a conventional steam-turbine power plant. Overall efficiencies as high as 50 to 60% are anticipated, although, with the exception of the Soviet Union, large-scale developmental projects have been minimal [14-18].\*

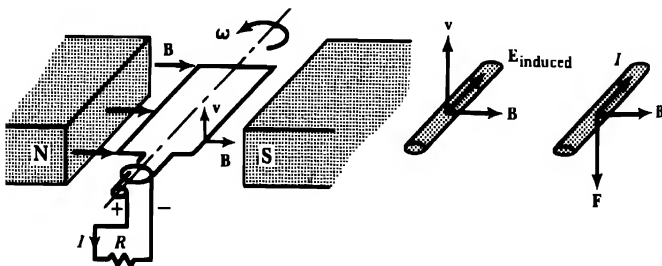
Conventional electric generation depends upon the motion of a conductor in a magnetic field as in the conceptually simple generator of Figure 5.20. An induced electric field,  $\mathbf{E}_{\text{induced}}$ , a vector quantity perpendicular to both the velocity,  $\mathbf{v}$ , and the magnetic field,  $\mathbf{B}$ , results.

$$\mathbf{E}_{\text{induced}} = \mathbf{v} \times \mathbf{B} \quad \text{V/m}$$

The movement of charges, electrons for the metallic conductor, results in a terminal charge accumulation that gives rise to an opposing electric field which, for an ideal conductor, cancels the induced field. A voltage is developed as shown in Figure 5.25, which in turn results in a load current for the resistance  $R$ . Concurrently, the conductor current in a magnetic field produces a force on the conductor. The force, as the expression for the induced electric field, involves a vector product.

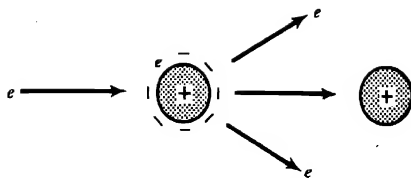
$$\mathbf{F} = \mathbf{I} \times \mathbf{B} \quad \text{n/m length}$$

The resultant force,  $\mathbf{F}$ , is in a direction opposite to the motion of the conductor. Work must therefore be done on the conductor in an amount equal to the energy dissipated by the resistance (assuming losses are negligible). This two-step conceptual



**FIGURE 5.25** *An Elementary Electric Generator.*

\*Reference 18 contains a summary of U.S. research projects.



**FIGURE 5.26 An Electron Ionizing Collision**

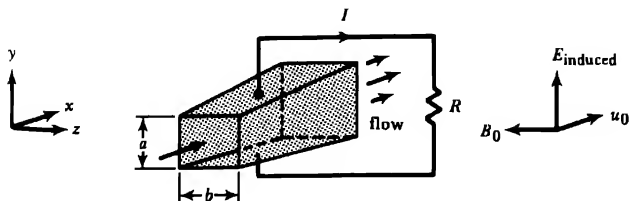
process accounts for the induced voltage and the reaction force resulting from the load current.

In an MHD generator a high-velocity ionized gas or plasma serves as the moving conductor. Since the gas is a direct product of combustion, the mechanical linkage of a rotating shaft is eliminated. At high temperatures, a few free electrons of the gas will have sufficient kinetic energy to ionize a neutral atom through a collision, as indicated in Figure 5.26. For the normal constituents of combustion, electron energies ( $\frac{1}{2}mv^2$ ) of 12 to 15 eV (electron volts) are necessary for ionization. Unfortunately, even at temperatures of 2500 to 3000°C, the ionization achieved is insufficient for a useful generator. This necessitates introducing, by a process known as *seeding*, molecules such as potassium or cesium which have lower ionization energies.

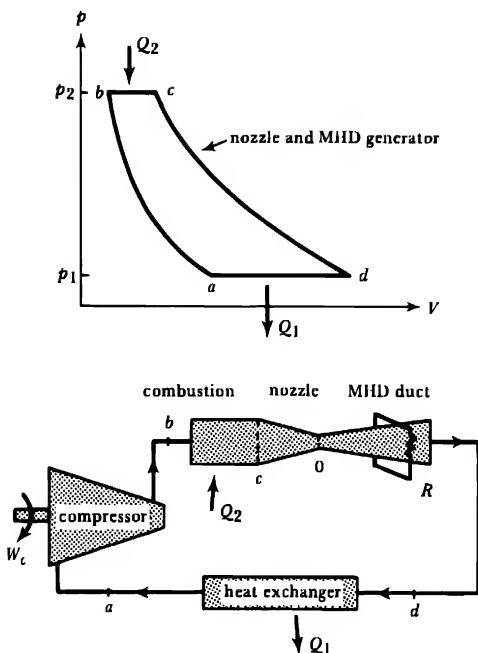
Consider the MHD duct of Figure 5.27 with the coordinate system shown. The flow is along the  $x$  axis with a velocity of  $\mathbf{u}_x u_0$ . The magnetic field is along the  $z$  axis and is directed in the  $-z$  direction ( $\mathbf{B} = -\mathbf{a}_z B_0$ ). The induced electric field may be obtained from the cross product of the velocity and the magnetic field.

$$\begin{aligned}\mathbf{E}_{\text{induced}} &= \mathbf{v} \times \mathbf{B} = \mathbf{a}_x u_0 \times (-\mathbf{a}_z B_0) \\ &= \mathbf{a}_y u_0 B_0 \\ E_{y\text{induced}} &= u_0 B_0\end{aligned}$$

This field results in an upward motion of positive ions and a downward motion of electrons. If the load current is zero, charges will accumulate on the upper and



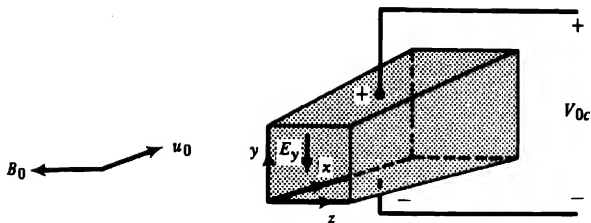
**FIGURE 5.27 An MHD Duct.**



**FIGURE 5.28** *An Ideal Closed Cycle MHD Generator.*

lower plates until the field produced by these charges cancels the induced field. With a load resistance, the current,  $I$ , is a result of the plasma current. The plasma current interacts with the magnetic field producing a force which tends to retard the gas flow. In many respects, the MHD duct is similar to the gas turbine of the Brayton cycle (in Section 3). During the adiabatic expansion of  $c$  to  $d$ , electrical, rather than mechanical, energy is produced. Since a portion of the electrical output would be needed to drive the compressor, an ideal MHD cycle has the same maximum theoretical efficiency as that of the ideal gas-turbine cycle (Figure 5.28). The waste heat,  $Q_1$ , of proposed MHD generators will be at a relatively high temperature. The low-temperature heat exchanger could thus be the boiler of a steam-turbine cycle. Used in this fashion, a combined cycle MHD generator and steam turbine are expected to result in overall efficiencies in excess of 50%.

Between the combustion chamber and the MHD duct,  $c$  to  $0$ , is a nozzle, within which a portion of the internal energy of the gas is converted to kinetic energy.



**FIGURE 5.29 An Open-Circuited MHD Duct.**

If the combustion chamber and nozzle are considered as a single unit, a steady-state, steady-flow equation follows.

$$h_b + Q_2 = h_0 + \frac{1}{2}Mu_0^2$$

Point  $b$  is considered the input, and point 0 is the exit. A term,  $\frac{1}{2}Mu_0^2$  is included to account for the kinetic energy associated with the high exit velocity of the gas (the input kinetic energy has been assumed negligible). The following is thus obtained for an ideal gas.

$$\begin{aligned} \frac{1}{2}Mu_0^2 &= Q_2 - (h_0 - h_b) \\ &= Q_2 - \frac{n+2}{2}R(T_0 - T_b) \end{aligned}$$

The temperature at the entrance to the combustion chamber,  $T_b$ , depends, for an ideal gas compressor, upon the pressure ratio and the ambient temperature,  $T_a$ .

$$T_b = T_a(p_2/p_1)^{2/(n+2)}$$

A specified value of  $Q_2$ , a quantity that depends upon the combustion process, yields a relationship between  $T_0$  and  $u_0$ . The higher the temperature, the lower the velocity. Since a high temperature is desired for a high ionization rate and a high velocity is desired to maximize the induced voltage, and hence, the output power, a compromise between temperature and velocity is necessary. To avoid supersonic turbulence, however, a velocity somewhat less than that of sound is necessary.

The Lorentz force equation may be used to determine the force on the moving charges of the ionized gas. Assuming that all currents are zero, as is the case for the open-circuited generator of Figure 5.29, the following is obtained for the force on a charge of  $q$  coulombs.

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$F = q(E_y + u_0 B_0)$$

Positive ions in the absence of an electric field experience an upward force, whereas the force acting on negatively charged electrons is downward. For an open circuit, these charges accumulate on the plates, resulting in a net positive charge on the upper plate relative to that of the lower plate. Charges will continue to accumulate until an equilibrium condition is achieved, that is, until an electric field is developed that results in a zero net force.

$$\begin{aligned}F_y &= 0 = q(E_y + u_0 B_0) \\E_y &= -u_0 B_0\end{aligned}$$

Since  $E_y$  is a negative quantity, it corresponds to a downward-directed electric field (the direction of a force on a positive test charge). The field thus tends to oppose further charge accumulation. The open-circuit voltage,  $V_{0c}$ , is hence equal to  $u_0 B_0 a$ ,  $a$  being the height of the duct.

$$V_{0c} = u_0 B_0 a$$

For magnetic fields of 2 to 5 T (tesla, or webers per square meter) and a velocity of 1000 m/s (comparable to the speed of sound for anticipated temperatures and pressures), a voltage of 2 to 5 kV results for a one-meter duct height ( $a = 1$  m).

To produce electric power, a load current and hence a plasma current must be generated. The plasma current depends upon the motion of positively charged ions and free electrons. The charge currents, however, depend upon velocity, with the velocity due to the more massive ions being considerably less than that due to the electrons. Therefore, only the current of the free electrons will be significant.

$$q = -e = -1.6 \times 10^{-19} \text{ coulombs}$$

It will also be assumed, due to the construction of the MHD duct in Figure 5.27, that  $E_z = 0$ . The  $z$  component of force,  $F_z$ , will also be zero, since  $\mathbf{v} \times \mathbf{B}$  for a  $z$  directed magnetic field will have no  $z$  component. If  $v_x$  and  $v_y$  are the components of the electron's drift velocity relative to the velocity of the gas, the following is obtained.

$$\begin{aligned}F_x &= -e(E_x - v_y B_0) \\F_y &= -e[E_y + (u_0 + v_x) B_0]\end{aligned}$$

For conditions typical of an MHD generator, the drift velocity of electrons is proportional to the force. Mobility,  $\mu$ , however, is usually specified in terms of an electric field.

$$\mathbf{v} = -\mu \mathbf{E}$$

The minus sign indicates that an electron drifts in the opposite direction of the field. Since  $-e\mathbf{E}$  is the force on an electron, the following may be written.

$$\mathbf{v} = -(\mu/e)e\mathbf{E} = (\mu/e)\mathbf{F}$$

While the above relation was derived in terms of a force due to an electric field, it is equally valid for forces arising from a combination of electric and magnetic fields.

$$v_x = (\mu/e)F_x \quad v_y = (\mu/e)F_y$$

Two simultaneous equations are thus obtained for the velocity components.

$$v_x = -\mu(E_x - v_y B_0)$$

$$v_y = -\mu[E_y + (u_0 + v_x)B_0]$$

If the single collector of Figure 5.27 is used, the axial component of the electric field,  $E_x$ , tends to be shorted out by the collector.

$$E_x = 0$$

$$v_x = \mu v_y B_0$$

$$v_y = -\mu[E_y + (u_0 + \mu v_y B_0)B_0]$$

$$v_y = -\frac{\mu(E_y + u_0 B_0)}{1 + \mu^2 B_0^2}$$

$$v_x = -\frac{\mu^2 B_0(E_y + u_0 B_0)}{1 + \mu^2 B_0^2}$$

The plasma current density (current per unit area) depends upon the velocity of the electrons, the density of free electrons,  $n_e$ , and the charge of the electron,  $-e$ .

$$J_x = -n_e e v_x \text{ amperes/square meter}$$

$$J_y = -n_e e v_y$$

$$J_x = \frac{n_e e \mu^2 B_0 (E_y + u_0 B_0)}{1 + \mu^2 B_0^2}$$

$$J_y = \frac{n_e e \mu (E_y + u_0 B_0)}{1 + \mu^2 B_0^2}$$

When  $E_y$  is equal to  $-u_0 B_0$  (the open-circuit condition), both  $J_x$  and  $J_y$  vanish.

For a load resistance, the load current is equal to the current density,  $J_y$ , times the area of the collector,  $bd$ .

$$I_L = bdJ_y$$

The load voltage,  $V_L$ , owing to the internal impedance of the MHD generator, will be less than the open-circuit voltage,  $V_{0c}$ . Introducing a loading factor,  $K$ , a quantity necessarily less than one, the following is obtained.

$$V_L = KV_{0c} = Ku_0 B_0 a$$

$$E_y = -V_L/a = -Ku_0 B_0$$

Expressions for both the load current and the power,  $P_L$ , may then be obtained.

$$I_L = \frac{n_e e \mu u_0 B_0 (1 - K) bd}{1 + \mu^2 B_0^2}$$

$$P_L = I_L V_L = \frac{n_e e \mu u_0^2 B_0^2 K (1 - K) abd}{1 + \mu^2 B_0^2}$$



The maximum power,  $P_{L\max}$ , corresponds to a maximum in the factor  $K(1 - K)$ , occurring when  $K = \frac{1}{2}$ .\*

$$P_{L\max} = \frac{n_e e \mu u_0^2 B_0^2 a b d}{4(1 + \mu^2 B_0^2)}$$

The load power depends upon the electron's mobility,  $\mu$ , as well as upon the electron density,  $n_e$ . Electron mobility for conditions typical of an MHD generator will be on the order of .1 to 10 meters/second per volt/meter. For magnetic fields of 2 to 5 T,  $\mu^2 B_0^2$  will be comparable if not large compared to unit. This factor, being in the denominator of the power expression, tends to reduce the load power. A segmented electrode generator, Figure 5.30, however, does not have this factor in the denominator of its expression for output. In a segmented generator, a return path for the axial current density,  $J_x$ , is lacking. Therefore, while an axial field may exist, the axial current density must vanish.

$$J_x = 0 \quad v_x = 0$$

Using the original expression for  $v_y$ , in which  $v_x$  is now zero, the following is obtained. The subscript 1 is used for the first segment.

$$v_y = -\mu(E_y + u_0 B_0)$$

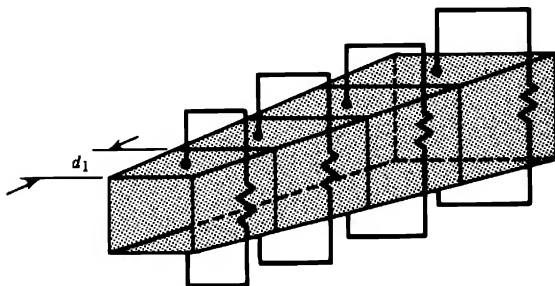
$$J_y = -n_e e v_y = n_e e \mu (E_y + u_0 B_0)$$

$$I_{L1} = n_e e \mu u_0 B_0 (1 - K) b d_1$$

$$P_{L1} = n_e e \mu u_0^2 B_0^2 K(1 - K) a b d_1$$

$$P_{L1\max} = \frac{1}{4} n_e e \mu u_0^2 B_0^2 a b d_1$$

If the total length of the duct is  $d$ , ( $d = d_1 + d_2 + \dots$ ), an expression for power identical to that of the numerator for the nonsegmented generator is achieved.



**FIGURE 5.30 A Segmented Electrode MHD Generator.**

\*When  $K = 0$ ,  $V_L = 0$  and when  $K = 1$ ,  $I_L = 0$ .

$$P_{L\max} = \frac{1}{4} n_e e \mu u_0^2 B_0^2 a b d$$

Since the output of a segmented generator may be considerably greater than the output of the nonsegmented version, a segmented design is normally preferable.

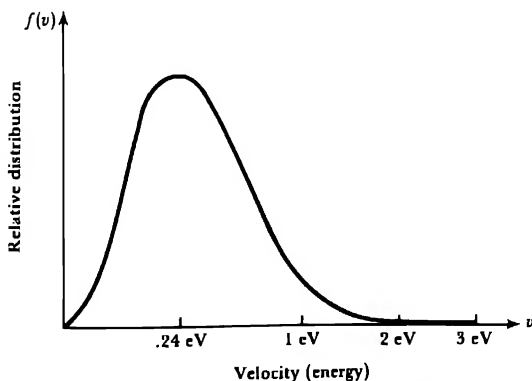
To obtain reasonable output powers, a free electron density greater than that associated with combustion alone is necessary. Since the product of the mobility and the density is involved, the conductivity of the plasma,  $\sigma$ , is important.

$$\sigma = n_e e \mu \quad \text{mhos/m}$$

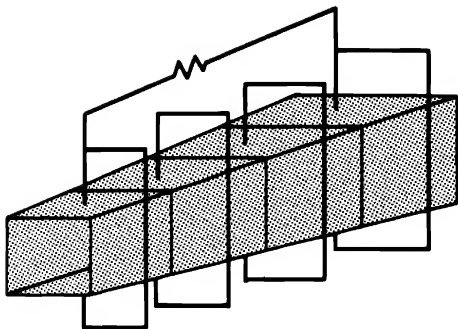
Reasonable values of conductivity, that is, values greater than one mho/m, are necessary for significant power generation. The electron density depends upon the rate at which electron ionizing collisions with molecules occur. This in turn depends upon the free electrons' velocity distribution. For free electrons in thermal equilibrium with molecules of the gas, an approximate Maxwellian distribution results. In terms of the speed of an electron,  $v$ , the following distribution,  $f(v)$ , in which  $m$  is the mass of the electron is obtained.

$$f(v) = 4\pi n_e v^2 \left( \frac{m}{2\pi k T} \right)^{3/2} e^{-mv^2/2kT}$$

The distribution function,  $f(v)$ , corresponds to the number of electrons with a speed of  $v$  in a velocity interval of one meter per second. Figure 5.31 is a plot of  $f(v)$  for a temperature of 2500°C (2773 K). Electron energies ( $\frac{1}{2}mv^2$ ) are indicated on the horizontal scale in electron volts. Very few electrons have energies greater than a few electron volts. Ionization of molecules having ionization energies of 10 to 15



**FIGURE 5.31** *Relative Electron Velocity Distribution for a Temperature of 2500°C.*



**FIGURE 5.32 A Hall-Effect MHD Generator.**

eV is insufficient for a practical generator. Alkali metals such as cesium (3.89 eV) and potassium (4.34 eV), which have considerably lower ionization energies, have a much higher likelihood of being ionized. Introducing a small quantity of these atoms during combustion, typically 1 to 2%, results in values of conductivity commensurate with reasonable output powers ( $\sigma > 10$  mho/m). Seeding, however, complicates the cycle since the seed materials need to be recovered. If an MHD generator is used in conjunction with a steam-turbine cycle, a seed precipitator preceding the stack is required. Both economic and pollution considerations preclude releasing the seeded gas to the atmosphere.

The generators of Figures 5.27, 5.29, and 5.30 rely on electric fields and currents perpendicular to the gas flow and are known as *Faraday generators*. Figure 5.32 illustrates another configuration, a Hall-effect generator, which depends upon axial components of the electric field and current. To maximize output, the segmented transverse electrodes are short-circuited, resulting in a zero value for  $E_y$ . The load for this generator is between the end electrodes.

MHD generators produce direct current, the voltage and current being determined by the duct configuration and the plasma parameters. A direct to alternating current converter is thus necessary if the electrical energy output is intended for use by utilities. Several practical considerations tend to limit the immediate introduction of MHD generators. Incomplete combustion tends to reduce both the expected gas temperature and velocity in the duct. Desired conditions can be achieved either by preheating the air prior to combustion or by introducing excess oxygen. Energy expenditures for producing oxygen or preheating the air tend to reduce the overall efficiency. Power is also required to produce the high magnetic fields although superconducting magnets could greatly reduce this loss. Duct materials need to be either able to withstand high combustion temperatures or to be suitably cooled. Unlike in the gas turbine, however, mechanical stresses are

minimal. Since plasma wall losses can be significant, reasonable size ducts are necessary to increase the volume to wall ratio. As a consequence, generators with thermal powers less than a few hundred megawatts appear impractical.

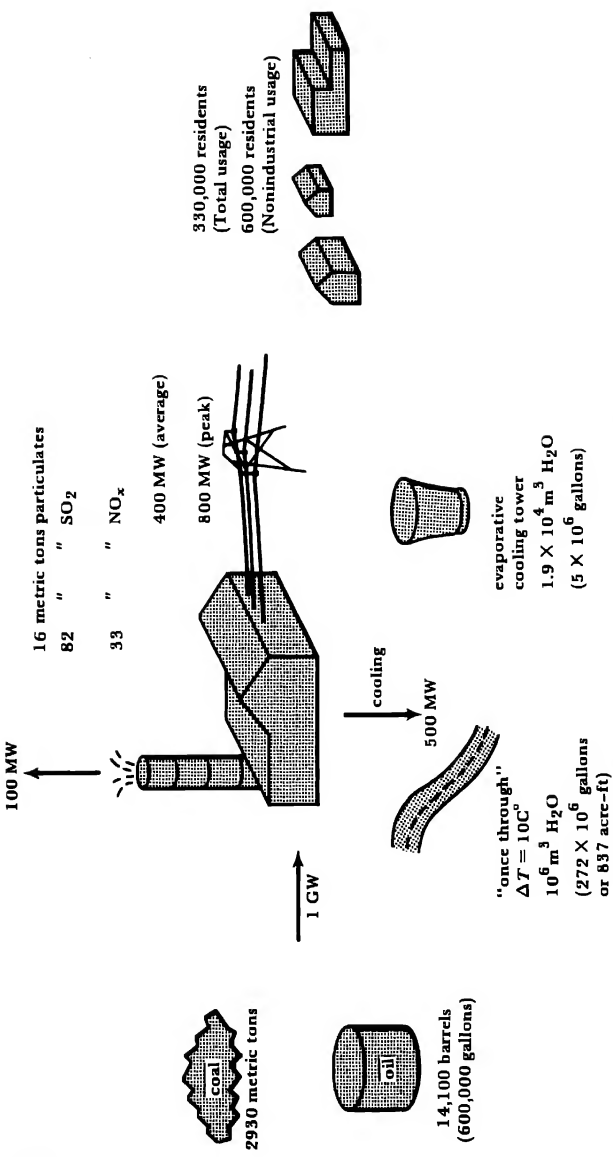
A more complete analysis of an MHD generator is obviously beyond the scope of this brief section. For example, the reaction on the gas flow by the plasma current ( $\mathbf{J} \times \mathbf{B}$ ) has been omitted though it must be considered in the duct's design and in estimating its efficiency. Several energy conversion texts have sections covering MHD generators [19-24] and numerous others are devoted exclusively to MHD generation [25-28]. Two problems are included at the conclusion of the chapter to provide a quantitative perspective.

## 6. ENVIRONMENTAL CONSIDERATIONS

Electric power plants, especially large modern facilities, have a significant environmental impact over a very substantial area. To obtain a quantitative estimate of this impact, an electric generating plant with an average thermal heat input of one gigawatt ( $10^9$  W) will be considered. Assuming an efficiency of 40%, an efficiency corresponding to that of modern plants, the average electrical output power is 400 MW. Based upon an average U.S. per capita electrical energy consumption rate of approximately 1.2 kW, this facility is sufficient to supply all electrical energy requirements, both direct and indirect, for 330,000 people. Based upon the ratio of installed capacity to the average consumption rate, total generating capacity needs to be approximately 2.3 times that inferred by the average rate of 1.2 kW per capita. The power plant thus needs to be capable of supplying a peak electric power of 920 MW which, assuming an unchanged conversion efficiency, requires a peak thermal input of 2.3 GW. This power plant with a peak electrical capacity of 920 MW is comparable in size to large single-unit generators now in operation and is typical of numerous facilities that are either under construction or have been planned.

Residential usage of electrical energy for the United States in 1979 accounted for 33% of that supplied by utilities (Figure 5.3). An average power of 400 MW would therefore be sufficient for the residential usage of 1 million people. If both residential and commercial usage is considered (55.9% of total consumption), this results in a population base of 600,000 people. Excluding industrial usage, the U.S. average consumption rate of electrical energy is approximately  $\frac{2}{3}$  kW per capita, and it is also the power requirement for those in a not heavily industrialized urban area. The 400 MW electric power plant (average power) is thus sufficient to supply a medium-sized urban area of 600,000 people.

Figure 5.33 provides the relevant data for such a power plant. An assumed boiler efficiency of 90% results in a stack loss of 100 MW. Since the electrical output for an overall efficiency of 40% is 400 MW, a cooling or waste heat removal of 500 MW is required. The fossil fuel requirements are significant. For one day's operation,  $24 \times 10^9$  Wh or  $8.19 \times 10^{10}$  Btu of thermal energy is necessary. Assuming a heat value of  $28 \times 10^6$  Btu per metric ton for coal, this would mean 2930 metric tons of coal, that is, 29 railroad cars of coal, each with a capacity of 100 tons. A



**FIGURE 5.33 One Gigawatt (Thermal) 40% Efficient Electric Power Plant: Daily Quantities.**

moderate-sized train (58 coal cars) would thus be required every other day to supply the plant with coal. If oil is used as a fuel ( $5.8 \times 10^6$  Btu/bbl), 14,100 bbl (nearly 600,000 gal) would be needed each day. Based upon a per capita usage of 1.2 kW, the daily coal requirement is nearly 9 kg and that of oil is 1.8 gal. It is primarily the centralized aspect of power plants rather than its per capital rates that result in its large impact.

The simplest method of removing the waste heat is through a system using a flow of water to directly cool the condenser. Since large quantities of water are necessary, this method is suitable only for plants adjacent to rivers or large bodies of water. A cooling rate of 500 MW requires the daily removal of  $4.1 \times 10^{10}$  Btu or  $10^{13}$  cal of waste heat. For a temperature increase of ten Celsius degrees,  $10^9$  kg of water are required. This constitutes a daily rate of one million cubic meters of water ( $272 \times 10^6$  gal or 837 acre-ft) or an average flow rate of  $12 \text{ m}^3/\text{s}$ . While the water is not used in the sense of being used up, large flow rates are necessary. "Once through" cooling systems for large power plants are impractical in water-sparse regions. In addition, if the growth of electrical energy consumption within the United States continues, natural runoff will be insufficient for inland power plants.

An alternative waste heat removal scheme which is, despite its additional expense, becoming increasingly popular, is the cooling tower [29-32]. A cooling tower transfers the waste heat to the atmosphere either by the evaporation of water (a wet tower) or through direct conduction (a dry tower). Natural draft towers can be very large (over 100 meters in diameter and 100 to 150 meters in height). Although forced draft towers are considerably smaller, large motor-driven fans are necessary in them to provide sufficient air flow.

Since a wet cooling tower, either natural or forced draft, cools through evaporation, considerably less water is required than for a "once through" system. The evaporated water, however, is "lost" to the atmosphere. While the heat of evaporation depends to a small extent upon ambient air conditions, a cooling of approximately 540 cal/g of water is realized. A daily water consumption of only  $20,000 \text{ m}^3$  (5 million gal) would thus be required for the 1 GW example, which corresponds to a flow rate of  $.22 \text{ m}^3/\text{s}$ —less than 2% of the water needed for a nonevaporative cooling system. Dry cooling towers require no water, but they tend to be larger, more expensive, and, owing to their poorer heat transfer, result in a small reduction of the power plant's overall efficiency.

**TABLE 5.2 Estimated 1977 Emissions for U.S. Electrical Utilities and Average Emissions per Unit of Thermal Energy**

Pollutant	$10^6$ tons per year	g/Wy
SO <sub>2</sub>	19.4	30.1
Particulates	3.7	5.7
CO	0.3	0.47
HC	0.1	0.16
NO <sub>x</sub>	7.8	12.1

SOURCE: Emission data from reference 33.

Large electric utility power plants also emit considerable quantities of air pollutants, as indicated by Table 5.2 [33]. The emissions per unit of energy of this table are based upon the total thermal energy used for electricity generation in 1977. While the emissions are due to all fuel types, sulfur dioxide and particulate emissions for natural gas are normally very small, and hence the emissions for the combustion of coal and oil tend to result in higher per unit energy emissions than indicated. The daily emissions of the 1 GW example, based upon the average emissions for 1977, are tabulated in Table 5.3. Included in this table is the dilution volume necessary to achieve the primary air quality standards of Table 5.3. The last quantity of this table,  $R$ , was calculated assuming the pill-shaped dilution volume of Figure 5.34 that has a height of 2 km.

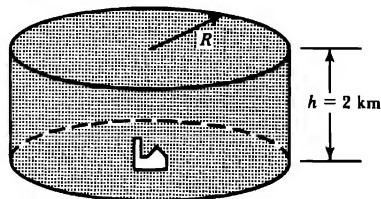
**TABLE 5.3 Daily Emissions of 1 GW (Thermal) Plant Based Upon Average U.S. Emissions for 1977**

Pollutant	Daily emission <sup>a</sup> (metric tons)	Primary standard <sup>b</sup> ( $\mu\text{g}/\text{m}^3$ )	Dilution volume ( $\text{km}^3$ )	$R$ (km)
SO <sub>2</sub>	82	80	1025	12.8
Particulates	16	75	213	5.8
NO <sub>x</sub>	33	100	330	7.2

<sup>a</sup> Average value based upon average emissions of Table 5.2.

<sup>b</sup> Table 3.2.

The region affected by the atmospheric pollution depends primarily upon the dispersal time of the pollutants, a time that is more or less comparable to one day. In general, however, other sources of emissions, particularly those due to the automobile, must also be considered. The pill-shaped dispersal volume should therefore be considered only as a very rough approximation of the actual emission dispersal process. Natural dispersal is usually insufficient to achieve desirable air quality standards. National emission standards, Table 5.4, have thus been adopted, which are applicable to all new power plants [34]. Based upon average figures for 1977,



**FIGURE 5.34 Pill-shaped Dilution Volume.**

a substantial reduction in emissions is required. In addition, state and locally enacted emission regulations affect many presently operating power plants.

Particulate emissions consist of inert fuel materials, fly ash, and combustibles that are released unburnt, as soot, owing to inefficient combustion. The technology for removing particulate emissions is considerably better developed and more readily available than that needed to remove gaseous emissions of sulfur dioxides and nitrogen oxides. Four devices used to remove particulates from flue gases are illustrated in Figure 5.35. The bag filter consists of a fabric bag which is periodically shaken to remove entrapped particles, which then settle to the bottom of the collector. The electrostatic precipitator utilizes a high-voltage electrode to ionize the flue gas. Ions tend to attach themselves to the particles, which are then attracted to the outer grounded electrode. Periodic tapping of the electrode is used to free collected material. Both bag filters and precipitators are highly effective (99%) in the removal of small particles that affect visibility and are responsible for adverse health effects.

**TABLE 5.4 Standards of Performance for Fossil-Fuel-Fired Steam Generators**

	<i>Fuel type</i>	<i>Standard based upon heat input<sup>a</sup></i>	<i>Alternative units</i>	<i>Percentage of 1977 emissions</i>
Sulfur dioxide	Liquid	.8 lb/10 <sup>6</sup> Btu	10.9 g/Wy	36
	Solid	1.2 lb/10 <sup>6</sup> Btu	16.4 g/Wy	54
Particulate matter <sup>b</sup>	All	.1 lb/10 <sup>6</sup> Btu	1.4 g/Wy	25
Nitrogen dioxide	Gaseous	.2 lb/10 <sup>6</sup> Btu	2.7 g/Wy	22
	Liquid	.3 lb/10 <sup>6</sup> Btu	4.1 g/Wy	34
	Solid	.7 lb/10 <sup>6</sup> Btu	9.6 g/Wy	79

<sup>a</sup> Maximum two-hour average.

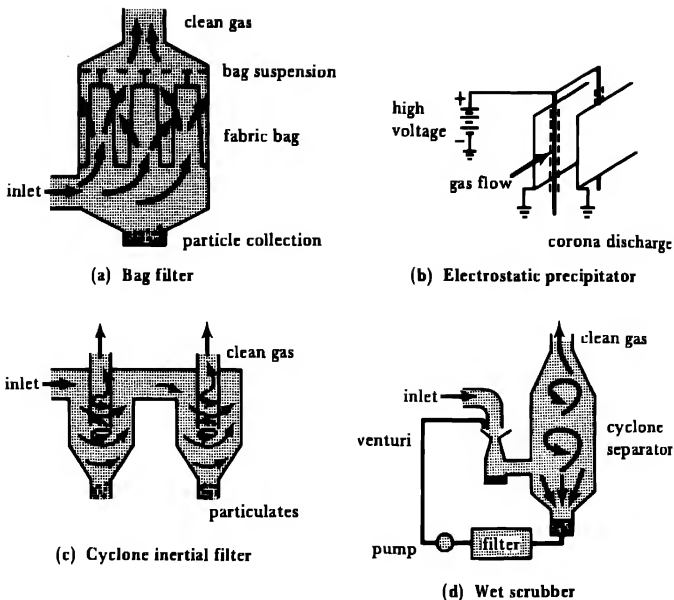
<sup>b</sup> Opacity no greater than 20%, except that 40% permissible for up to two minutes of any hour. The effect of uncombined water vapor is excluded from the opacity standard.

SOURCE: Data from reference 34.

A less complex but also less effective filter is the cyclone inertial collector. Inertial and drag forces are used to separate the particulates from the gaseous components. This collector, although particularly effective in the removal of large particles, is limited in its ability to remove small particles. The last particulate removal device of Figure 5.35 is the wet scrubber. Particles entrapped by the fluid are separated along with the fluid in a cyclone inertial filter. Water used as a scrubbing fluid is highly effective in particulate removal.

Sulfur dioxide flue gas removal schemes, unfortunately, are less developed than particulate filters [35-37]. The present reduction of sulfur dioxide emissions has been accomplished through the use of fuels with a low sulfur content. Since supplies of low sulfur fuels are very limited, this is purely a short-term solution. One scheme for reducing sulfur dioxide emissions consists of injecting limestone (CaCO<sub>3</sub>) along with fuel into the furnace. At high temperatures, the sulfur dioxide reacts with the limestone to form calcium sulfate (CaSO<sub>4</sub>) which can then be removed by a wet





**FIGURE 5.35 Flue Gas Particulate Removal Devices.**

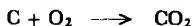
scrubber. Limestone injection, however, is not effective enough in removing sulfur dioxide to be used for many of the high sulfur content fuels.

Scrubbing flue gases with a slurry of limestone, however, does appear a viable sulfur dioxide removal process for high sulfur fuels. This method avoids the furnace injection of limestone and the associated problems. In the scrubbing processes, the sulfur dioxide reacts with the slurry, forming calcium sulfates and sulfides which are then removed with the scrubbing slurry. Since the scrubbing process concurrently removes particulates, this appears to be a particularly attractive scheme.

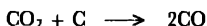
An alternative process for reducing sulfur dioxide emissions is to remove the sulfur from the fuel before combustion. In order to conserve limited supplies of oil and gas, large quantities of coal will undoubtedly be used for electric power plants, making sulfur removal from coal very important. Large-scale usage of coal for proposed mammoth generating complexes may, in itself, result in an unacceptable environmental impact [38, 39]. Coal gasification appears an attractive means of producing a low-energy content clean gas, free of sulfur, for electric generation [40–43]. In addition to low-energy gas production, pilot plants are also being built to produce a high-energy content natural gas substitute from coal. While a high-energy content gas is necessary for efficient pipeline distribution, the low-energy fuel produced by a much simpler process is adequate for use at the gasification site. Gasification

would thus be done at the site of the electric power plant.

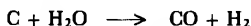
Coal gasification is not a new process. "Power gas" was used in the nineteenth century for specialized applications that required a clean fuel. When heated, entrapped hydrogen and hydrocarbons are liberated from the coal. Heating can most readily be accomplished through combustion.



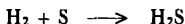
If the available oxygen is insufficient, the carbon dioxide can react with carbon to produce carbon monoxide.



While hydrogen is liberated from the coal, additional hydrogen can be formed by passing steam through the coal bed.



The combustible constituents of the coal gas are thus primarily carbon monoxide and hydrogen. Since air is used as a source of oxygen, large quantities of nitrogen that reduce the heating value of the gas are also present. A further reaction between hydrogen and sulfur is important.



Hydrogen sulfide may be removed from the gas by a reaction with limestone.



This reaction is accomplished by scrubbing the gas with a limestone or dolomite ( $\text{CaCO}_3$  and  $\text{MgCO}_3$ ) slurry. Scrubbing also removes other undesirable particulates from the coal gas, thus producing a clean-burning fuel.

A low-heating-value fuel, 120 to 160 Btu/ft<sup>3</sup> (as compared to natural gas, on the order of 1000 Btu/ft<sup>3</sup>), is produced by this process, which is adequate for a steam-turbine boiler or a gas turbine. Since coal cannot be used directly for gas turbines, this increases the utility of coal. The advantages of high-efficiency combined gas-turbine steam-turbine cycles can thus be realized with a coal-derived fuel. A concurrent benefit of the coal gasification sulfur-removal process is that sulfur is removed in the form of a sulfide,  $\text{CaS}$ , as opposed to that associated with flue gas removal, in which a sulfate,  $\text{CaSO}_4$ , is obtained. Elemental sulfur, a salable product, may readily be obtained from the sulfide.

Coal gasification and the subsequent combustion of the gas can, in effect, be carried out in a single reactor: a fluidized-bed combustion system. This system is fueled with coal particles of up to two centimeters in size as opposed to the finely pulverized coal used in conventional boilers. A turbulent, relatively high-velocity air stream (a few meters per second) tends to suspend the coal particles during combustion. By suitably controlling the coal rate and the air flow, an incomplete combustion reaction occurs yielding a gas rich in carbon monoxide. The boiler is generally located right above the fluidized-bed combustion region. Through the use of a secondary supply of air at the boiler, the carbon monoxide gas is oxidized

resulting in an efficient heat transfer to the boiler tubes. One advantage of fluidized-bed combustion is the ease of ash removal. The ashes produce a heavy agglomerate which falls from the combustion region and is carried off by a moving grate. Emissions of sulfur oxides can be significantly reduced by injecting crushed limestone, together with the coal, into the fluidized-bed combustion region. During combustion, the sulfur of the coal reacts with the limestone forming calcium sulfate which, along with the ash agglomerates, falls from the combustion region. Flue-gas scrubbing is thus not necessary for a fluidized-bed combustion system.

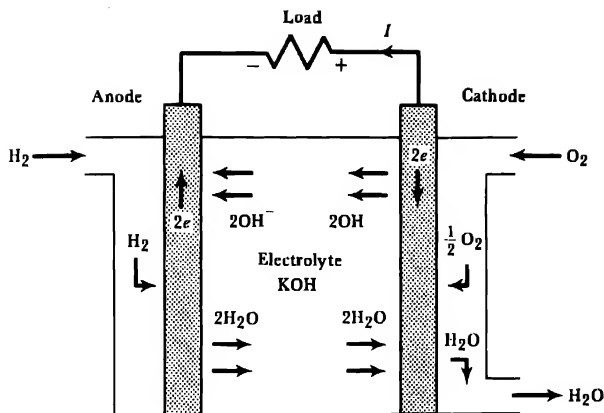
Nitrogen oxide emissions are primarily the result of the combustion process in which oxygen combines with the nitrogen present in the air. Since the oxidation of nitrogen increases with temperature, a low flame temperature is important. Slow burning, in which air is continually mixed with fuel along the path of the flame (tangential firing) in a conventional boiler, tends to reduce the combustion temperature and hence the formation of nitrogen oxides. Since combustion temperatures required for efficient fluidized-bed combustion systems tend to be lower than those of conventional boilers, these combustion systems tend to also have relatively low emissions of nitrogen oxides.

## 7. FUEL CELLS

In a fuel cell the chemical energy of the fuel is converted directly to electrical energy. Not only is the mechanical-energy transfer eliminated (as for MHD energy conversion), but the high-temperature combustion process is also unnecessary. As a consequence, many of the pollutants normally formed at high temperatures are avoided. The fuel cell having no moving parts (except for auxiliary equipment such as circulating pumps) is a potentially viable, low polluting, high-efficiency device. The hydrogen-oxygen fuel cell, the most advanced type, has been extensively used for space applications. For these cells, energy storage is in the form of liquid hydrogen and oxygen, which, when combined in the fuel cell, produces electrical energy. Terrestrial applications, however, require the use of more readily available fuels, such as coal or hydrocarbons. A substantial developmental effort has been directed toward producing low-cost fuel cells that use fossil fuels (the cost of cells produced for space applications has been \$100,000/kW and more). The development of this type of fuel cell has been considerably more difficult than that of hydrogen-oxygen cells.

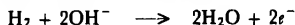
The basic concept of a fuel cell can be traced back to Sir William Grove (1839). It was not until 1932, however, that Francis Thomas Bacon (a descendant of the seventeenth-century Bacon) demonstrated a working hydrogen-oxygen fuel cell. During the 1950s, several fuel cells capable of producing substantial output powers were developed. A detailed coverage of fuel cells is included in many of the texts on direct energy conversion referred to in the discussion of MHD energy converters [19-24]. Other references are also available [44, 45], and there is an excellent brief survey of fuel cells written by Crowe and published by NASA [46].

Figure 5.36 is a schematic diagram of a hydrogen-oxygen fuel cell. Porous electrodes are required to bring the gaseous reactants into contact with the electrolyte.

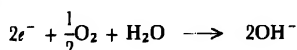


**FIGURE 5.36** *Schematic Representation of a Hydrogen-Oxygen Fuel Cell.*

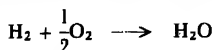
potassium hydroxide for the cell shown. At the electrodes, an electron interchange occurs which results in an external load current. Hydrogen, the fuel, combines with the hydroxial ions of the electrolyte at the anode.



The electrons, due to the external current, are transferred to the cathode where they react with the oxygen, the oxidant.



A water molecule is the net product of the overall reaction.

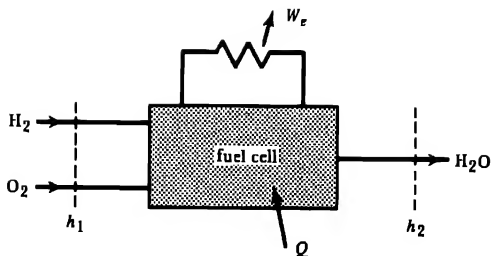


The fuel cell reaction differs from that of combustion in that the oxidation involves an electron transfer via an external circuit. A considerable portion of the reaction energy is thus obtained in the form of electrical energy, the remainder being heat.

Since each molecule of hydrogen gas,  $\text{H}_2$ , results in the transfer of two electronic charges, the charge transfer due to a kilogram-mole of gas may be obtained by multiplying the charge of the two electrons by Avogadro's number,  $N_0$ .

$$\begin{aligned} q &= 2eN_0 \\ &= 1.93 \times 10^8 \text{ coulombs} \end{aligned}$$

This is equivalent to a current of one ampere for  $1.93 \times 10^8$  s (approximately six



**FIGURE 5.37** *Steady-Flow Process of a Fuel Cell.*

years). The useful electrical energy,  $W_e$ , delivered to the load for each kilogram-mole of hydrogen gas depends upon the potential achieved,  $V$ .

$$W_e = qV$$

A fuel cell may be considered a steady-flow process (similar to that of Figure 5.15) in which the output work is electrical energy (Figure 5.37).

The output work and input heat are related to the enthalpy of the reactants,  $h_1$ , and the reaction product,  $h_2$ .

$$h_1 + Q = h_2 + W_e$$

$$W_e = h_1 - h_2 + Q$$

For a hydrogen-oxygen fuel cell,  $Q$  is negative, that is, heat is produced as a result of the reaction. The electrical energy is therefore less than the enthalpy change,  $h_1 - h_2$ , and hence less than the thermal energy produced in a simple combustion process in which the mechanical work is zero.

The electrical energy produced by the cell may be calculated from another thermodynamic property of the reactants, the Gibbs function (often referred to as the *free energy*). For a reversible, isothermal, constant pressure process, the work is equal to the change in the Gibbs function which, for a temperature of 25°C, yields the following.

$$\begin{aligned} W_e &= g_1 - g_2 = 56.7 \times 10^6 \text{ cal}/(\text{kg-mole of H}_2) \\ &= 2.37 \times 10^8 \text{ J}/(\text{kg-mole of H}_2) \end{aligned}$$

Knowing the electrical energy, the cell potential may be obtained.

$$\begin{aligned} V &= W_e/q \\ &= 1.23 \text{ V} \end{aligned}$$

Losses due to electrode resistances, polarization of the electrolyte, and depletion of the electrolyte at the electrodes decrease the electrical energy and hence the output voltage of the cell. The above potential, which was calculated ignoring losses, is therefore the open-circuit voltage.

The maximum electrical energy,  $W_e$ , may be compared to the energy of combustion,  $h_1 - h_2$ .

$$\begin{aligned} h_1 - h_2 &= 68.3 \times 10^6 \text{ cal/(kg-mole of H}_2\text{)} \\ &= 2.86 \times 10^8 \text{ J/(kg-mole of H}_2\text{)} \end{aligned}$$

The maximum efficiency of a cell is the ratio of  $W_e$  and  $h_1 - h_2$ .

$$\eta_{\max} = \frac{W_e}{h_1 - h_2} = .83$$

A hydrogen-oxygen fuel cell has a maximum efficiency of 83%; a minimum of 17% of the reaction energy is in the form of heat. Since the output is electrical energy, the thermodynamic losses of a conventional heat cycle are avoided.

The fuel cells used for the Apollo missions were similar to the cell of Figure 5.36. The cell was designed to operate at a pressure a few times the atmospheric pressure and at a temperature of 260°C. At room temperature, the electrolyte, a water solution of 75% potassium hydroxide, was solid. A complex start-up procedure was necessary to bring the electrolyte up to the operating temperature. Sintered nickel electrodes with a high porosity on the electrolyte side (to provide good electrolyte contact) and with a low porosity on the side in contact with the gaseous reactant were used. Accurate pressure differentials were maintained to prevent the electrolyte from flooding the gas passages. A complete unit consisting of 31 series-connected cells produced a voltage of 27 to 31 V and a power of 560 to 1400 W and weighed 111 kg. Three units operating in parallel produced the required electrical energy as well as potable water for the crew.

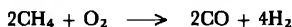
The Gemini fuel cells utilized a solid electrolyte (also referred to as an *ion exchange membrane*). While solid electrolytes tend to have higher resistivities than liquid electrolytes, they can be made very thin. Fibrous transport channels were used to absorb and carry off the water. This type of cell collected the reaction products within itself—a process suitable only for limited durations which are typical of space flights. With the solid electrolyte and fibrous water absorber, controlled pressurization was unnecessary. A fuel cell unit consisted of three stacks, each containing 32 cells, and produced powers up to one kilowatt with the 31-kg unit. Two units were required for each flight.

Hydrogen and oxygen can be produced by the electrolysis of water or by a high-temperature catalytic dissociation of water. The latter process, very inefficient at present, may be improved in the future. Manufactured hydrogen could be used as a synthetic fuel for terrestrial applications, directly replacing natural gas and, if liquified, could provide a substitute for petroleum [47, 48]. The widescale usage of hydrogen as a replacement for natural gas and petroleum has often been referred to as a *hydrogen economy*. Hydrogen-oxygen fuel cells could then be used for localized

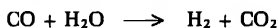
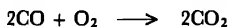
generation of electricity. Electrolysis of water, however, requires large quantities of electrical energy. The viability of hydrogen as a synthetic fuel, if produced by electrolysis, is therefore dependent upon the availability of large quantities of low-cost electrical energy. A greater potential for terrestrial fuel cells is offered by the direct conversion of fossil fuels to electricity.

Common hydrocarbon fuels such as methane (the main combustible constituent of natural gas) and petroleum derivatives are much less reactive than hydrogen and are therefore more difficult to oxidize. In addition, many hydrocarbon fuels, when oxidized, result in undesirable by-products which interfere with the operation of fuel cells. Since in addition to water, carbon dioxide is also a product of the oxidation of a hydrocarbon, an acidic electrolyte is necessary to avoid the formation of carbonates which inhibit the fuel cell reaction. Either expensive catalytic electrodes or high temperatures are necessary for the direct oxidation of hydrocarbon fuels.

An alternative to direct oxidation is to re-form the hydrocarbon fuel into a more readily oxidizable fuel, hydrogen. For natural gas, methane may be converted to hydrogen by a reaction with either oxygen or steam.



The resultant carbon monoxide may be further oxidized to form carbon dioxide.



The hydrogen produced by this reaction may then be directly used in a fuel cell. Since the use of liquified oxygen is impractical for most terrestrial applications, a fuel cell using air as the oxidant is desirable for an economically viable electric power source. The development of a self-contained fuel reformer and air-fuel cell has been the aim of a gas utility consortium, TARGET (Team to Advance Research for Gas Transformation). Pratt and Whitney Aircraft has been responsible for the developmental work. A 12½-kW electrical unit approximately the size of a conventional furnace which provides heat as well as electricity from natural gas has been tested. Phosphoric acid immobilized in a matrix electrode separator forms the electrolyte. Details of the electrode construction and catalyst have not been released.

An alternative fuel cell study has been carried out by Westinghouse. The goal of this project was to produce a high-temperature (1000°C) fuel cell suitable for producing electric power from coal in a central generating facility [49, 50]. A low-energy fuel gas described in the preceeding section would first be produced from the coal. Sulfur compounds which would interfere with the operation of the fuel cell would then be removed by scrubbing the gas. The resultant fuel would be a mixture of hydrogen and carbon monoxide; air would be used as the oxidant. Calcia-stabilized zirconia would form the electrolyte,  $(\text{ZrO}_2)(\text{CaO})$ . The fuel electrode would be either cobalt- or tin-stabilized zirconia cermet, while the air electrode would be either tin-doped indium oxide or antimony-doped strontium oxide. The studies conducted indicated that a 60% overall conversion efficiency was possible.

The technical feasibility of fuel cells has been established, but further developmental efforts are needed to substantially reduce their cost. Proposals for governmental funding provide for the continuation of a reasonable effort in the development of fuel cells.

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## PROBLEMS

1. Assume an electric power plant has a thermal efficiency of 40%. The heat energy and corresponding fuel to produce one kilowatt-hour of electrical energy are desired.
  - (a) Determine input heat energy in Btu.
  - (b) Determine, assuming coal to be the fuel, the quantity (in kilograms) necessary.
  - (c) Determine, in gallons, the quantity of oil that would be necessary. (One barrel of oil is equivalent to 42 gal.)
  - (d) Determine, in cubic feet, the natural gas necessary.
  - (e) Suppose the power plant had an efficiency of only 30%. By what percentage would its fuel requirements increase?
2. Consider the Rankine cycle of Figure 5.7.
  - (a) Determine for the upper pressure of 2000 psia the pressure in atmospheres and in newtons per square meter.
  - (b) Determine, assuming the ideal gas law is valid, the volume in cubic meters per kilogram-mole of the point 4' ( $T = 1000^{\circ}\text{F}$ ). Water vapor can be assumed to behave as a gas with six degrees of freedom. What is this volume in cubic feet per pound of water vapor? The experimentally derived value is  $.3935 \text{ ft}^3/\text{lb}$ .
  - (c) Path 4' to 5 ( $p = 1 \text{ psia}$ ) is an adiabatic expansion. Determine, using the pressure-volume relation for an ideal gas, the turbine expansion ratio, that is, the ratio of the volume of point 5 to that of 4'. The actual volume of point 5 is somewhat larger than that calculated by ignoring the phase change ( $258 \text{ ft}^3/\text{lb}$ ).
3. A common English energy unit for steam calculations is Btu per lb of water. Determine the conversion factor to convert the English units to joules per kilogram of water. Determine the equivalent energy units for the cycle of Figure 5.7. What is the work output, in watts, for a steam rate of one kilogram per second?
4. Consider the steam cycle of Figure 5.10. Determine the steam rate in pounds

per hour necessary for a mechanical output power of 1.0 GW. Assume individual turbines are used. What is the power output of the high-pressure turbine (4' to 4'')? What is the output of the low-pressure turbine?

5. Dry air with an atomic weight approximately 29 contains 21% by volume of oxygen,  $O_2$ . Since the other constituents of air, primarily nitrogen, do not contribute to the combustion process, only the heat associated with the combustion of oxygen is obtained. Assume an initial temperature of  $20^\circ C$  and atmospheric pressure.
  - (a) Calculate the heat of combustion of carbon with air. (See Chapter 3.) Express the result in joules per kilogram-mole of air.
  - (b) What is the heat of combustion for methane?
  - (c) Assume the exit velocity of a combustion chamber is negligible. Calculate, assuming the heat of combustion is equal to the change in enthalpy, the combustion temperature for carbon and methane. Ignore the change in gas composition due to combustion products and assume air behaves as a gas with five degrees of freedom.
  - (d) What is the volume expansion ratio assuming the pressure remains constant?
6. Consider the ideal closed-cycle gas turbine of Figure 5.12, with a pressure ratio of 16. Point (a) corresponds to ambient conditions, that is,  $20^\circ C$  and atmospheric pressure. The working fluid is air.
  - (a) Determine the compressor exit temperature,  $T_b$ .
  - (b) What is the heat input per kilogram-mole of air,  $Q_2$ , assuming the complete combustion of methane with air (use result of previous problem)? What would  $T_c$  be for this heat input? The entrance and exit velocities can be considered negligible.
  - (c) A more reasonable upper temperature is  $1200^\circ C$  for  $T_c$ . Determine the heat input,  $Q_2$ , for this temperature. Use this temperature for the remaining parts of the problem.
  - (d) Determine the exit temperature,  $T_d$ . What is the turbine work output?
  - (e) What is the compressor input work? What is the net output work? What fraction of the turbine output is needed to drive the compressor?
  - (f) What is the overall efficiency of the turbine, that is,  $W/Q_2$ ? Compare this result with that obtained from the expression derived in the text.

$$\eta = 1 - (p_1/p_2)^{2/(n+2)}$$

7. An ideally reversible compressor is used to compress 1 kg/min of air. Assume air behaves as an ideal gas with five degrees of freedom. The ambient pressure,  $p_A$ , is 1.01 bar and the ambient temperature,  $T_A$ , is  $20^\circ C$  (1 bar =  $10^5$  n/m<sup>2</sup>). The exit pressure of the compressor is eight times the ambient pressure.
  - (a) What is the temperature of the air after being compressed?
  - (b) What is the power, in kilowatts, necessary to drive the compressor? What is the rate at which available work is delivered by the compressed air?
  - (c) The compressed air is cooled, by means of a heat exchanger, to the ambient temperature while its pressure remains unchanged. What is the rate at which heat is removed from the air?

- (d) What is the rate at which available work is delivered by the cooled compressed air?
  - (e) Suppose an ideal system is used to recover the available work of the cooled compressed air. What is the rate at which heat, at a temperature of  $T_A$ , needs to be absorbed by the ideal system if the available work is to be fully recovered?
8. A throttling valve is used with the compressor of Problem 7 to reduce the compressed air pressure to 5 bars. While throttling is irreversible, the change in enthalpy of the air remains unchanged.
- (a) What is the rate at which available work is lost as a result of the throttling process? What is the rate at which available work is transferred by the compressed air after being throttled?
  - (b) An ideal reversible turbine is used which exhausts air at atmospheric pressure. What is the output power of the turbine? What is the rate at which available work is lost by the turbine exhaust?
  - (c) An ideal system which allows for a heat transfer at the ambient temperature is used to extract mechanical power from the compressed air. Determine the maximum output power for the system and corresponding rate of heat transfer.
9. A compressed air system using a compressor and a turbine is used for transferring mechanical power. The input power is 2 kW and the pressure ratio for the air is five. The ambient pressure is 1.01 bar and the ambient temperature is  $20^\circ\text{C}$  ( $1 \text{ bar} = 10^5 \text{ n/m}^2$ ).
- (a) Assume the compressor and turbine are ideal reversible devices. What is the temperature of the compressed air and what is its flow rate expressed in kilogram-moles per second?
  - (b) Assume the compressor is 80% efficient and the turbine is ideal. What is the temperature of the compressed air and its flow rate for this condition? What is the rate at which available work is transferred by the compressed air expressed in kilowatts? What is the output power of the turbine if the air is exhausted at the ambient pressure? What is the rate at which available work is "lost" if the turbine is exhausted to the atmosphere?
  - (c) Assume both the compressor and turbine are 80% efficient. Repeat part (b) for this condition.
10. A district heating system supplies water at a temperature of  $95^\circ\text{C}$ . A  $20^\circ\text{C}$  cooling occurs before the water reaches its destination. The return temperature is  $20^\circ\text{C}$ , the same as that of the ambient environment.
- (a) Heat is supplied at the rate of  $P(\text{thermal})$  to the system. What is the minimum rate at which available work needs to be supplied to provide the same quantity of thermal energy?
  - (b) What is the fraction of the thermal energy lost by the distribution system? What is, in terms of  $P(\text{thermal})$ , the rate at which available work is lost?
  - (c) At the destination of the system, a perfectly insulated heat exchanger is used to provide a domestic supply of hot water at a temperature of  $50^\circ\text{C}$ . The domestic water enters at a temperature of  $20^\circ\text{C}$  and the district return is

- at 20°C. What must be the ratio of the two flow rates? What must be the rate at which thermal energy is transferred to the domestic water? What is the rate at which available work is transferred to the domestic water?
- (d) A furnace with a 15% flue loss is used to power the district heating system. What is the overall first-law efficiency in supplying the domestic hot water? What is the second-law efficiency.
11. High-energy electrons are required to generate ions in an MHD electric generator. An expression for a Maxwellian electron distribution,  $f(v)$ , is given in the text and indicated in Figure 5.31.
- (a) Show that this distribution function integrated over all speeds (0 to  $\infty$ ) is indeed  $n_e$ .
- (b) Assume the temperature of the electron gas is 2500°C (2773 K). Determine the fraction of the electrons that have kinetic energies sufficient to ionize cesium (3.89 eV). (Note: The integral that needs to be evaluated can be, if integrated by parts, transformed to a standard probability integral.)
- (c) Determine, again for a temperature of 2500°C, the fraction of the electrons that have sufficient kinetic energies to ionize potassium (4.34 eV).
12. For an MHD generator, a gas with a high temperature and velocity is desired. Assume a compression ratio of three is utilized. The entrance conditions correspond to ambient conditions, that is, a temperature of 20°C and atmospheric pressure. Air is the working fluid and the cycle is that of Figure 5.23.
- (a) What is  $Q_2$ , assuming the heat is a result of the combustion of methane (see Problem 5)?
- (b) Determine the compressor exit temperature,  $T_b$ .
- (c) Assume the exit velocity is 1000 m/s, a velocity just slightly less than sonic velocity for these conditions. What is the exit temperature?
- (d) Assume a flow rate of one kilogram-mole per second. What is the heat input power? What is the power supplied to the compressor?
13. Based upon the per unit of energy emission data of Table 5.2, what is the average quantity of each pollutant per kilowatt-hour of electricity for 1977? Assume an overall conversion efficiency of  $\frac{1}{3}$ . What is the yearly per capita quantity of pollutant emissions for an average per capita consumption rate of 1.2 kW?
14. Consider the electric power plant of Figure 5.33.
- (a) What are the daily carbon dioxide emissions if the 1 GW input power is due entirely to the combustion of carbon?
- (b) A limestone slurry is to be used to reduce the sulfur dioxide emissions. Assume the limestone,  $\text{CaCO}_3$ , combines with the sulfur dioxide forming calcium sulfide,  $\text{CaSO}_3$ , and carbon dioxide. What is the daily quantity of limestone required to remove all sulfur dioxide? What is the daily quantity of waste sulfide?
- (c) By how much does the sulfur dioxide removal increase the carbon dioxide emissions of the plant?
- (d) Complete removal of sulfur dioxide is impractical. Suppose the sulfur dioxide emissions are reduced to that permitted by the emissions standards of Table 5.4 for solid fuels. What is the daily quantity of sulfur dioxide

that must be removed by the slurry? Repeat parts (b) and (c) for this condition.

15. Since the demand for electrical energy is not uniform throughout a given day, a generating capacity in excess of the average power is necessary. Alternatively, storage of electrical energy is a possibility. Assume the average output is  $P_{av}$  and the daily variation about the average can be approximated by a sinusoid with a peak to peak amplitude  $\alpha$  of  $P_{av}$  and a period of one day.
  - (a) Determine the energy that would need to be stored by a lossless storage system if the power plant output is held constant at  $P_{av}$ .
  - (b) If  $P_{av} = 1$  GW, determine the storage capacity in kilowatt-hours.
  - (c) With an energy storage system, a smaller capacity power plant is required. Assume a generating facility costs \$1000/kW of capacity. What is the value of the storage system, expressed in dollars per kilowatt-hour of capacity.

## CHAPTER 6

# *Nuclear Energy*

## *Fission*

### 1. INTRODUCTION

Nuclear energy, as is well known, is an outgrowth of the atomic bomb development of World War II. Energy derived from the splitting of the atom in a controlled reaction was viewed in the late 1940s as a nearly unlimited source of power. "Responsible men spoke of atomic power so cheap it wouldn't pay to meter it" [1]. While statements of scientists and government officials tended to be more restrained, they did little to dampen these unrealistically high expectations.

As a result of the urging of Albert Einstein in 1939, the United States embarked on a nuclear development program. Under the direction of Enrico Fermi, a natural uranium controlled nuclear reaction was obtained on December 2, 1942 at the University of Chicago. Due to wartime secrecy, very few were aware of this historic occasion. Excellent descriptions of this and subsequent events by Lapp [2] and Calder [3] are available and provide fascinating reading.

In 1972, 30 years after the first chain reaction, nuclear energy provided only 1% of the total energy used in the United States. The development of nuclear power was and continues to be both slow and costly.

In 1979, 71 reactors with a combined generating capacity of 53.7 GW, produced 11.5% of the electrical energy in the United States [4]. In that year, nuclear power accounted for 3.5% of overall energy usage. As of 1980, 115 reactors with a combined generating capacity of 128 GW were either under construction or being planned. If all these facilities are completed, the nuclear generating capacity at the end of the 1980s will be 182 GW. Based upon the rate at which electricity was used in 1979, nuclear power plants could supply about 40% of the electrical demand. About

40% of the world's nuclear generating capacity is in the United States. In 1978, the world's installed capacity of about 130 GW accounted for approximately 2.8% of the world's energy consumption.

While the growth in nuclear generating capacity in the United States during the 1970s was impressive, the rate of expansion has been less than that anticipated in 1970. Projections of 1200 to 1500 GW of capacity for the year 2000 were then common [5]. This projected capacity for nuclear reactors was twice the total 1979 generating capacity of all types of facilities. By 1977, however, a capacity of 400 to 600 GW for 2000 was thought to be more probable. At the beginning of the 1980s, it is doubtful that a nuclear generating capacity of 400 to 600 GW for 2000 will be realized. Not only have construction costs of reactors been increasing at a much more rapid rate than inflation, but the acceptance of nuclear power by the public, especially in the aftermath of the March 1979 Three Mile Island accident, is not assured. Furthermore, the growth in demand for electrical energy is expected to be much less than in the past. Many of the proposed new generating facilities, nuclear reactors as well as fossil-fueled plants, may not be needed.

No longer does one hear talk of the low cost of nuclear energy for, at present, nuclear reactors for electrical energy generation are at best marginally competitive with fossil-fueled plants. Present discussion has shifted from its low cost to that of a Faustian bargain. Even the then director of the AEC Oak Ridge National Laboratory, Alvin M. Weinberg, referred in 1971 to nuclear energy in such a context [6].\* The main cost of this nearly inexhaustible source of energy is the cost of maintaining long-term social institutions that will oversee the storage of the very slowly decaying fission waste products. These products, due to their radioactivity, need to be isolated from the environment for many millenia. At present, there appears to be no means of avoiding their nearly indefinite storage.

Nuclear energy depends upon the equivalence of mass and energy predicted by Einstein's 1905 theory of relativity. Very few formulas are as well known.

$$E = mc^2$$

The energy equivalent of one kilogram of mass is thus  $9 \times 10^{16}$  J ( $c^2$ ). For a world energy usage rate of  $8.1 \times 10^{12}$  W, the conversion of one kilogram of mass to energy would be sufficient for 3 h. A daily conversion rate of only 8 kg would be equivalent to the present fossil fuel consumption.

For the United States, 2.5 kg per day would supply a total consumption rate of  $2.6 \times 10^{12}$  W, whereas .77 kilogram of matter would be sufficient to generate all electricity used. Even considering that only a small fraction (less than  $\frac{1}{1000}$ ) of fissionable material such as uranium can be converted to energy, the fuel requirement is still exceedingly small. Taking this factor into account, less than one metric ton

\* As of January 1975, the Atomic Energy Commission (AEC) was eliminated and in its place two new organizations were established. The Nuclear Regulatory Commission (NRC) was formed to replace the regulatory structure of the AEC, while the Energy Research and Development Administration (ERDA) was formed to oversee not only nuclear energy development but other energy forms as well. The functions of ERDA were taken over by the Department of Energy which was established in 1977.



of uranium is sufficient for a day's electricity generation. For a comparison, 2.4 million metric tons of coal would be required to produce the same energy.

The very minute quantity of fuel required for nuclear reactions as compared to chemical reactions undoubtedly contributed to the early overly optimistic projections. While nuclear fuels are reasonably abundant, the limitation of fission reaction is its waste products. Thus, instead of being limited by procurement, as in the case of dwindling fossil fuel reserves, nuclear energy is limited by an unprecedented disposal problem.

## 2. EXCESS MASS AND BINDING ENERGIES

Nuclear energy depends upon either splitting or combining atoms to result in products which have a mass that is slightly less than that of the original reactants. Nuclear energies can thus be predicted from changes in atomic masses or weights. By definition, the carbon-12 isotope has an atomic mass of precisely 12.00000.\* The average mass of the individual nucleons of carbon, that is, the six protons and six neutrons, is therefore one amu (atomic mass unit). A kilogram of one amu nucleons thus contains Avogadro's number,  $N_0$ , of nucleons. Based upon the carbon system, the following kilogram mass is obtained for one amu and for one  $^{12}_6\text{C}$  atom.

$$\begin{array}{ll} 1 \text{ amu} & 1/N_0 = 1/6.0225 \times 10^{26} = 1.66 \times 10^{-27} \text{ kg} \\ 12 \text{ amu} & (12)(1.6603 \times 10^{-27}) = 19.9 \times 10^{-27} \text{ kg} \end{array}$$

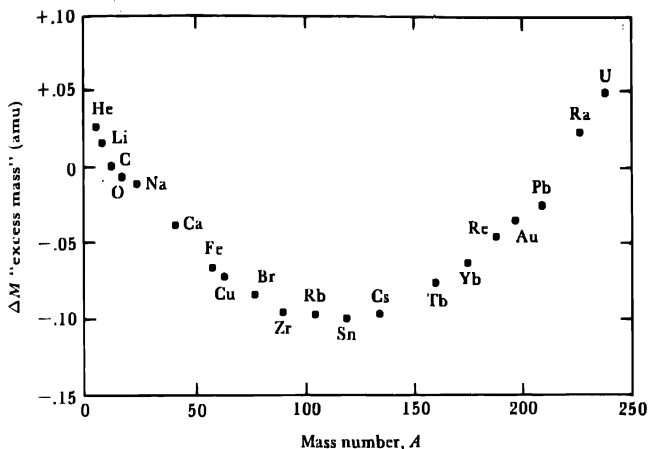
If the average mass of nucleons were the same for all atoms, the atomic mass would be an integer equal to the mass number of the atom. Even correcting for the mass of the orbital electrons ( $\frac{1}{1823}$  amu per electron), this is not the case. Both the very light and very heavy atoms have a mass in excess of that of the atoms in the central region of the periodic table. Since extremely accurate techniques are available for determining atomic masses, an excess mass which depends upon mass differences may readily be determined. For convenience, the following notation will be introduced.

$A$	mass number (number of nucleons: protons plus neutrons)
$Z$	atomic number (number of protons)
$M(A, Z)$	measured mass of a particular isotope

An excess mass quantity can be defined as being the actual mass less the "expected" value, based upon the assumption that individual nucleons have a mass of one amu, as is the case for carbon-12.

$$\Delta M = M(A, Z) - A$$

\*An older chemical system of atomic weights used oxygen, defined as having an atomic weight of 16.00. Naturally occurring oxygen, however, consists of a mixture of three isotopes,  $^{16}\text{O}$ ,  $^{17}\text{O}$ , and  $^{18}\text{O}$ . As a consequence, the atomic weight of oxygen-16 (99.759% abundance) differs from an integer weight of 16. The slight differences between the two systems can be significant when calculating energies dependent upon mass differences.



**FIGURE 6.1** "Excess Mass" Relative to Average Mass of a Carbon-12 Nucleon.

Figure 6.1. is a plot of "excess mass" in which the most abundant naturally occurring isotopes were used.

Elements with mass greater than 12 and less than approximately 220 have an average nucleon mass less than that of carbon. Both the very light and very heavy elements have greater nucleon masses. For fission, a large element such as uranium is split into two smaller atoms, while for fusion, the topic of the next chapter, light elements are combined to produce one larger atom. In both cases a decrease in mass results.

An approximate energy value for the fission of a uranium atom may be obtained from Figure 6.1. If the products have comparable atomic mass, the change of mass for the two fission products is approximately twice .14 amu, or .28 amu. The expected energy release may readily be calculated.

$$\begin{aligned}
 E &= \Delta Mc^2 \\
 &= (.28 \text{ amu}) (1.66 \times 10^{-27} \text{ kg/amu}) (3 \times 10^8)^2 \\
 &= 4.18 \times 10^{-11} \text{ J}
 \end{aligned}$$

Expressed in terms of electron volts ( $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ), a value of 260 MeV is obtained. A more accurate calculation based upon binding energies predicts an energy of approximately 200 MeV.

While the excess mass is conceptually enlightening, binding energies are more useful. For atoms other than normal hydrogen,  $^1_1\text{H}$ , the total mass of the nucleons

and orbital electrons when free or independent of each other is greater than their mass when combined to form the atom. Therefore, if atoms could be formed directly from free neutrons and protons, an energy release corresponding to the decrease in mass would occur. Conversely, a like quantity of energy is necessary to reduce an atom to its constituents.

Rather than a mass of one amu, the free neutron and proton mass numbers are slightly greater than unity [9].

$$M_n = 1.008665 \text{ amu} \quad (\text{neutron})$$

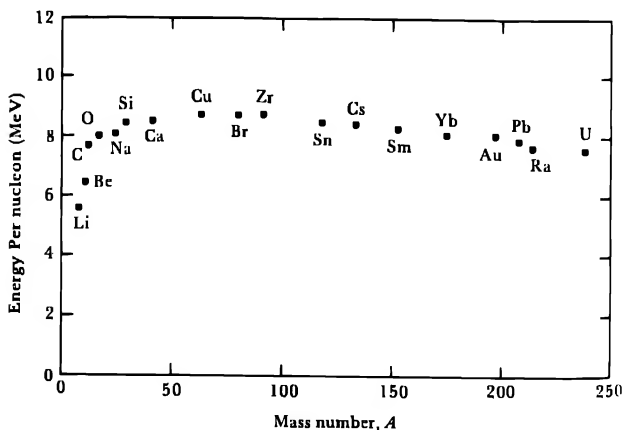
$$M_p = 1.007277 \text{ amu} \quad (\text{proton})$$

$$M_H = 1.007825 \text{ amu} \quad (\text{hydrogen atom})$$

The difference,  $M_H - M_p$ , is essentially the electron mass which is approximately  $\frac{1}{1836}$  of that of a proton. Carbon,  $^{12}_6\text{C}$ , has a nucleus of six neutrons and six protons as well as six orbital electrons. The mass of a hydrogen atom,  $M_H$ , is that of a proton and electron since the binding or ionization energy of orbital electrons is exceedingly small and can be ignored compared to nuclear binding energies. Using the above values, the mass of the free constituents may be calculated.

$$\begin{aligned} 6M_n + 6M_H &= 6(M_n + M_H) \\ &= 12.09894 \text{ amu} \end{aligned}$$

Since the mass of  $^{12}_6\text{C}$  is 12.000000, the mass "lost" is .09894 amu. Also, since one amu corresponds to an energy ( $mc^2$ ) of  $14.9 \times 10^{11} \text{ J}$  or 931 MeV, the energy associated



**FIGURE 6.2** *Average Binding Energy in Megaelectronvolts per Nucleon.*

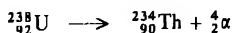
with .09894 amu is 92.1 MeV. This quantity of energy would thus be necessary to reduce  $^{12}_6\text{C}$  to free neutrons and hydrogen atoms. The average binding energy for each nucleon is therefore one-twelfth of 92.1 MeV or 7.68 MeV.

Figure 6.2 is a plot of the binding energy per nucleon for several elements. The fissionable isotope of uranium,  $^{235}_{92}\text{U}$ , has an approximate binding energy of 7.6 MeV/nucleon. If, as the result of a fission reaction, comparable size fragments result ( $A = 100$  to 125), the average nucleon binding energy of the products is 8.5 MeV. This corresponds to an energy "loss" of .9 MeV per nuclide or 212 MeV per atom. A more accurate calculation predicts a slightly smaller fission energy, approximately 200 MeV.

### 3. RADIOACTIVE DECAY

The most abundant naturally occurring isotope of uranium is that with a mass number of 238 ( $^{238}_{92}\text{U}$ ). The nucleus of this isotope contains 146 neutrons ( $238 - 92 = 146$ ) and hence has a neutron to proton ratio of 1.587. This is the largest neutron-proton ratio for naturally occurring isotopes. Figure 6.3 is a plot of the average neutron to proton ratio of the naturally occurring isotopes. Despite considerable scattering of points, a tendency for the neutron to proton ratio to increase with atomic number for numbers greater than 20 is apparent.

Uranium, as well as other elements with atomic numbers above that of lead (82), is unstable, that is, it tends to decay to a stable element with a smaller neutron to proton ratio. Lead,  $^{206}_{82}\text{Pb}$ , an end product of the  $^{238}_{92}\text{U}$  chain of decay, has 124 neutrons and thus its neutron to proton ratio is 1.512. Uranium-238 decays by emitting an alpha particle which is a combination of two neutrons and two protons. This is identical to the nucleus of the common isotope of helium,  $^4_2\text{He}$ . Upon combining with two electrons, an alpha particle is indistinguishable from a neutral helium atom. Alpha decay reduces the atomic number of the uranium atom by two (owing to the two protons of the emitted alpha particle) and the mass number by four (two protons plus two neutrons). For the decay of  $^{238}_{92}\text{U}$ , the following is thus obtained.

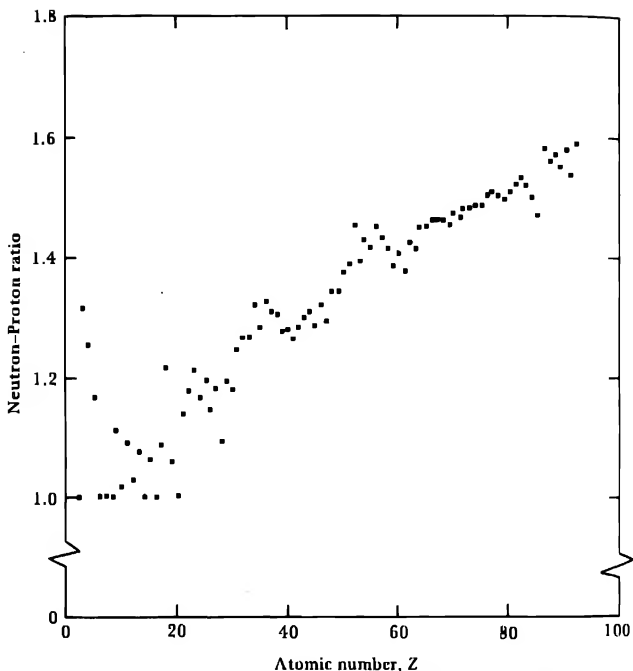


Thorium, with an atomic number of 90, is the result of this decay.

Thorium-234 is also unstable, although it decays by emitting a beta particle, that is, a nuclear electron. This emission, in effect, increases the number of protons by one. Since the quantity of nucleons remains unchanged, the number of neutrons must necessarily decrease by one. Beta decay thus increases the atomic number by one and leaves the mass number (the sum of the neutrons and protons) unchanged. Again, the neutron to proton ratio is decreased and the following is obtained for the beta decay of  $^{234}_{90}\text{Th}$ .



The minus superscript ( $\beta^-$ ) is used to distinguish this type of decay from a much



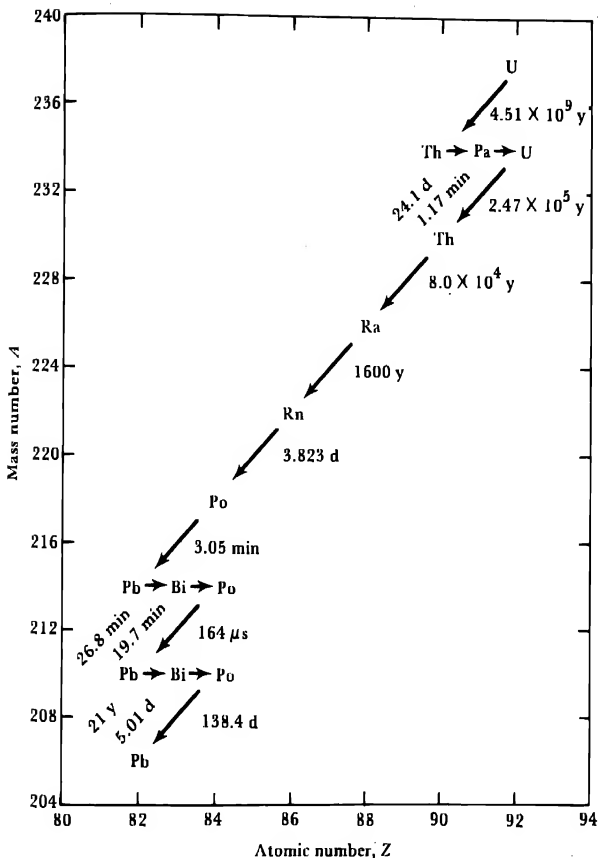
**FIGURE 6.3** *Average Neutron to Proton Ratio of the Naturally Occurring Isotopes.*

more rarely observed decay, the emission of a positron ( $\beta^+$ ). Protoactinium-234 is also unstable and decays, again by emitting a beta particle.\*



While the result of the decay of protoactinium-234 is a new isotope of uranium, the nucleon of this isotope contains four fewer neutrons than that of the initial isotope. Since this and successive isotopes are also unstable, they proceed to decay radioactively by emitting either an alpha or beta particle until a stable isotope of lead,  ${}_{82}^{206}\text{Pb}$ , is obtained. The principal decay paths of this process are shown in Figure 6.4. Naturally occurring uranium-235, and thorium-232, as well as an artificially created isotope, neptunium-237, result in unique decay chains. (See Problems 3, 6, and 7 at the end of this chapter.)

\*Since the protoactinium-234 formed as a result of the thorium-234 decay is in a nuclear excited state, a relatively rare (.15%) transition to the nonexcited state may occur at this point. This isotope also decays to uranium-234 by emitting a beta particle



**FIGURE 6.4** Radioactive Decay Chain of Uranium-238 (Half-Life Data from Reference 7).

Both alpha and beta decay can be accompanied by the emission of gamma rays. A gamma ray is a form of electromagnetic radiation similar to electrically produced x rays. A subsequent section will more fully discuss gamma radiation and its biological effects.

The rate of decay of a particular radioactive isotope is found to be proportional to the number of atoms,  $N$ , present. Using  $\lambda$  for the proportionality constant, the following is obtained for the time derivative of  $N$ .

$$\frac{dN}{dt} = -\lambda N$$

Since the decay of an atom reduces the quantity of atoms that have not yet decayed, a minus sign is necessary. Dividing by  $N$ , a slightly different form is obtained.

$$-\frac{1}{N} \frac{dN}{dt} = \lambda$$

The fraction of atoms which decay in one second (left-hand side of equation) is equal to a constant, that is, it is independent of the fraction which has already decayed. The probability of decay of an individual atom is independent of its past history as well as the decay of its neighbors.

Integrating the decay equation, an exponential decay is obtained for  $N$ .

$$N = N_0 e^{-\lambda t}$$

The constant of integration,  $N_0$ , is the quantity of atoms at the initial time of  $t$  being equal to zero. While  $\lambda$  specifies the decay rate, usually the half-life, that is, the time necessary for one-half of a collection of atoms to decay, is specified. Using  $T_{1/2}$  to specify half-life, it may be related to  $\lambda$  through the following equations.

$$N = \frac{1}{2} N_0 = N_0 e^{-\lambda T_{1/2}}$$

$$e^{-\lambda T_{1/2}} = \frac{1}{2}$$

$$e^{\lambda T_{1/2}} = 2$$

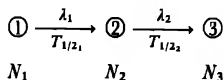
$$\lambda T_{1/2} = .693$$

$$T_{1/2} = .693/\lambda$$

Since expressions for exponential decay and growth differ by only a minus sign, it is not surprising that similar relations are obtained for half-lives and doubling times.

Included in Figure 6.4 are the half-lives of the transitions. The half-life of uranium-238,  $4.51 \times 10^9$  years, is on the order of the age of the universe. Half of the uranium-238 that may have been present  $4.51 \times 10^9$  years ago has since decayed. Other half-lives are considerably less than that of uranium-238, sufficiently less that the decay of these isotopes may be considered nearly instantaneous when measured on a  ${}^{238}_{92}\text{U}$  time scale. The decay rate of the chain is therefore primarily determined by the initial isotope of uranium since the various isotopes do not begin their decay until formed from the preceding isotopes.

Mathematically, successive decays may be treated by considering an initial isotope ① which decays to an intermediate isotope ② which in turn decays to a third isotope. The nature of the decay process (alpha or beta) is unimportant since only the half-lives determine the decay rates.



The terms  $N_1$ ,  $N_2$ ,  $N_3$  are the quantities of the respective isotopes. The decay rate of the first isotope is dependent upon only its natural decay rate.

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

This equation is identical to that already considered.

$$N_1 = N_{01}e^{-\lambda_1 t}$$

An initial value for  $N_1$  of  $N_{01}$  has been assumed.

The rate of change of  $N_2$  depends not only upon its natural decay but also upon the decay of the initial isotope, since the second isotope is the product of this decay.

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

Using the solution for  $N_1$ , the following equation results.

$$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_{01}e^{-\lambda_1 t}$$

Part of the solution for  $N_2$  is the solution of the complementary equation, that is, the equation obtained by setting the right-hand side to zero.

$$\frac{dN_2}{dt} + \lambda_2 N_2 = 0$$

$$N_2 = Ae^{-\lambda_2 t}$$

The quantity  $A$  is an arbitrary constant which will be evaluated after obtaining a complete solution by introducing an initial condition. The particular solution may be obtained by assuming a solution with the same time dependence as the driving term (right-hand side) of the original equation.

$$N_2 = Be^{-\lambda_1 t}$$

After substituting this assumed solution into the original equation and rearranging terms, a value for  $B$  is obtained.

$$-\lambda_1 Be^{-\lambda_1 t} + \lambda_2 Be^{-\lambda_1 t} = \lambda_1 N_{01}e^{-\lambda_1 t}$$

$$(-\lambda_1 + \lambda_2)B = \lambda_1 N_{01}$$

$$B = \frac{\lambda_1 N_{01}}{\lambda_2 - \lambda_1}$$

$$N_2 = \frac{\lambda_1 N_{01}}{\lambda_2 - \lambda_1} e^{-\lambda_1 t}$$

The sum of these two solutions is hence a solution of the original equation.



$$N_2 = Ae^{-\lambda_2 t} + \frac{\lambda_1 N_{01}}{\lambda_2 - \lambda_1} e^{-\lambda_1 t}$$

Specifying an initial condition for  $N_2$  determines the value of  $A$ . If the second isotope is wholly a result of the decay of the first isotope, its initial value will be zero.

$$A = -\frac{\lambda_1 N_{01}}{\lambda_2 - \lambda_1}$$

$$N_2 = \frac{\lambda_1 N_{01}}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

For successive decays in which the initial decay rate is very slow, such as that of the uranium-thorium-neptunium chain, the successive decay rates may be shown to be approximately equal to the initial rate. The decay rate of the second isotope is  $\lambda_2 N_2$ .

$$\begin{aligned}\lambda_2 N_2 &= \frac{\lambda_1 \lambda_2 N_{01}}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \\ &= \frac{\lambda_1 \lambda_2 N_{01}}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} (1 - e^{-(\lambda_2 - \lambda_1)t})\end{aligned}$$

For  $\lambda_2 \gg \lambda_1$ , that is, the half-life of the initial isotope is much greater than the second isotope, the following approximate result is obtained.

$$\lambda_2 N_2 \simeq \lambda_1 N_{01} e^{-\lambda_1 t} (1 - e^{-\lambda_2 t})$$

When  $t \gg 1/\lambda_2$  which is equivalent to  $t \gg T_{1/2}$ , the half-life of the second isotope, the decay rate obtained is indeed equal to that of the initial isotope.

$$\lambda_2 N_2 \simeq \lambda_1 N_{01} e^{-\lambda_1 t} = \lambda_1 N_1$$

Since the half-lives of the intermediate isotopes formed by the decay of uranium-238 are much less than the initial half-life of  $^{238}_{92}\text{U}$ , the successive decay rates are essentially equal to that of uranium.

An interesting element of the uranium-238 radioactive decay chain is that of radium-226. Its half-life of 1600 years is sufficiently long that the element may be chemically separated from uranium ore before appreciably decaying and sufficiently short that it still has a relatively high rate of activity. Radium was discovered in 1898 by Pierre and Marie Curie in their attempt to ascertain the nature of uranium radioactivity. A unit of radioactive decay, the curie (Ci), was originally defined in terms of the natural decay rate of one gram of radium. Since radium-226 has an atomic weight of approximately 226, 226 kg of radium contain Avogadro's number ( $N_0 = 6.02 \times 10^{26}$ ) of atoms. One gram therefore consists of  $2.66 \times 10^{21}$  atoms. Knowing the half-life of radium to be 1600 years,  $\lambda$  and the decay rate may be calculated.

$$\lambda = \frac{.693}{T_{1/2}} = 1.375 \times 10^{-11} \text{ s}^{-1}$$

$$\text{decay rate} = \lambda N$$

$$= 3.66 \times 10^{10} \text{ disintegrations per second}$$

For convenience, the curie has been redefined as  $3.7 \times 10^{10} \text{ s}^{-1}$ . The curie is thus a measure of the radioactivity of a sample. The unit does not, however, specify the type of disintegration involved or the energy of the resultant emissions. While samples of different isotopes may have equal curie values of radioactivity, their lethality to biological systems may not necessarily be comparable.

The alpha decay of radium produces radon gas. Radon-222, with a half-life of just under four days, decays to form polonium-218. Polonium and its decay products are known as radon "daughters." The successive isotopes formed prior to  $^{210}_{82}\text{Pb}$  have very short half-lives and hence decay very rapidly. Radon, being gaseous, has proven to be particularly hazardous when mining and processing uranium ore. Radon and its "daughter" products, when inhaled, can lodge on the sensitive tissues lining the lungs. The decay of an original radon atom results in a succession of decays each of which irradiates the tissue. As a consequence, uranium miners often develop lung cancer, which may manifest itself several years after exposure. With sufficient mine ventilation, however, this hazard can be significantly reduced.

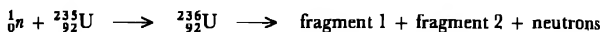
Another problem associated with radon "daughters" involves uranium mine tailings. These tailings being readily available have, unfortunately, been used as land fill for commercial and residential construction. Radon produced by the decay of the radium remaining in the tailings permeates the structures and subjects its inhabitants to the same injury as that sustained by uranium miners. The hazards associated with uranium mining and tailings are particularly well treated in a well-documented book by Metzger [8].

#### 4. FISSION CHAIN REACTION

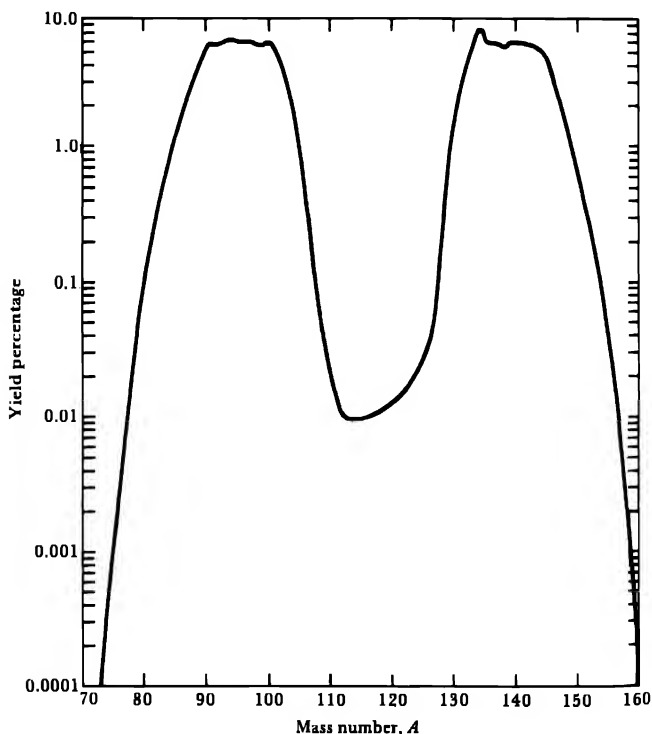
The probability of spontaneous fission by a uranium atom, that is, fission caused by no apparent external cause, is exceedingly minute. For one kilogram of uranium (235 or 238), less than one spontaneous fission per second can be expected. Even with a fission energy of 200 MeV or  $3.2 \times 10^{-11} \text{ J}$ , this is a negligible power. One fission per second is merely  $3.2 \times 10^{-11} \text{ W}$ .

Fission of uranium-235, as well as the artificially produced isotopes uranium-233 and plutonium-239, may be induced by the capture of a neutron. Fission of these isotopes can occur with neutrons of any energy. Slow or thermal neutrons, however, have a greater likelihood of being absorbed. On the other hand, uranium-238, the naturally abundant isotope of uranium, will fission only as a result of the absorption of a neutron with a kinetic energy of approximately one MeV or more. The latter event is known as a *fast fission process*, whereas fission that results from thermal or slow neutrons is known as a *thermal fission process*. Since thermal neutron fission has the highest likelihood, commercial reactors have relied on this process. The breeder reactor, the subject of a subsequent section, depends upon fast fission processes.

The capture of a neutron by a fissionable isotope such as uranium-235 results in the formation of a highly unstable isotope, uranium-236. Fission of  ${}^{236}_{92}\text{U}$  normally yields two fragments and several neutrons.



In addition to the neutrons and fragments, fission is accompanied by  $\gamma$  and  $\beta$  radiations and a neutrino emission. Most of the energy, 165 to 170 MeV of a total of approximately 200 MeV, appears as kinetic energy of the fragments. The kinetic energy of the neutrons, the radiation energy, and the subsequent decay energy of the fragments, account for the difference. The neutrino, a very difficult particle to detect, has a negligible rest mass and no electric charge. (While its existence was postulated



**FIGURE 6.5 Thermal Neutron Fission-Product Yield of Uranium-235 (Reference 10).**

by Pauli in 1927, it was not experimentally observed until 1953). More than 40 combinations of fission fragments are possible. As shown in Figure 6.5, the fragments obtained from the fission of uranium-235 have mass numbers between 72 and 160 [9, 10]. Fission fragments with mass numbers centered about 95 and 139 are the most likely (approximately 6.5% or 65 isotopes from each 1000 fission events). Since two fragments occur, the total yield will be twice the number of fission events. The probability of equally massive products, it may be noted, is very small (.01%).

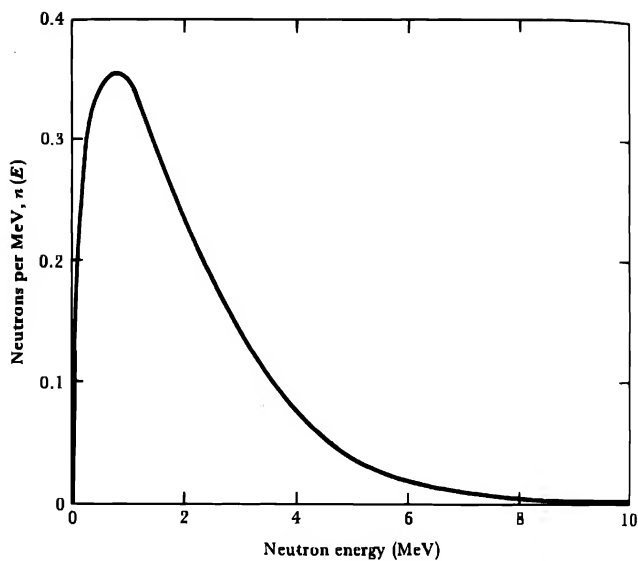
For a given fission event, the sum of the mass numbers of the fragments plus the number of neutrons must be equal to the initial mass number plus one, to account for the captured neutron. Atomic numbers must either balance or differ only by that which can be accounted for by a beta emission. On the average, approximately 2.5 neutrons are released by the thermal fission of uranium-235. Ignoring a beta emission, an average of 141.5 neutrons ( $236 - 2.5 - 92$ ) must be divided between the fission fragments. The average neutron to proton ratio for these products, 1.54, is thus considerably greater than that of the naturally occurring isotopes with mass numbers between 72 and 160. Therefore fission fragments are highly unstable and, as a consequence, radioactively decay to form stable isotopes. These products, especially those with moderately long half-lives, are the biologically hazardous reactor waste products which will be discussed more fully in Section 8.

If, on the average, at least one of the neutrons produced results in a subsequent fission, a self-sustained fission reaction can occur. Fission by thermal neutrons results in highly energetic neutrons with a measured energy density that can be approximated by the Watt equation [11].

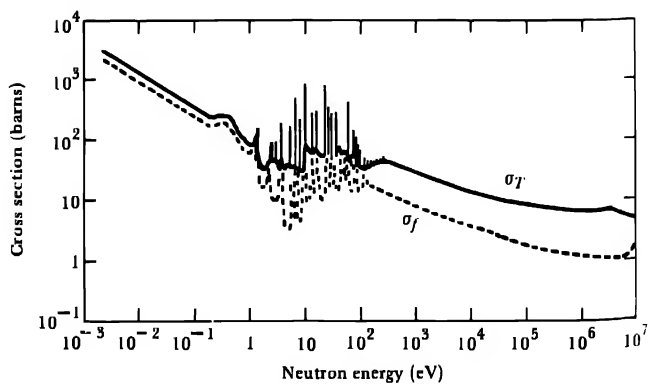
$$n(E) = \sqrt{\frac{2}{\pi e}} e^{-E} \sinh \sqrt{2E}$$

Energy,  $E$ , is expressed in megaelectron volts, and  $n(E) dE$  is the probability that a neutron will have an energy between  $E$  and  $E + dE$ . Integrating  $n(E) dE$  over all energy values yields one. A plot of  $n(E)$ , Figure 6.6, shows that the most likely energy is between .5 and 1 MeV and that neutron energies above 1 MeV are probable. The probability of neutrons with energies below one eV, the range desired for thermal fission of uranium-235, is exceedingly minute.

The probability of interaction between particles, such as a neutron and a uranium atom, is often expressed in terms of a cross section. The total cross section, as well as that for fission of uranium-235, is given in Figure 6.7 [12-14]. The probability of a neutron causing a fission event depends upon its kinetic energy and tends to increase with decreasing neutron energy. The minimum average energy of a neutron corresponds to the situation in which the neutron is in thermal equilibrium with its environment. As for a gas molecule, a thermal neutron will have on the average, a kinetic energy of  $\frac{3}{2}kT$ . For a temperature of 293 K (20°C), this corresponds to .038 eV. The microscopic fission cross section,  $\sigma_f$ , for this condition is approximately 500 b (barns), one barn being equal to  $10^{-28} \text{ m}^2$ . An interaction probability for a neutron depends not only upon the microscopic cross section of the target atom but also upon the number of atoms present (uranium-235 atoms for this case). Nuclear cross sections, it will be noted, are smaller than the atomic cross sections (discussed



**FIGURE 6.6** *Energy Distribution of Uranium-235 Fission Neutrons.*



**FIGURE 6.7** *Total ( $\sigma_T$ ) and Fission ( $\sigma_f$ ) Cross Sections for Uranium-235 (Reference 13).*

in the MHD section in the previous chapter) by several orders of magnitude. Nevertheless, the mathematical and physical significance of the cross-section concept remains unchanged.

The total cross section,  $\sigma$ , for a unit volume (one  $\text{m}^3$ ) is  $\sigma N$ , where  $N$  is the volume density of the target atoms. For a neutron flux density of  $I$  neutrons per square meter per second which has an initial value of  $I_0$ , an exponential decrease with distance results.

$$I = I_0 e^{-\sigma N x} \text{ neutrons/m}^2\text{-s}$$

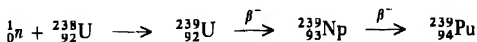
When  $x = 1/\sigma N$ , the neutron flux is reduced to a value of  $1/e$  of its value at  $x = 0$ . This distance is also the mean free path of a neutron.

$$l = 1/\sigma N$$

Based upon the mass density of uranium, its volume density,  $N$ , is approximately  $4.8 \times 10^{28}$  atoms/ $\text{m}^3$ . For an interaction cross section of 1 b ( $10^{-28} \text{ m}^2$ ), the mean free path is .208 m or 20.8 cm.

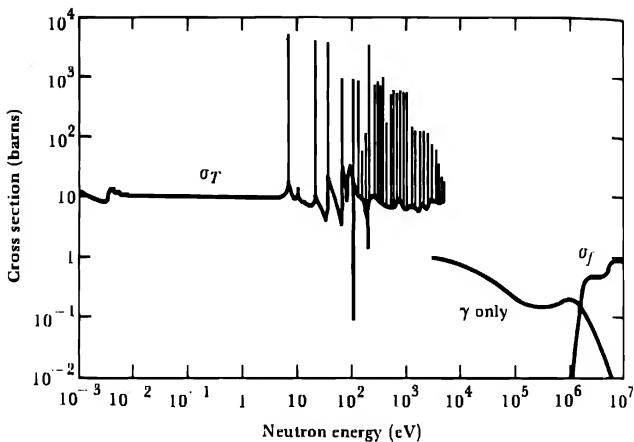
At thermal neutron energies, the average fission cross section of uranium-235 is approximately 500 b, resulting in a mean free path of only .04 cm, which represents a high probability of neutron fission. Since only .71% of naturally occurring uranium is uranium-235, only one out of each 140 atoms is  $^{235}_{92}\text{U}$ . Therefore, the fission macroscopic cross section of  $^{235}_{92}\text{U}$ ,  $\sigma N$ , is considerably reduced, and the corresponding mean free path is on the order of 5.8 cm. For thermal neutron energies, uranium-238 has a relatively small cross section of approximately 10 b. Unfortunately, exceedingly large cross-section resonances occur at higher energies (see Figure 6.8). For the high energy fission neutrons to have a high probability of being absorbed by uranium-235, they must first be slowed down or thermalized. The energy transition results in intermediate energies that may correspond to the resonance energies of uranium-238.

Capture by uranium-238 of neutrons with energies less than 1 MeV results in the formation of a new isotope of uranium which through beta decay results in plutonium-239.\*



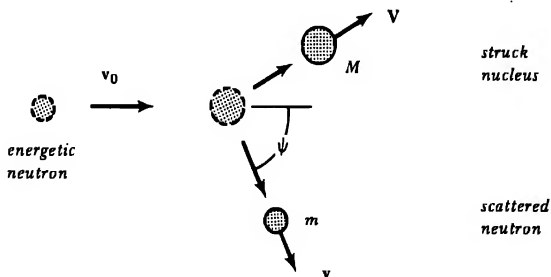
Plutonium-239, like uranium-235, is fissionable by thermal neutrons. If this is the dominant process, however, insufficient neutrons will be available to sustain a fission chain reaction. Therefore, the high energy fission neutrons of Figure 6.6 must be slowed to thermal velocities without an appreciable number being lost through capture by the uranium-238 present. While it is possible to eliminate or reduce the quantity of uranium-238 atoms present (enrichment), the separation of uranium isotopes is costly. Concentrations of uranium-235 greater than a few percent are seldom used in thermal reactors.

\* The beta decay of  $^{239}_{92}\text{U}$  has a half-life of 23.5 min and that of neptunium-239 is 2.35 days.



**FIGURE 6.8** Total ( $\sigma_T$ ) and Fission ( $\sigma_f$ ) Cross Sections of Uranium-238 (Reference 13).

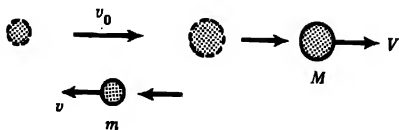
High energy neutrons are slowed down through elastic collisions primarily with moderator nuclei nominally at rest. For such a “billiard ball” type collision, both kinetic energy and momentum are conserved.



$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 \quad (\text{kinetic energy})$$

$$m\mathbf{v}_0 = m\mathbf{v} + M\mathbf{V} \quad (\text{momentum})$$

An interesting case is a head-on collision in which the neutron scattering angle,  $\psi$ , is  $\pi$  or  $180^\circ$ .



$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 \quad (\text{kinetic energy})$$

$$mv_0 = -mv + MV \quad (\text{momentum})$$

Since the velocities are along the same line as  $v_0$ , a scalar equation for momentum, with the appropriate sign for the velocity of the scattered neutron results. Eliminating the struck nucleus velocity,  $V$ , an expression relating the scattered neutron velocity to its initial velocity is obtained.

$$m(v_0^2 - v^2) = MV^2$$

$$m(v_0 - v) = MV$$

$$\frac{v}{v_0} = \frac{M/m - 1}{M/m + 1}$$

Since the mass of a neutron is approximately one amu, while that of the nucleus,  $M$ , is approximately its mass number,  $A$ , the ratio  $M/m$  is simply  $A$ . The ratio of the resultant kinetic energy,  $E$ , to that before the collision,  $E_0$ , is simply  $(v/v_0)^2$ .

$$\frac{E}{E_0} = \left( \frac{A - 1}{A + 1} \right)^2$$

With  $A = 1$ , all energy is transferred to the struck nucleus for this case (a head-on hit). The smaller the mass number, the greater the energy loss of the neutron. Scattering at an angle other than  $180^\circ$  increases the ratio  $E/E_0$  and hence decreases the neutron energy loss. It may readily be shown that  $E/E_0 = 1$  for  $\psi = 0$ , a solution corresponding to a complete "miss." A general solution involves finding the energy ratio for all angles and then averaging over all angles to obtain an average energy ratio and hence average energy loss.

Ideally, a moderator should have a low mass number to maximize the energy absorbed by each collision, and concurrently have a very small absorption cross section. As for the case of uranium-238, neutrons absorbed by the moderator reduce the quantity available to sustain fission. Typical moderators include graphite (carbon), heavy and light water, beryllium and beryllium oxide, and organic materials. For thermal neutrons, carbon has a scattering cross section of 4.8 b compared to an absorption cross section of only .0034 b. The probability of an energy reducing scattering collision is



thus 1400 times that of being absorbed. On the average, however, over 100 scattering collisions are necessary to thermalize 2 MeV neutrons. The corresponding cross sections for normal hydrogen are 38 b and .332 b, while those for deuterium ( ${}^2\text{H}$ ) are 7 b and .00046 b. Oxygen also has a very low absorption cross section of .00019 b. To thermalize 2 MeV neutrons with a water moderator, approximately 35 scattering collisions are necessary in heavy water, whereas 19 are sufficient in light water. Since the constituents of heavy water have very small absorption probabilities, heavy water is, except for its high cost, an ideal moderator. Due to the relatively small cross sections of all moderators and the large number of collisions necessary to slow megaelectronvolt neutrons to thermal velocities, large volumes of moderators are necessary. Reactors must therefore be reasonably large.

The Fermi reactor established that a fission chain reaction was possible. Since enriched or concentrated uranium-235 was not available in 1942, the Fermi reactor used natural uranium as fuel. Graphite was used as a neutron moderator. Theoretical calculations had shown that a uniform core, that is, a core with uranium distributed uniformly throughout the graphite (homogeneous core), would absorb more neutrons than would allow for a chain reaction. Szilard and Fermi proposed using small lumps of fuel, uranium oxide, interdispersed in the graphite moderator (heterogeneous core).

Fission results in fast neutrons being born in the fuel. With small lumps, neutrons have a high probability of leaving the fuel with energies greater than those corresponding to the resonance cross sections. In the graphite moderator, the neutrons are thermalized. With a large ratio of moderator to fuel, very few neutrons re-enter the fuel before being thermalized. When thermal neutrons do finally enter the fuel, the large fission cross section of uranium-235 results in a high probability of fissioning. In order to regulate the reaction, cadmium control rods were inserted into the fuel-graphite pile. Cadmium has a particularly high neutron cross section, 2450 b for thermal neutrons. Only by removing the rods from the pile was a chain reaction possible. Inserting the rods had the effect of shutting off the reaction. The term *atomic pile*, often misapplied to all reactors, stems from the pile-like structure of the Fermi reactor.

## 5. THERMAL NEUTRON REACTORS

Numerous texts that treat both the nuclear theory necessary for the understanding of reactors [15-19] and the theoretical and constructional aspects of reactors [20-29] are available. In discussing reactors, a neutron multiplication factor,  $k$ , which is the average number of fission events that may be expected for each initial fission, is important. The corresponding factor for an infinite size reactor,  $k_{\infty}$ , is often expressed as the product of four probabilities ("four-factor reactor equation"). When neutron leakage for a finite size reactor is included, a modified equation results.

For a self-sustained reaction, the multiplication factor must necessarily be at least one, that is, on the average, at least one neutron produced by each fission must result in a subsequent fission. Highly energetic neutrons, neutrons with energies in excess

of 1 MeV such as produced by fission, can result in the fission of uranium-238. If  $\nu$  is the average number of neutrons per thermally induced fission, the fast fission factor is defined such that  $\nu\varepsilon$  is the ultimate quantity of neutrons resulting from the original event. If fast neutron fission does not occur,  $\varepsilon$  is equal to one. Normally  $\varepsilon$  is not much greater than one due to the relatively small fast fission cross section of uranium-238 (Figure 6.8). While being slowed down or thermalized, some neutrons are lost, primarily due to the large resonance cross section of uranium-238. The probability of escaping capture,  $p$ , is a quantity less than one. If losses from the core are ignored,  $\nu\varepsilon p$  thermal neutrons result, on the average, for each thermal neutron induced fission.

Not all thermal neutrons result in a fission reaction, since some neutrons may be captured by the moderator and other materials of the reactor. The ratio of thermal neutrons captured by the fuel to those captured by fuel as well as all other effects is  $f$ . Finally, not all neutrons captured by the fuel result in fission. This loss may be accounted for in terms of the thermal cross section for fission,  $\sigma_f$ , and the total absorption cross section  $\sigma_a$ . The ratio of  $\sigma_f/\sigma_a$  is therefore the probability of a captured neutron causing fission. Ignoring core leakage, a multiplication factor for an infinite size core is obtained.

$$k_{\infty} = \nu\varepsilon pf(\sigma_f/\sigma_a)$$

where  $k_{\infty}$  = multiplication factor for an infinite size core

$\nu$  = number of neutrons per  $^{235}_{92}\text{U}$  fission

$\varepsilon$  = fast fission factor of  $^{238}_{92}\text{U}$

$p$  = resonance escape probability

$f$  = thermal neutron utilization factor

$\sigma_f$  = cross section for fission

$\sigma_a$  = total cross section for absorption

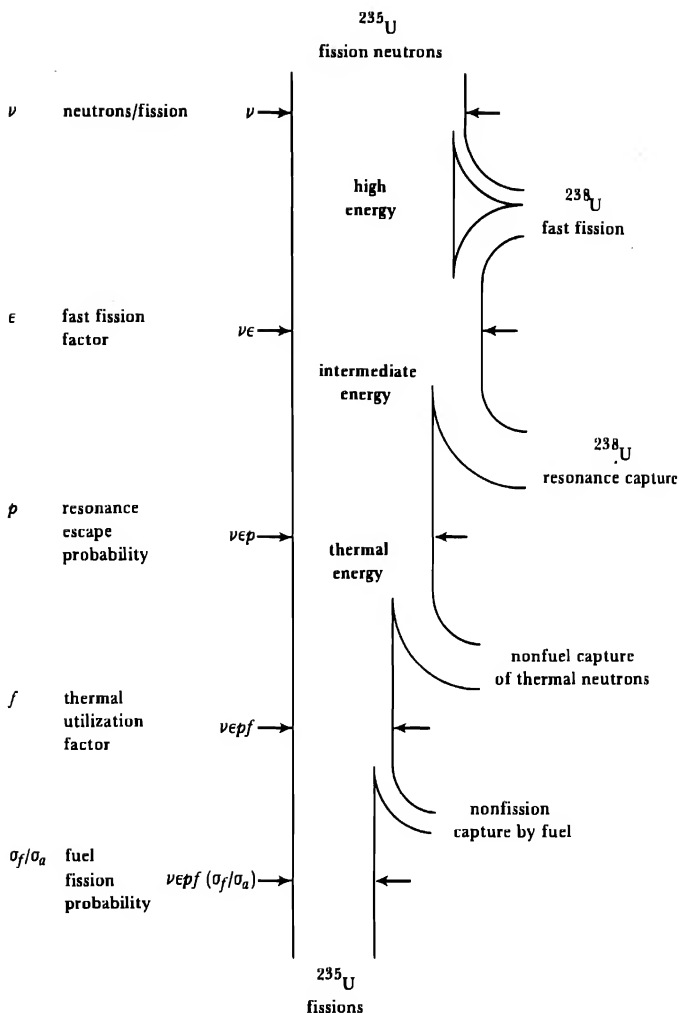
Neutron utilization is depicted in Figure 6.9. Since  $\sigma_a$  in the thermal region is very large for uranium-235, it is essentially equal to the total cross section which includes nonabsorptive elastic collisions, that is, it is essentially equal to  $\sigma_T$  of Figure 6.7.

The ratio of fast neutrons generated by each thermal neutron captured by the fuel,  $\nu(\sigma_f/\sigma_a)$ , is usually designated by  $\eta$ , the thermal fission multiplication factor.

$$k_{\infty} = \eta\varepsilon pf$$

A finite reactor results in neutron leakage losses. To reduce losses, the reactor core is surrounded by a neutron reflector made of moderator type materials which, through scattering of neutrons, tend to return a portion of neutrons that would otherwise have been lost. If  $l_f$  is the leakage probability of fast neutrons,  $1 - l_f$  is the probability of a neutron not being lost. Likewise, if  $l_t$  is the leakage probability of thermal neutrons,  $1 - l_t$  is the nonleakage probability. To obtain the multiplication factor for a finite size core, the nonleakage probabilities must be included.

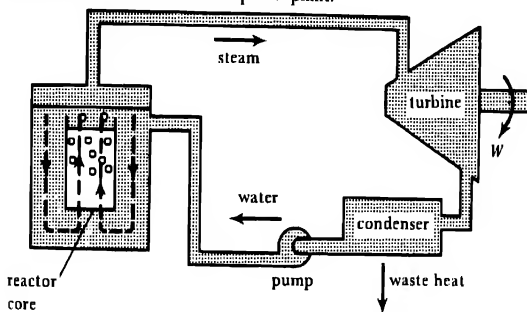
$$k = \eta\varepsilon pf(1 - l_f)(1 - l_t) = k_{\infty}(1 - l_f)(1 - l_t)$$



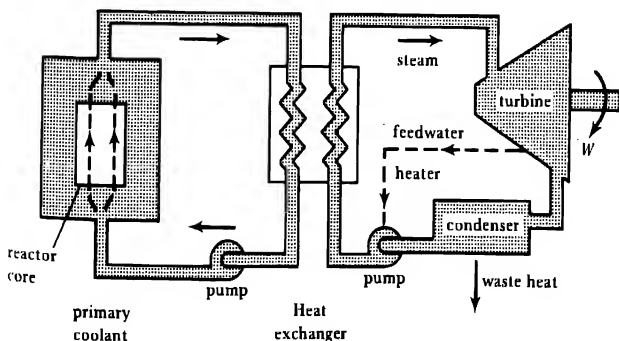
**FIGURE 6.9** Symbolic Representation of Average Neutron Utilization for an Infinite Reactor.

In order to minimize leakage, relatively large reactor cores are normally required. While this is ideal for electric power generation, it can limit the utility of nuclear power for other applications.

Due to the very low power level of the original Fermi reactor, cooling of the core was unnecessary. A power reactor, on the other hand, is designed to produce large quantities of heat in order to generate steam either directly or indirectly, which in turn is used to drive a turbine. Water is the coolant for both the boiling water reactor, BWR, and the pressurized water reactor, PWR. In addition to serving as a heat transfer fluid, the water also functions as a neutron moderator. Figure 6.10 is a schematic representation for both of these reactors. The core, either alone (BWR) or in conjunction with a heat exchanger (PWR), replaces the boiler of a conventional fossil-fueled electric power plant.



Boiling water reactor (BWR)



Pressurized water reactor (PWR)

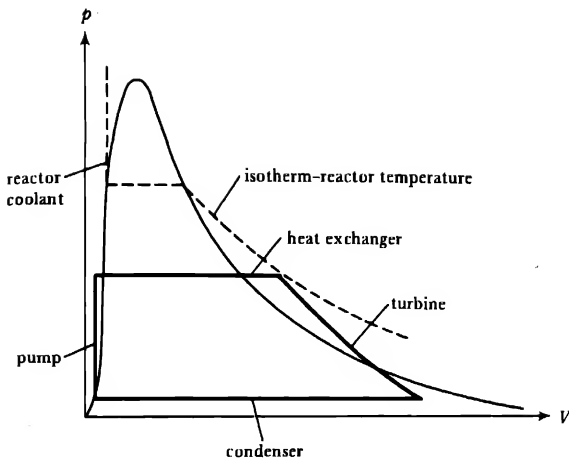
**FIGURE 6.10** Water Cooled and Moderated Reactors.

The fuel is usually enriched uranium, that is, uranium with a  $^{235}_{92}\text{U}$  concentration greater than that of naturally occurring uranium (.71%).\* Enrichment of a few percent is typical. Fuel rods with diameters less than an inch are also common, although other fuel element shapes such as slabs may be used. A protective cladding which not only insures the integrity of the elements but which confines the fission fragments or wastes is also used. Since low neutron absorption by the cladding is necessary, alloys of beryllium and zirconium, as well as stainless steel are used. Interspersed with the usually vertically suspended fuel elements are movable high neutron-absorbing control rods made of alloys of cadmium or boron. Since the cooling water also functions as a neutron moderator, a loss of coolant is also a loss of moderator. Without sufficient moderator, the reactor will no longer be critical and will thus tend, except for the residual heat generated by the decay of the accumulated waste products, to shut itself off. A water-cooled and moderated reactor thus has a built-in safety control.

The upper temperature of both the boiling water reactor and the pressurized water reactor is limited by the temperature corresponding to the critical point for water. The highest temperature that water can exist in a liquid state is  $374^{\circ}\text{C}$  ( $705^{\circ}\text{F}$ ) (Figure 4.20). Owing to pressure considerations, coolant temperatures are usually on the order of only  $250$  to  $320^{\circ}\text{C}$  (approximately  $500$  to  $600^{\circ}\text{F}$ ). Low reactor temperatures necessarily limit the thermal efficiency of nuclear-electric power plants. Efficiencies for water-cooled reactors are usually 30% or less. As 70% of the initial heat appears as waste heat extracted by the condenser,  $2\frac{1}{3}$  units of waste result for each unit of useful output (70%/30%). Since a modern high-efficiency fossil-fueled plant will have an efficiency on the order of 40%, only 1.5 units of waste heat for each unit of output will be generated. When stack losses (10%) are considered, a 30% efficient nuclear plant requires nearly twice the condenser cooling of a modern nonnuclear plant.

For the boiling water reactor, no heat exchanger is necessary, as the reactor coolant is used to drive the turbine. The boiling water of the core can result in voids (steam) in the coolant around excessively hot fuel elements. Since voids reduce the neutron moderation and hence the localized fission rate, a degree of automatic fuel temperature control is achieved. An inherent danger of boiling water reactors is the danger of depositing radioactive wastes in the turbine, should a fuel rod rupture. In addition, due to neutron activation, the cooling water becomes mildly radioactive. Both  $^{16}\text{O}$ , the abundant isotope of oxygen, and  $^{17}\text{O}$ , .037% abundant, absorb neutrons and form  $^{17}\text{N}$  and  $^{18}\text{N}$ , respectively. Both nitrogen isotopes radioactively decay back to oxygen by a beta emission. Fortunately the half-lives for both isotopes are only a few seconds. Within a few hours after shutdown, the radioactivity will have ceased. As a further precaution, a continuously operating purification system is normally employed to remove radioactive wastes that might otherwise collect in the water.

\* Fuels produced by breeder reactors, uranium-233 and plutonium-239, may, if sufficient quantities are available, be used.



**FIGURE 6.11** *Pressure-Volume Diagram for Pressurized Water Reactor (Scales Distorted).*

Boiling does not occur in the pressurized water reactor under normal operating conditions. Higher pressures, however, necessitate heavier vessels. The heat exchanger or steam generator adds to the cost and reduces the thermodynamic efficiency. On the positive side, however, the reactor coolant is isolated from the turbine. Superheated steam, as shown by Figure 6.11, can be produced by the heat exchanger. Feedwater heating, as used in conventional power plants, can be used to increase the thermodynamic efficiency of the turbine cycle.

If heavy water,  $D_2O$ , is used, a pressurized water reactor using natural, non-concentrated uranium fuel is possible. Deuterium has an exceedingly low thermal neutron absorption cross section, .00046 b as compared to .322 b for normal hydrogen. While heavy water is expensive, uranium enrichment, also an expensive process, is avoided.

For both types of reactors, only a small portion of the initial fuel charge can be consumed before reprocessing is necessary. Fission reduces the fuel available which in turn reduces the macroscopic fission cross section,  $N\sigma_f$ . Furthermore, the waste products tend to result in "fuel poisoning." Many fission fragments are neutron absorbers and, hence, reduce the neutrons available for the fission chain reaction. Two fission products,  $^{135}Xe$  and  $^{149}Sm$ , have particularly large thermal neutron absorption cross sections,  $2.71 \times 10^6$  b and  $4.08 \times 10^4$  b, respectively.

Before proceeding to other types of reactors, uranium fuel requirements and reserves will be determined. Each fission event produces approximately 200 MeV

**TABLE 6.1 Uranium-Coal Comparison (one thermal kilowatt-hour)**

	Uranium $1.12 \times 10^{17}$ fissions	Coal	Coal-Uranium ratio
$^{235}\text{U}$	$4.4 \times 10^{-5}$ g		
U-natural	$6.2 \times 10^{-3}$ g		
$\text{U}_3\text{O}_8$	$7.3 \times 10^{-3}$ g		
	.048¢ <sup>a</sup>	.4¢ <sup>b</sup>	6.25
Ore	2.9 g <sup>c</sup>	120 g	41

<sup>a</sup>  $\text{U}_3\text{O}_8$  at \$30/lb

<sup>b</sup> \$30/ton

<sup>c</sup> 5 lb/ton

or  $3.2 \times 10^{-11}$  J of energy. Therefore,  $3.12 \times 10^{10}$  fissions are required to produce one joule or watt-second of energy. One kilowatt-hour of thermal energy thus requires the fission of  $1.12 \times 10^{17}$  uranium-235 atoms. Since 235 kg of  $^{235}\text{U}$  contain Avogadro's number ( $6.02 \times 10^{26}$ ) of atoms,  $4.4 \times 10^{-5}$  g are "burned" (see Table 6.1). To produce the same energy from coal, approximately 120 g (based upon 28 million Btu/metric ton) are required. On a mass basis, the energy content of uranium-235 is 2.7 million times that of coal. Naturally occurring uranium consists of three isotopes:  $^{238}\text{U}$ ,  $^{235}\text{U}$ , and  $^{234}\text{U}$ . The abundance of the fissionable isotope is only .71% ( $^{234}\text{U}$  is a mere .006%). Therefore, 140 times as much naturally occurring uranium is required. In considering the oxide of uranium that is extracted from the ore,  $7.28 \times 10^{-3}$  g of  $\text{U}_3\text{O}_8$  are required for one thermal kilowatt-hour. Costs of  $\text{U}_3\text{O}_8$  (1979) were less than \$30/lb. At \$30/lb,  $7.28 \times 10^{-3}$  g represents a cost of .048¢, which is about one-eighth the cost of coal. Uranium oxide averages about 5 lb/ton of presently utilized "black oxide" ores. Therefore, 2.9 g of ore are required. Based upon ore requirements and equal efficiencies, uranium ore is 41 times as effective as coal. The impact of extracting uranium, for a specified energy, is then only about one-fortieth that of coal.\* In addition, since only a few pounds of each ton of uranium ore are used, nearly a ton of crushed tailings are left at or near the mill site for each few pounds of uranium oxide extracted.

The cost of uranium oxide depends upon the ore concentration from which it is obtained. Uranium reserves therefore depend upon the price one is willing to pay. The data of Table 6.2 reflects the 1979 price-reserve relationship for uranium oxide [4]. At \$30/lb,  $\text{U}_3\text{O}_8$  reserves of 690,000 tons (627,000 metric tons) are anticipated. In terms of energy, this is equivalent to a thermal energy of  $8.6 \times 10^{13}$  kWh or  $9.8 \times 10^9$  kW, which, for a generating efficiency of 30%, is  $2.9 \times 10^9$  kW of electrical energy. In 1979, the average electric power generated by nuclear

\* If coal is utilized at a 40% thermal efficiency, compared to a 30% thermal efficiency for water-cooled reactors, the ratio of coal to uranium ore is approximately 31. The impact of extracting uranium is thus only about one-thirtieth that of coal for a specified output energy. Not all the fissionable uranium isotope,  $^{235}\text{U}$ , is normally utilized. This loss, however, tends to be offset by the gain associated with the breeding of plutonium and its subsequent fission in a reactor.

**TABLE 6.2 Uranium Resources<sup>a</sup> (thousand short tons,  $U_3O_8$ )**

Class	Forward cost category <sup>b</sup>	
	\$30 or less	\$50 or less
Reserves	690	920
Potential resources	1980	3225
Probable	1005	1505
Possible	675	1170
Speculative	300	550
Total	2670	4145

<sup>a</sup>January 1, 1979 (Reference 4).

<sup>b</sup>Forward costs are those costs yet to be expended and do not include sunk costs, taxes, profit, or amortization of existing capital equipment, and therefore they do not represent prices at which  $U_3O_8$  will be marketed.

reactors was 31 GW. The uranium oxide reserve would, for this power level, last about 94 years. For the installed generating capacity of 182 GW that is anticipated by the end of the 1980s, the reserve would last only about 24 years (assuming a 2/3 load factor for the reactors). At a cost of \$50/lb, reserves increase by about one-third and if the potential resources of Table 6.2 should eventually be recovered, a 500% increase occurs.

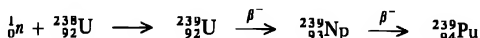
Lieberman, however, through an analysis similar to that discussed in Chapter 2 for estimating crude oil reserves, has arrived at a much lower estimate of the quantity of uranium oxide that may eventually be extracted in the United States [30]. Uranium oxide is discovered, as is petroleum, through exploratory drilling in geological formations that are likely to contain the resource. Discoveries per foot of drilling have shown an exponential decline similar to that of Figure 2.16 for crude oil. Furthermore, it was found that cumulative discoveries and production of uranium oxide could accurately be approximated with logistic functions. A value of 630,000 tons was obtained by Lieberman for  $Q_\infty$ , the ultimate recoverable quantity of uranium oxide, based upon a 1976 forward cost of \$8/lb or less (comparable to a 1979 cost of \$30/lb). Since cumulative production, as of 1976, was 270,000 tons, according to this estimate only 360,000 tons remained to be produced and, since 273,000 tons had already been discovered, only 87,000 tons remained to be discovered. In addition, a quadrupling in price did not quite double  $Q_\infty$  for domestic reserves of uranium oxide. If the estimates of this study should prevail, very little of the  $U_3O_8$  of the "potential resource" category of Table 6.2 will be recovered.

The cost of  $U_3O_8$ , it should be noted, has only a small effect on the overall cost of nuclear-produced electrical energy. For a \$30/lb cost and a generating efficiency of 30%, the fuel cost is only .16¢/kWh of electrical energy. Even for a cost of \$100/lb, the fuel cost is only .53¢/kWh. Although not insignificant, these fuel costs are only a small portion of overall electric generation and distribution costs.



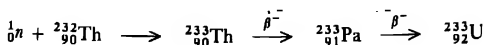
## 6. FAST NEUTRON BREEDER REACTORS

As pointed out in Section 4, the capture of neutrons by uranium-238 in thermal neutron reactors results in a new fissionable isotope, plutonium-239.



As uranium-235, the only naturally occurring fissionable isotope, plutonium-239 may be used as a reactor fuel. All thermal reactors using either natural or enriched uranium produce small quantities of plutonium-239 which, when the fuel elements are reprocessed, can be chemically separated from the remaining mixture of uranium isotopes and fission wastes. While plutonium-239 is suitable for a reactor fuel, it is only through the use of reactor-produced neutrons that substantial quantities may readily be obtained.

Uranium-238 is known as a fertile isotope since, with the absorption of a neutron, a fissionable or fissile isotope is obtained. Naturally occurring thorium-232 may also be converted to a fissile isotope of uranium ( ${}_{92}^{233}\text{U}$ ).



The half-life of protoactinium-233 is 27 days. Since naturally occurring thorium is predominantly thorium-232, thorium represents a potential nuclear fuel. In a breeder reactor, fission neutrons are used not only to sustain the chain reaction but also to convert fertile isotopes to fissionable fuels. High neutron utilization efficiencies are thus necessary to produce reasonable quantities of fuels. A large U.S. development effort is being expended to produce breeder reactors which will eventually consume both  ${}_{92}^{235}\text{U}$  and the abundant isotope  ${}_{92}^{238}\text{U}$ . This will, in effect, increase the energy content of naturally occurring uranium by a factor of 140. In terms of energy reserves, those represented by uranium would increase 140-fold. Utilizing uranium-238, oxide costs as high as \$100/lb would represent a negligible portion of final electricity costs. The impetus for the development of large breeder reactors that will generate more fuel than they consume stems from a concern with eventual ore costs.

Not only is ore cost an important factor, but the enrichment of uranium is both a difficult and expensive process. Since  ${}_{92}^{235}\text{U}$  and  ${}_{92}^{238}\text{U}$  are chemically identical, separation must be based upon the very small mass difference, an atom of  ${}_{92}^{238}\text{U}$  being only 1.3% more massive than that of  ${}_{92}^{235}\text{U}$ . The most common means of separating uranium isotopes is by gaseous diffusion of uranium fluoride at low pressures. Since fluorine has only one naturally occurring isotope (mass number of 19), the mass difference of molecules of  $\text{UF}_6$  is due entirely to the mass numbers of the uranium isotopes. The mass ratio of  ${}^{238}\text{UF}_6$  to  ${}^{235}\text{UF}_6$  may readily be determined.

$$\frac{m_{238}}{m_{235}} = \frac{238 + 6(19)}{235 + 6(19)} = \frac{352}{349} = 1.0086$$

The average kinetic energy of a molecule depends only upon temperature and is equal to  $\frac{3}{2}kT$  (three degrees of freedom).

$$\frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}kT$$

Since for equal temperatures,  $\langle v^2 \rangle$  is inversely proportional to mass, the average speed,  $v$ , is inversely proportional to the square root of the mass.

$$v_{235}/v_{238} = \sqrt{m_{238}/m_{235}} = \sqrt{1.0086} = 1.0043$$

Molecules of the lighter isotope will thus tend to be moving slightly faster than those of the heavier isotope (on the average, .43% faster). Therefore, the lighter molecules will tend to strike a specified wall area at a slightly greater rate per unit of concentration than the heavier molecules. As a consequence, uranium-235 molecules tend to diffuse through a porous barrier at a slightly greater rate relative to their concentration than those of uranium-238. Diffusion through a porous barrier thus results in a slight enrichment of the uranium. By repeating this process a sufficient number of times (usually hundreds), any desired degree of enrichment may be achieved. Enrichment of uranium is expensive due to the large capital investment necessary for the construction of diffusion plants. Present U.S. enrichment costs, however, do not completely reflect actual costs since construction and developmental costs have been borne primarily by military expenditures for nuclear weapons.

The enrichment of uranium by gaseous diffusion is also very energy intensive. Approximately 5% of the electrical energy produced by a lightwater reactor is, in effect, required for the enrichment of its fuel [32]. Other enrichment processes are being studied and developed [32-36]. The gas centrifuge, a fairly well developed separation process, also uses gaseous uranium fluoride. In a centrifuge, separation is effected by the slight difference in the mass-dependent centrifugal forces for the uranium isotopes. Even though very high rotational speeds are required and many centrifugal stages must be utilized, the energy required is only about 4% of that required for the gaseous diffusion process. Another separation process relies on the selective excitation and subsequent ionization of uranium isotopes by laser beams. The ionized atoms can readily be separated from the nonionized ones by an electric field. While large-scale commercialization of this process is not expected before 1990, when fully developed it is expected to be considerably cheaper than either gaseous diffusion or centrifuges. Since highly concentrated uranium-235, suitable for weapons, can readily be produced by the laser enrichment process, its development could result in further proliferation of nuclear weapons.

Efficient utilization of neutrons is necessary if a reactor is to produce at least as much fuel as it consumes. On the average, each fission event must result in one neutron that is absorbed by a fertile isotope as well as one that maintains the chain reaction. The process of moderating the high energy fission neutrons in a thermal reactor requires numerous moderating collisions. The likelihood of an absorbing collision, either by the moderator or the structural materials of the reactor, increases as the lifetime of a neutron (the time between the fission birth and the subsequent fission it induces) increases. Fast neutron breeder reactors minimize the neutron's

lifetime by using high energy fission neutrons with little or no moderation to sustain the chain reaction. An increase in  $\eta$ , the neutron multiplication factor, is an added benefit of fast neutron fission.

Recall that  $\eta$  depends both upon  $\nu$ , the number of neutrons produced by each fission event, and upon the ratio of fission and absorption cross-section areas,  $\sigma_f/\sigma_a$ .

$$\eta = \nu(\sigma_f/\sigma_a)$$

The prime advantage of thermal reactors is that the fission cross section is very large (500 to 600 b) for thermal neutrons. For fast neutrons, that is neutrons with energies on the order of one MeV,  $\sigma_f$  is only one to two barns. The absorption cross section,  $\sigma_a$ , is likewise much smaller for fast neutrons. Of importance, however, is that the ratio of cross sections,  $\sigma_f/\sigma_a$  (necessarily less than one), is greater for fast neutrons than for thermal neutrons. Concurrently,  $\nu$  is also slightly greater for fast neutrons. As may be seen from the data of Table 6.3,  $\eta$ , especially for  $^{239}\text{Pu}$ , is significantly larger for fast neutrons than for thermal neutrons.

**TABLE 6.3 Comparison of Fast and Thermal Constants**

	Thermal			Fast		
	$^{233}\text{U}$	$^{235}\text{U}$	$^{239}\text{Pu}$	$^{233}\text{U}$	$^{235}\text{U}$	$^{239}\text{Pu}$
$\sigma_f$ barns	500–600	500–600	700–800	2.2	1.4	1.8
$\eta$	2.3	2.1	2.1	2.4	2.2	2.6
$C_{\text{max}}$	1.3	1.1	1.1	1.4	1.2	1.6

Since one neutron, on the average, is necessary to sustain the fission reaction, a maximum of  $\eta - 1$  neutrons, assuming no losses, is available for breeding. The conversion ratio,  $C$ , is defined as the number of fissionable isotopes produced by each neutron captured by the fuel.

$$C = \eta - 1 - L$$

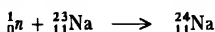
$$C_{\text{max}} = \eta - 1$$

The maximum possible conversion is obtained when  $L$ , the loss, is zero. For a conversion ratio of one, a reactor generates as much fuel as it consumes. Reprocessing involves an input of only fertile material, that is,  $^{238}\text{U}$  or natural uranium, to maintain a uranium-plutonium cycle. If  $C$  is greater than one, a net gain of fuel during operation of the reactor is achieved. Rates of gain which will result in a doubling of the reactor fuel in a period of 10 to 20 years are envisioned for advanced breeder reactors. Since plutonium-239 has the largest value of  $\eta$  and  $C_{\text{max}}$ , it is the ideal fuel for a breeder reactor.

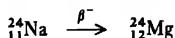
For little or no moderation of the neutrons, water cooling of the reactor core

is excluded. Also, as a result of the small microscopic fast neutron cross sections, high fuel concentrations are necessary to achieve reasonable macroscopic cross sections ( $N\sigma$ ). Fuel inventories for fast neutron reactors are hence considerably greater than for thermal reactors. Cooling of breeder reactors is by gas or liquid metals. Cooling by gases requires very high flow rates owing to their low specific heat. Nevertheless, reactors using helium, which is chemically inert and only mildly affected by high neutron flux densities, are being built and tested. The main U.S. breeder program is focused on the liquid-metal-cooled fast breeder reactor, LMFBR. Both sodium and potassium have very small moderating ratios and small absorption cross sections. Sodium, which has a reasonably low melting point of  $98^{\circ}\text{C}$  and a very high boiling point of  $892^{\circ}\text{C}$ , has received the most attention. Unlike water, low coolant pressures are possible with sodium since reactor temperatures are considerably less than its boiling point. Sodium, unfortunately, is extremely corrosive and becomes highly radioactive when exposed to high neutron flux densities.

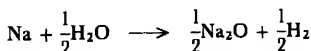
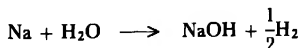
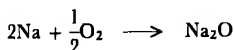
The naturally occurring isotope of sodium with a mass number of 23 absorbs a neutron to produce a radioactive isotope, sodium-24.



Sodium-24 decays by a  $\beta$  emission with a half-life of 15 hours.

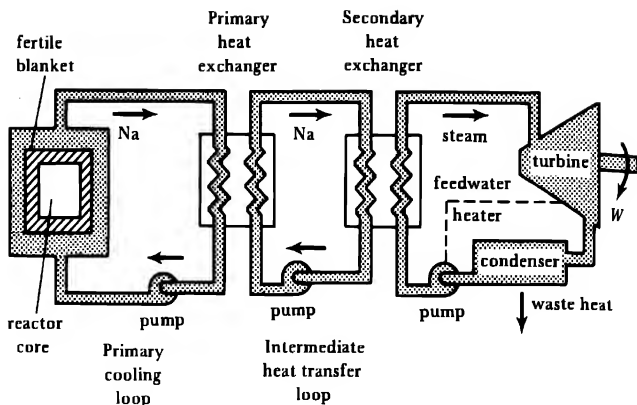


Owing to its short half-life, sodium-24 is highly radioactive. Even though the neutron cross section for sodium-23 is very small (on the order of .001 b for .25 MeV neutrons), the sodium is recirculated through the core and hence is repeatedly exposed to the high neutron flux densities. Sodium also reacts violently with both air (oxygen) and water.



These reactions are all highly exothermic. The resultant hydrogen of the sodium-water reaction will react with any oxygen present or with the sodium, producing water or NaH, respectively. Both of these reactions release additional energy. Within the reactor, an inert argon blanket is used to cover the sodium. Owing to both the induced radioactivity of sodium and its reaction with water, extremely reliable heat exchangers are necessary.

Figure 6.12 is a schematic diagram of a liquid-metal-cooled fast breeder reactor. An intermediate nonradioactive sodium heat transfer loop is used to isolate the radioactive sodium of the reactor core from the steam turbine portion of the plant.



**FIGURE 6.12** *Schematic Diagram of an LMFBR Power Plant.*

Reactor coolant temperatures of  $600$  to  $650^{\circ}\text{C}$  ( $\sim 1100$  to  $1200^{\circ}\text{F}$ ) are considerably higher than the temperatures possible for water-cooled reactors, but they are still much less than the boiling temperature of sodium. These reactor coolant temperatures result in steam temperatures on the order of  $540^{\circ}\text{C}$  ( $1000^{\circ}\text{F}$ ). Despite the unavoidable temperature drop of the intermediate heat transfer loop, thermal efficiencies comparable to fossil-fuel plants may be achieved. Surrounding the reactor core is a blanket of fertile material, which, for breeding plutonium-239, is natural uranium. Fast neutron fission of uranium-238 and plutonium-239 of the blanket results in a small contribution to the total power of the reactor.

Other forms of breeder reactors are also being built or proposed, which include gas-cooled reactors using helium and steam. Also being considered are fluid-fueled reactors in which a liquid fuel serves as the coolant. Basically, the reactor is simply a "bulge" in the fuel loop that is large enough to result in a unity value for the neutron multiplication factor. Surrounding the reactor bulge will be a blanket of fertile material in which excess neutrons are used for breeding. Also proposed for thorium breeding is a molten salt reactor which utilizes thermal neutrons for the fission process. In addition to breeder reactors, converters which produce fuel at a rate less than the rate at which it is being consumed are being built. Reasonably high conversion ratios are possible with efficiently designed thermal reactors. Uranium-233 obtained from the fertile isotope of thorium has a particularly high value of  $\eta$  for thermal neutrons. Numerous and varied reactor schemes are thus possible. Long-term operation of these novel reactors will, however, be necessary to ascertain their practicality.

## 7. RADIATION

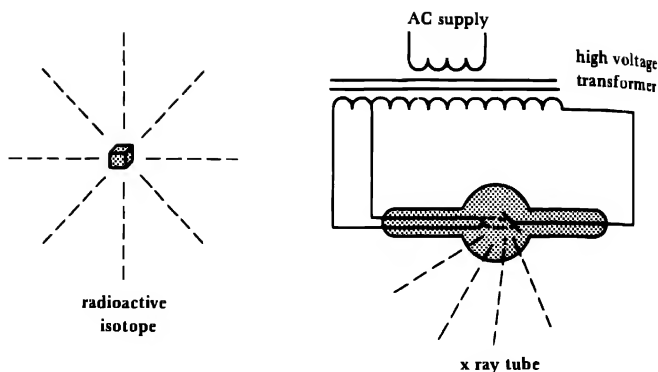
Nuclear reactors are a source of large quantities of potentially dangerous radioactive isotopes. Unavoidable are fission waste products, which are unstable due to their excessive neutron to proton ratios. Within the core, neutron absorption by structural materials results in induced radioactivity. Furthermore, plutonium-239, which may be present either as a fuel or as a result of breeding, is also a potential hazard.

The unit of radioactivity, the curie, which is the approximate decay rate of one gram of radium ( $3.7 \times 10^{10}$  dis/sec), has been introduced. The curie does not, however, distinguish between the type of decay or the decay energy. Biological damage depends upon both type and energy. The discussion of the decay of uranium-238 and its intermediate decay products of radium and radon included a treatment of beta and alpha decay. This emission may or may not be accomplished by the emission of a gamma ray, that is, a high energy photon. Less likely emissions include that of a positron ( $\beta^+$ ), a positively charged electron. A positron, having an extremely short life (order of microseconds), combines with an electron and the two particles are annihilated. The emission of two gamma-ray photons with energies equal to the mass equivalent of the particles (approximately .51 MeV) accompanies annihilation. Beta and alpha emissions often result in atoms with excited nuclear states, which, when they decay, result in gamma-ray emissions. In addition to fission that produces large quantities of neutrons, a few isotopes decay by the emission of neutrons.

Since gamma rays are extremely penetrating, they are the greatest biological hazards of radiation originating outside the human body. Alpha particles have a range of only a few centimeters in air and are completely stopped by the outer skin tissue. While beta particles have a range on the order of a meter in air, most of their energy is absorbed primarily by the skin. Sensitive internal organs are thus shielded from the effects of both alpha and beta particles emitted by external sources.

Concurrent with the discovery and isolation of naturally occurring radioactive isotopes was the discovery of x rays by Roentgen in 1895. High energy photons indistinguishable from those associated with radioactive decay were produced by bombarding a metal plate with high velocity electrons (Figure 6.13). While neither gamma- nor x-ray radiation produces any direct physiological sensation, delayed cancerous growths often result from high radiation dosages. The results of early indiscriminate x-ray usage, often merely for cosmetic effects, are described in detail by Schubert and Lapp [37].

The first radiation standards were concerned with the radiation intensity of gamma or x-rays. Since biological damage is approximately proportional to the time integral of the radiation intensity, the exposure unit, the roentgen (R), was defined as a measure of the time-integrated radiation effect. For both a radioactive isotope and an x-ray tube, the radiation intensity is inversely proportional to the square of the distance from the source due to the spreading of the radiation field. High energy photons traveling through air tend to ionize molecules. Ionization is the result of photon-electron collisions which may impart enough energy to an electron for it to escape the atom. While the ionization energy for nitrogen and



**FIGURE 6.13** External Gamma- and X-Ray Sources.

oxygen is on the order of 14 eV for valence electrons, a photon loses, on the average, 34 eV of energy for each ionizing collision. Both ionizing collisions with nonvalence electrons and the kinetic energy of the resultant free electrons account for the experimentally observed loss of 34 eV. Ionization by high energy photons is referred to as *Compton scattering*.

The unit of radiation dosage was first introduced by the Radiological Congress held in Stockholm in 1928. Due to the convenience of measuring ionizing effects (with ionization chambers), the roentgen was defined as the quantity of radiation that produced one statcoulomb ( $\frac{1}{3} \times 10^{-9}$  coulombs) of ionized charges of either sign in one cubic centimeter of dry air at standard conditions (atmospheric pressure, 0°C). Since each ion pair results in a free charge of  $1.6 \times 10^{-19}$  coulombs, the quantity of ions produced and, hence, the energy absorbed for an exposure of one roentgen may readily be calculated.

$$N = \frac{\frac{1}{3} \times 10^{-9}}{1.6 \times 10^{-19}} = 2.08 \times 10^9 \text{ ion pairs}$$

$$E = (34 \text{ eV}) N = 7.083 \times 10^{10} \text{ eV}$$

$$= 1.133 \times 10^{-8} \text{ J} \quad (\text{one cubic centimeter})$$

Alternatively, the roentgen may be expressed in terms of the energy absorbed by a unit mass of air. At standard conditions, a kilogram-mole of an ideal gas occupies a volume of 22.4 m<sup>3</sup>. The density of air (atomic weight approximately 29) is thus 1.293 kg/m<sup>3</sup>. This results in an absorbed energy density of  $8.76 \times 10^{-3} \text{ J/kg}$ . Usually the roentgen is expressed in terms of ergs (one erg is equivalent to  $10^{-7} \text{ J}$ ) per gram.

$$1 \text{ R} = 87.6 \text{ ergs/g} \quad (\text{air})$$

Dividing the charge produced in one cubic meter,  $\frac{1}{3} \times 10^{-3}$  coulombs, by the density of air, an alternative expression for one roentgen is obtained.

$$1 \text{ R} = 2.58 \times 10^{-4} \text{ coulombs/kg (air)}$$

In 1962 the International Commission on Radiological Units (ICRU) redefined the roentgen in terms of the above relation. It is numerically equal to the original definition.

While the roentgen is a measure of the energy intensity of radiation, biological damage depends upon energy absorption. Absorption depends upon both the biological material and the type of radiation. The radiation-absorbed dose, the rad, is defined such that one rad is equal to an absorbed dose of 100 ergs per gram of material (.01 J/kg). Even though the roentgen is defined for air and the rad for any material, the units are often, due to their approximate equivalence, used interchangeably. Energy absorption for gamma- and x-ray radiation depends upon the Compton scattering of the incident photons by orbital electrons. Therefore, energy absorption is proportional to the electron density, which is equal to the nuclear proton density. Variations in electron density per unit mass are therefore dependent upon the small variations in neutron to proton ratios. Hence, electron densities per unit mass are approximately equal. An exposure in air of one roentgen (87.6 ergs/g) is almost equivalent to an absorption of one rad (100 ergs/g) by a biological material placed in the same position and for the same interval of time. Unlike the roentgen, however, the rad is defined for all types of radiation.

The energy associated with radiation exposures is exceedingly small, as illustrated by the following comparison. One calorie or 4.18 J of energy is necessary for a 1 C° temperature increase for water. Since one rad is equivalent to  $10^{-5}$  J/g, an absorbed dose of  $4.18 \times 10^5$  rads would be necessary to produce a one degree thermal effect. An absorption of only 400 to 500 rads over the whole body in a short time interval is considered lethal for 50% of those so exposed. The thermal effect of this dosage is only one-thousandth of a degree, an effect which, if produced by a nonradioactive energy source, would be completely benign. However, ionizing radiation usually destroys the complex molecular and cellular structure of living materials.

Gamma- and x-ray radiation is the main cause of biological damage that results from external radiation. Radioactive isotopes deposited within the body, however, result in internal radiation damage. The emission of alpha and beta particles by internal sources is important since the energy of these particles is absorbed wholly within the body. To relate the effect of these sources, as well as the effects of other sources of radiation, such as neutrons that are produced by fission and high energy protons of cosmic radiation, the radiation dose equivalent man, the rem, was introduced. The dose in rem is equal to that in rad multiplied by a dimensionless quality factor,  $QF$ , which depends upon the type of radiation.

$$\text{rem} = \text{rad} \times QF$$

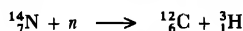
Quality factors for various types of radiation are listed in Table 6.4. A radiation dose for a mixed source is equal to the sum of the dosages in rem of the individual radiations.



**TABLE 6.4 Quality Factors  
for Various Sources of  
Radiation**

<i>Radiation</i>	<i>QF</i>
Gamma and x-rays	1
Beta particles	1
Thermal neutrons	3
Fast neutrons	10
Protons	10
Alpha particles	10

All living materials are subjected to the effect of background or natural radiation. Cosmic rays are extremely energetic radiations from outer space composed primarily of protons with very high energies ( $> 1000$  MeV). While partially attenuated by the earth's atmosphere, cosmic rays can produce secondary radiation by the interaction of the primary particles with the atmosphere. At sea level, cosmic radiation results in an absorbed dose of approximately .05 rem or 50 mrem (millirem) per year. An approximate doubling of this dose occurs for every kilometer increase in elevation. Another source of background radiation is the radiation due to naturally occurring radioactive isotopes. In addition to the heavy radioactive isotopes of thorium and uranium, several other radioactive isotopes occur naturally (Table 6.5). With the exception of the first two isotopes, tritium (hydrogen-3) and carbon-14, the half-lives are comparable to the age of the universe. They could, therefore, have been formed during the early evolutionary stages of the earth. Tritium, with a half-life of 12.3 years, and carbon-14 (5700 years) would no longer exist if they were not being continually formed. Both are formed from nitrogen-14 through the interaction of secondary neutrons produced by cosmic rays in the upper atmosphere.



The average combined dose for terrestrial and cosmic radiation is 70 to 200 mrem/y at sea level. In addition to being dependent on altitude, the dose is also dependent on geographic location and may, in select areas, be several times the average value. Medical x rays also increase the dose an individual receives. A typical chest x ray results in a dose on the order of 50 mrem to the chest area. Diagnostic x rays result in an absorbed dose on the order of 100 mrem/y for an average U.S. citizen.

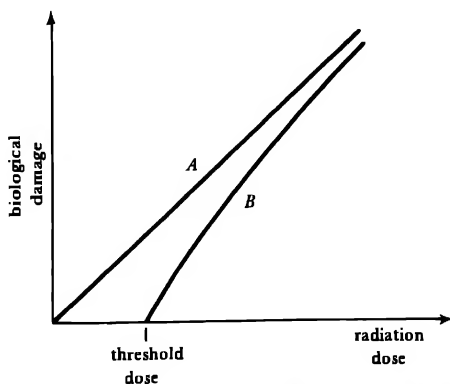
Since natural background and medical radiation results in exposures of 200 to 300 mrem/y, the average nonoccupational permissible additional exposure has been set at 170 mrem/y by the Federal Radiation Council. The question of a "safe" radiation dose, however, is a point of scientific contention. Part of the argument concerns the question of the existence of a threshold dose. Figure 6.14 shows two hypothetical relationships between biological damage and radiation dose, one

**TABLE 6.5 Naturally Occurring Radioactive Isotopes**

Element	Isotope	Abundance	$T_{1/2}$	Principal decay
Tritium (hydrogen-3)	$^3_1\text{T}$	—	12.3y	$\beta^-$
Carbon	$^{14}_6\text{C}$	—	5700y	$\beta^-$
Potassium	$^{40}_{19}\text{K}$	0.00118%	$1.28 \times 10^9\text{y}$	$\beta^-$ , $\beta^+$ and $\gamma$
Rubidium	$^{87}_{37}\text{Rb}$	27.85%	$5 \times 10^{10}\text{y}$	$\beta^-$
Lanthanum	$^{138}_{57}\text{La}$	0.089%	$1.1 \times 10^{11}\text{y}$	$\beta^-$ and $\gamma$
Samarium	$^{147}_{62}\text{Sm}$	14.97%	$1.1 \times 10^{11}\text{y}$	$\alpha$
Lutetium	$^{176}_{71}\text{Lu}$	2.59%	$2 \times 10^{10}\text{y}$	$\beta^-$ and $\gamma$
Rhenium	$^{187}_{75}\text{Re}$	62.93%	$10^{11}\text{y}$	$\beta^-$

relationship (curve *B*) displaying a threshold. Curve *B* implies that a radiation dose below the threshold will have a zero effect. If the threshold dose is greater than that due to background radiation, an increase in radiation dose up to that of the threshold would be completely harmless. If, however, no threshold exists, as implied by curve *A*, or if the threshold is considerably less than that produced by background radiation, any increase in radiation would result in an increase in biological damage. The presently accepted dose standard of 170 mrem/y, a value comparable to that received from background radiation and diagnostic x rays, infers that a doubling of the radiation dosage will have a minimal effect. The precise relation between damage and dose is unknown. A straight-line relation (curve *A*) infers a doubling of damage for a doubling of dose. Any increase in dose, however small, implies an increase in damage.

The preceding discussion raises a further question: To what extent are presently

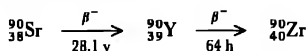


**FIGURE 6.14 Biological Damage as a Function of Radiation Dose.**

observed incidents of cancer and genetic effects due to radiation, as opposed to other causes? Radiation effects are, and may remain for many years, a matter of opinion, opinion based upon legitimate differences in interpretation of medical data. Most noteworthy of those that seriously question present standards are Lederberg, Gofman, and Tamplin [38-41]. The case of Gofman and Tamplin is particularly interesting, since it represents an adversary view from within the former AEC (Lawrence Radiation Laboratory). As a result of an article by Sternglass [42, 43], they were requested to prepare a rebuttal to Sternglass's contention that atmospheric nuclear testing resulted in the death of 400,000 babies. Gofman and Tamplin, on the basis of their studies, concluded the number was on the order of "only" 4000, a number still considerably greater than what is generally accepted by most nuclear scientists. Debate on a safe level of radiation will undoubtedly continue for many years.

Background radiation, due to cosmic rays and naturally occurring radioactive isotopes as well as medical x rays, results in external sources of radiation dosages. One of the naturally occurring radioactive isotopes, potassium-40, is also a significant source of internal radiation. Potassium comprises approximately .2% by weight of the human body. Therefore, potassium-40 is found within the body along with the normally occurring nonradioactive isotopes ( $^{39}\text{K}$  and  $^{41}\text{K}$ ) in proportion to its natural abundance of .00118%. For an average 70-kg person, the activity of potassium-40 is very small ( $\sim .01 \mu\text{Ci}$ ). Radioactive reactor waste products, if they are not isolated from the biosphere, can also result in isotopes that find their way into the human body and cause internal radiation damage. Due to their relatively long half-lives of approximately 30 years and to their biological activity, both strontium-90 and cesium-137 are potentially harmful. Other nuclear wastes have considerably shorter half-lives and will thus decay to nonlethal, stable isotopes in five to ten years.

Strontium-90 decays by emitting a  $\beta^-$  particle to yttrium-90 which, in turn, decays by a  $\beta^-$  emission to zirconium-90.\*



Since the half-life of the second transition is extremely short compared to the first one, the decay rate is determined by the 28.1 year half-life of strontium-90. Each disintegration of strontium therefore results in two beta particles. Strontium, having two valence electrons, is chemically similar to calcium and finds its way along with calcium into the bone structure of humans. Significant quantities of strontium-90 were first produced by atmospheric nuclear testing. After being carried from the test site by high altitude winds, strontium-90 was carried to the earth's surface by rain and snow. Both grass and food crops withdrew the radioactive strontium as well as calcium from the soil. Through the food chain strontium-90 tends to concentrate in all living things. Cows feeding on grass concentrate strontium-90 along with calcium. Hence for humans, the chief source of strontium-90 is milk, a food high in calcium.

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\* The decay of yttrium 90 has a very slight probability, .02%, of a lower energy  $\beta^-$  particle followed by a gamma emission.

Relatively large intakes of calcium are necessary for bone growth. In the growth process, however, radioactive strontium, if present, is deposited within the bones along with calcium. Within the bones and particularly within the sensitive marrow of the bones, strontium-90 is an intense source of beta particles. Furthermore, since individual calcium atoms tend to remain within the bones for 20 to 30 years, radioactive isotopes deposited along with calcium can result in high radiation doses because of the long time interval involved. Radioactive isotopes of cerium, praseodymium, barium, radium, and plutonium are also "bone seekers." Cesium-137, which also has a long half-life, tends to follow potassium and concentrate in the muscles. Unlike strontium-90, cesium-137 decays by the emission of a gamma ray along with a beta particle. Intense gamma rays can result in genetic effects, effects that may take several generations to observe.

The biological concentration of radioactive isotopes from nuclear testing is particularly well described by Commoner [44, 45]. In the past, the earth has been considered sufficiently large that undesired wastes could be "eliminated" through dilution. Biological processes can, however, for some radioactive isotopes, counteract the natural dilution mechanism.

## B. FISSION PRODUCTS

To establish a perspective on the "waste" problem, a reactor with a one gigawatt thermal power output will be considered. Its electrical output would be 300 to 400 MW for a 30 to 40% efficiency. Since reactors are presently being designed and constructed for electrical powers as great as one gigawatt, this example represents a moderately but not exceptionally large reactor. For each fission event, approximately 200 MeV or  $3.2 \times 10^{-11}$  J of energy is obtained. Therefore,  $3.125 \times 10^{10}$  fissions are necessary to produce one joule. Since one joule per second is a power of one watt, the same number of fissions per second are necessary for each watt of power.

$$1 \text{ watt} \quad 3.125 \times 10^{10} \text{ fissions/sec}$$

$$1 \text{ gigawatt} \quad 3.125 \times 10^{19} \text{ fissions/sec}$$

An operation of one year at an average thermal power level of 1 GW thus requires  $9.86 \times 10^{26}$  fission events. Dividing by Avogadro's number, it is found that 1.64 kg-moles, that is, 385 kg of uranium-235 or 391 kg of plutonium-239 is required.

Not all the uranium fuel in a reactor contributes to its output power. Neutrons may be absorbed by uranium-235 without resulting in a fission reaction ( $\sigma_f/\sigma_a < 1$ ). When this occurs, fuel is, in essence, lost since  $^{235}_{92}\text{U}$  is converted to  $^{236}_{92}\text{U}$ , an isotope which does not fission. Also, only a portion of the initial fuel loading of a reactor can be fissioned before it is necessary, due to the reduced fuel concentration (fuel "burn up"), to remove the fuel from the reactor. The plutonium-239 produced when a neutron is absorbed by uranium-238 contributes to the energy output of a reactor.

A uranium fuel enrichment of about 3% is common for a light-water reactor. Therefore, 97% of the uranium is uranium-238, the fertile isotope which through

the absorption of a neutron is transmuted to plutonium-239. While a breeder reactor is designed to optimize this process, a light-water reactor, owing to the large quantity of uranium-238 present, also produces a significant amount of plutonium-239. The "bred" plutonium-239 has a large fission cross-section area for thermal neutrons and hence it tends to fission. To illustrate this effect, it will be assumed that the output power, and hence the fission rate,  $R$ , for a reactor is constant. Let  $N_u$  be the total number of  $^{235}_{92}\text{U}$  atoms and  $N_p$  the number of  $^{239}_{92}\text{Pu}$  atoms of a reactor core. Since the fission rate for each species will tend to be proportional to its relative concentration, the following differential equation is obtained for the uranium-235 atoms.

$$\frac{dN_u}{dt} = -\frac{N_u R}{N_u + N_p}$$

If it is assumed that each fission event results, on the average, in the formation of  $C$  atoms of plutonium-239 ( $C < 1$  for a light-water reactor), the following is obtained for the plutonium-239 atoms.

$$\frac{dN_p}{dt} = -\frac{N_p R}{N_u + N_p} + CR$$

The first term on the right-hand side of the above relation is the loss due to fission and the second term is the conversion gain. Since the relative concentration of uranium-238 atoms is very large, the effect of a change in this concentration on the conversion factor,  $C$ , may be ignored. A time-dependent expression for total fissionable inventory,  $N_u + N_p$ , may be obtained

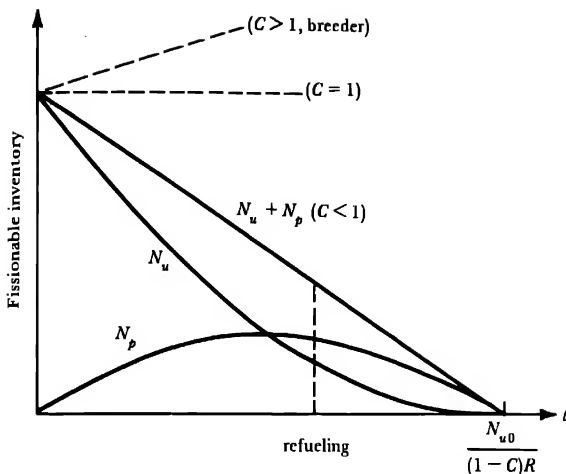
$$\begin{aligned}\frac{d}{dt}(N_u + N_p) &= -(1 - C)R \\ N_u + N_p &= -(1 - C)Rt + N_{u0}\end{aligned}$$

It was assumed, in doing the integration, that the initial inventory of fuel consisted of only uranium with the quantity of uranium-235 atoms being  $N_{u0}$ . For a conversion ratio that is less than one, the inventory of fissionable fuel,  $N_u + N_p$ , decreases linearly in time (Figure 6.15). For a conversion factor of one, fissionable isotopes are produced at the same rate as they are consumed whereas for a conversion factor that is greater than one, the fissionable inventory increases (a breeder reactor).

Solving the appropriate differential equation (using the expression obtained for  $N_u + N_p$ ), explicit functions for the quantities of uranium and plutonium atoms are obtained.

$$\begin{aligned}N_u &= N_{u0}[1 - (1 - C)Rt/N_{u0}]^{1/(1-C)} \\ N_p &= N_{u0}[1 - (1 - C)Rt/N_{u0}] \\ &\quad - N_{u0}[1 - (1 - C)Rt/N_{u0}]^{1/(1-C)}\end{aligned}$$

While the number of uranium-235 atoms continually declines, the number of plutonium-239 atoms initially increases, reaches a peak, and then, if fission continues,



**FIGURE 6.15** *Fissionable Inventory of a Reactor.*

declines. A conversion ratio,  $C$ , of .5 (a value for a typical light-water reactor) yields a particularly simple set of expressions for  $N_u$  and  $N_p$ .

$$N_u = N_{u0} [1 - Rt/(2N_{u0})]^2$$

$$N_p = \frac{1}{2} [1 - Rt/(2N_{u0})] Rt$$

$$N_u + N_p = N_{u0} - \frac{1}{2} Rt$$

The time at which the fissionable inventory,  $N_u + N_p$ , is reduced to one-half its initial value may be determined.

$$\frac{1}{2} N_{u0} = N_{u0} - \frac{1}{2} Rt$$

$$Rt = N_{u0}$$

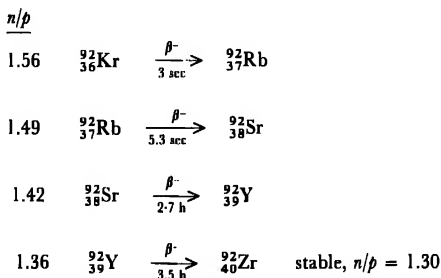
At this time, the inventories of the uranium-235 and plutonium-239 atoms are equal.

$$N_u = N_p = \frac{1}{4} N_{u0}$$

For example, the initial uranium fuel loading may have a 3% concentration of uranium-235 atoms. When the concentration of fissionable atoms falls to 1.5%, the inventory consists of equal concentrations (.75%) of uranium-235 and plutonium-239

atoms. At approximately this concentration, the fuel will be removed since there are insufficient fissionable isotopes to sustain a chain reaction. The total number of fission events, for this case, is equal to  $N_{u0}$  and the energy produced thus corresponds to the fission of the entire initial inventory of uranium-235 atoms. Furthermore, the spent fuel contains a significant quantity of fissionable plutonium-239.

The fuel rods of a reactor are designed to entrap the fission products. The experimentally determined fission product yield for the thermal neutron fission of uranium-235 may be obtained from Figure 6.5. Other fissionable isotopes, such as plutonium-239, have slightly different yields [9, 10]. The yield is also dependent to a small extent upon the energy of the fission-initiating neutron. Knowing the decay process of fission products (beta emission is by far the most common), the specific "waste" elements may be determined. For example, consider the isotopes with mass numbers of 92.

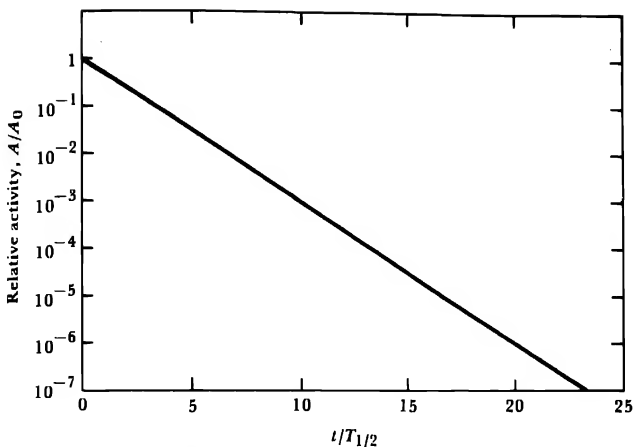


The above four radioactive isotopes decay by beta emission and hence result in a new isotope with the same mass number. Since the decay rates for these isotopes are relatively rapid, the yield after a brief decay interval will be entirely zirconium-92, the end product of this decay chain. Therefore, the yield of 6% for isotopes of mass number 92 implies that this quantity of zirconium-92 will result. The beta decay, it will be noted, reduces the neutron to proton ratio, the stable isotope having a ratio considerably less than that of the first isotope of the series (which is comparable to that of uranium-235,  $n/p = 1.55$ ).

The resultant radioactivity of an isotope depends upon its half-life and the duration over which it has been allowed to decay. Activity has been shown to be an exponentially decreasing function of time and its half-life has been defined as the time necessary for the activity to decrease to one-half of its initial value.

$$\begin{aligned}
 A &= A_0 e^{-\lambda t} \\
 e^{-\lambda T_{1/2}} &= \frac{1}{2}
 \end{aligned}$$

An expression for activity in terms of the half-life may readily be obtained.

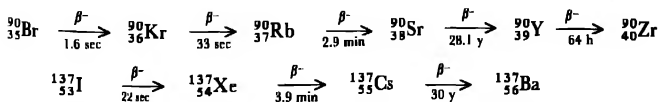


**FIGURE 6.16** *Relative Decay Activity Expressed in Terms of Half-Lives.*

$$A = A_0 e^{-\lambda T_{1/2}(t/T_{1/2})} = A_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

Figure 6.16 is a plot of relative activity,  $A/A_0$ . A time interval of approximately 20 half-lives is required to reduce the radioactivity of a particular quantity of an isotope to one-millionth ( $10^{-6}$ ) of its initial value. For yttrium-92 (half-life of 3.5 h), less than five days are required to reduce its activity by this amount. A comparable time is required for the strontium-92 of the same decay chain. Therefore, the radioactivity of the isotopes of this set of fission products will reach a negligible level within a couple of weeks after being removed from the reactor.

Not all waste products have such a rapid decay. Of particular interest are the isotopes with mass numbers of 90 and 137.\*



Both these decay chains have an isotope with a very long half-life: strontium-90 for the first and cesium-137 for the second. The isotopes rapidly decay (instantaneously compared to a 28.1- or 30-year half-life) to the isotopes with the long half-life. Since the yield for a mass number of 90 is 5.77%, this is the yield of strontium-90 which

\* Iodine-137 also decays by a neutron emission to xenon-136, a stable isotope.



results from the decay of bromine-90, krypton-90, and rubidium-90, as well as that which is the direct result of fission. Both yttrium-90 and zirconium-90 are wholly the result of decay of strontium-90. A similar decay process results in a yield of 6.15% for cesium-137.

Multiplying the fission rate ( $3.125 \times 10^{19} \text{ sec}^{-1}$  for the 1 GW example) by the fractional yield of .0577 for strontium-90 or .0615 for cesium-137 gives the approximate rate at which the respective isotopes are produced within a reactor. If this rate is represented by the symbol  $I$ , the time derivative of the number of atoms of the isotope,  $N$ , is equal to  $I$  minus the natural decay rate,  $\lambda N$ .

$$\frac{dN}{dt} = I - \lambda N$$

A solution for  $N$  and hence the decay rate,  $\lambda N$ , assuming an initial zero value of  $N$  for  $t = 0$ , is readily obtained.

$$N = (I/\lambda)(1 - e^{-\lambda t})$$

$$\lambda N = I(1 - e^{-\lambda t})$$

For isotopes with short half-lives,  $\lambda t$  becomes large before the wastes are removed from the reactor. The activity of these isotopes is  $I$ , that is, these isotopes decay as rapidly as produced. Assuming the reactor operates for an interval of  $t_0$  sec, the activity at that time is the following:

$$\lambda N(t_0) = I(1 - e^{-\lambda t_0})$$

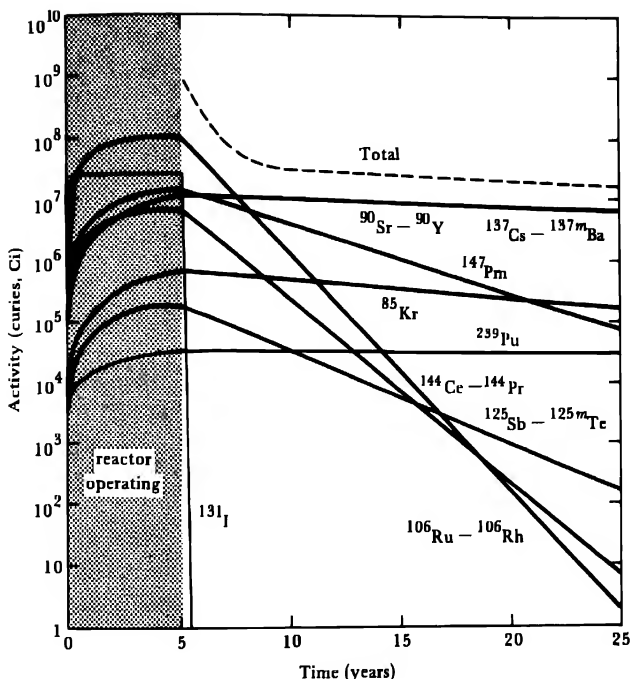
If the waste products are then removed from the reactor, a normal exponential decay is obtained for  $t > t_0$ .

$$\lambda N = \lambda N(t_0)e^{-\lambda(t-t_0)} = I(1 - e^{-\lambda t_0})e^{-\lambda(t-t_0)} \quad t > t_0$$

For a given value of  $I$ , isotopes with short half-lives (large values of  $\lambda$ ) will have high levels of activity, approximately  $I$ , within a reactor and will decay very rapidly when removed from the reactor. Isotopes with long half-lives (small values of  $\lambda$ ) will have lower values of activity within a reactor and decay much more slowly when removed from the reactor.

Yttrium-90, the decay product of strontium-90, decays with a half-life of 64 h, which compared to that of strontium-90 is nearly instantaneous. Therefore, each strontium-90 disintegration is accompanied by a yttrium-90 disintegration. The resultant activity, as measured in curies, is thus twice that calculated for strontium-90. The decay of cesium-137 also results in a similar radioactive process with the dominant decay process resulting in the formation of a metastable isotope of barium-137, which decays with a half-life of only 2.55 min by the emission of a gamma ray (an isomeric transition) to the stable barium-137 isotope.

Figure 6.17 presents the activity of significant waste products for a 1-GW reactor which operates for five years. In addition to strontium-90 and cesium-137, promethium-147 and krypton-85 contribute significantly to the long-term radioactivity of the waste products. The total activity, obtained by considering all fission-



**FIGURE 6.17 Waste Product Activity for 1-GW (Thermal) Reactor: Five-Year Operation,  $^{235}\text{U}$  Thermal Neutron Fission.**

produced isotopes, was obtained from the detailed calculations of Cember [46] for a one-megawatt reactor. Simply "storing" the wastes for a five- to ten-year period after removing them from the reactor results in relatively low levels of activity for all but the few long-half-life isotopes.

The mass of the radioactive isotopes, for this example, is not very great: 40 kg of strontium-90 and 65 kg of cesium-137. Each results in an activity of slightly more than ten million ( $10^7$ ) curies, which decays to only half of its initial value in approximately 30 years. To reduce the activity to ten curies (a factor of  $10^{-6}$ ), 20 half-lives, or 600 years, will be necessary. Since quantities of these isotopes measured not in curies but in microcuries (another factor of  $10^{-6}$ ) can be biologically harmful, storage times of 1200 years will be needed. A measure of the activity of these isotopes may be gleaned from the thermal energy produced as a result of the decay. Strontium-90 when removed from the reactor has an activity of  $2.1 \times 10^{17}$  dis/sec. The first decay is by the emission of a beta particle with average energy of .20 MeV, and the

second, that of yttrium-90, is by the emission of a .93 MeV (average) beta particle.\* When the beta particles lose their kinetic energy, each disintegration results in an average thermal energy of 1.13 MeV. Based upon the disintegration rate, the thermal power is 38 kW. Cesium-137 decays by emitting a beta particle with an average energy of .19 MeV. A metastable barium-137 isotope, which decays by a gamma emission with an energy of .66 MeV, is produced by more than 90% of the cesium-137 decays. A thermal power of 27 kW results if the gamma emission is assumed to be absorbed by the surroundings. Data for the isotopes of Figure 6.17, including the thermal power associated with decay processes, is summarized in Table 6.6. The thermal power of these isotopes, with the relatively long half-lives, accounts for over .1% of the reactor power at the end of the five years of operation. After five to ten years of storage, the bulk of the radioactivity and thermal power is due almost wholly to the strontium-90 and cesium-137.

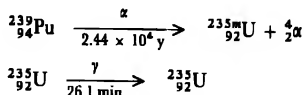
**TABLE 6.6 Waste Product Activity When Removed from 1-GW Thermal Reactor: Five-Year Operation,  $^{235}\text{U}$  Thermal Neutron Fission**

Decay	Activity curies	$T_{1/2}$	Thermal power
$^{144}\text{Ce} - ^{144}\text{Pr}$	$1 \times 10^8$	284 d	382 kW
$^{131}\text{I}$	$2.6 \times 10^7$	8.05 d	28 kW
$^{147}\text{Pm}$	$1.5 \times 10^7$	2.68 y	5.4 kW
$^{90}\text{Sr} - ^{90}\text{Y}$	$1.1 \times 10^7$	28.1 y	38 kW
$^{137}\text{Cs} - ^{137\text{m}}\text{Ba}$	$1.1 \times 10^7$	30 y	27 kW
$^{106}\text{Ru} - ^{106}\text{Rh}$	$6.2 \times 10^6$	1.01 y	26 kW
$^{85}\text{Kr}$	$7 \times 10^5$	10.8 y	1.0 kW
$^{125}\text{Sb} - ^{125\text{m}}\text{Te}$	$1.8 \times 10^5$	2.7 y	.07 kW

As has been discussed, strontium-90 concentrates, along with calcium, in the bone structure of living things. The kinetic energy of the emitted beta particles is absorbed by the bone structure and thus their energy contributes directly to an internal radiation dose. If the total "acceptable" bone radiation dose of 170 mrem per year is due wholly to strontium-90 concentrated within the bones, the quantity of strontium may readily be calculated. Since one rem is 100 ergs/g or  $10^{-2}$  J/kg, 170 mrem is equivalent to .0017 J/kg. Since each disintegration (average beta emissions of 1.13 MeV) results in an energy of  $1.8 \times 10^{-13}$  J,  $9.4 \times 10^9$  disintegrations are necessary for a 170-mrem dose. A yearly dose of 170 mrem implies 298 dis/sec or an activity of .008  $\mu\text{Ci}$ . Only a microscopic quantity of strontium-90,  $5.6 \times 10^{-5}$   $\mu\text{g}$ , is necessary to produce this activity. In terms of its potential biological hazard, the 40 kg of strontium-90 of the reactor example compared to a dose of  $5.6 \times 10^{-5}$   $\mu\text{g}$ , represents an enormous quantity.

\*A substantial portion of the energy for each beta emission is carried off by a neutrino which does not interact with the immediate environment.

The approximate activity of the plutonium-239 produced by a light-water reactor is also indicated in Figure 6.17. Plutonium-239 decays by the emission of an energetic alpha particle which is followed by a gamma emission by the excited metastable uranium ( $^{235m}\text{U}$ ) decay product.



Even though plutonium-239 has a reasonably long half-life which implies a low level of activity, its alpha emission is a serious internal radiation hazard. On exposure to air, plutonium spontaneously combines with oxygen to form an oxide. When inhaled, the oxide can deposit on the sensitive lung tissues. The high energy alpha particle (5.1 MeV) is absorbed by biological materials within a very short distance, typically 40 to 45  $\mu$ . A high radiation dose occurs within a sphere surrounding the plutonium particle that has a radius of only 40 to 45  $\mu$ . According to the reasoning of Tamplin and Cochran [47], it is this dose that is important in assessing the potential hazard of plutonium-239 rather than, as standards were originally devised, averaging the dose over the entire surface of the lungs. If their reasoning is correct, initial plutonium-239 safety standards allowed occupational levels of plutonium that were excessively high. While others disagree with this "hot particle" hypothesis, there is little disagreement about plutonium's being an extremely toxic substance [48-51].\*

The spent fuel, when removed from a reactor, contains, along with the fission products and plutonium-239, other heavy isotopes of plutonium, neptunium, curium, and americium. These heavy isotopes are usually referred to as the *actinides* (a classification for elements with atomic numbers of 89 or greater). Several of these radioactive reactor-produced isotopes (they do not occur in nature) have half-lives measured in thousands of years.

The plutonium-239 of the fuel removed from a reactor has a potential value since it can be chemically separated from the uranium and fission products. The recovered plutonium can, along with enriched uranium, be fabricated into new fuel rods. Less uranium or uranium with a lower enrichment can be utilized for "mixed oxide" fuel rods (oxides of plutonium and uranium are used). Ideally, the slightly enriched uranium that remains can also be recovered and the fission waste products with long half-lives can be separated from the less hazardous fission products [52].

Uranium fuels have been processed in the United States to obtain plutonium for weapons. In the process, 85 million gallons of high-level wastes, that is, wastes containing isotopes with long half-lives, have been generated [53]. These liquid wastes are stored in large (up to one million gallons each) holding tanks at three locations: the reprocessing plants at Hanford, Washington, the National Reactor

\* Plutonium was named after the outer planet of the solar system, Pluto. In Greek mythology Hades (Pluto was the Roman name of the god) was the god of the underworld, a god that had a dual domain: that of the earth's fertility and that of the dead.

Testing Station in Idaho, and the Savannah River Plant in South Carolina. While the specific radioactivity of the wastes is classified, Novick has estimated the activity of these wastes to be 100 Ci/gal [54] which implies a total activity of nearly ten billion ( $10^{10}$ ) Ci.

Fuel from commercial nuclear power plants in the United States has not been successfully reprocessed. A reprocessing plant, constructed in 1966 by Nuclear Fuel Services at West Valley, New York, was closed in 1972, leaving the State of New York with not only a highly contaminated plant but also 600,000 gal of liquid wastes [55, 56]. Furthermore, numerous employees may have received excessively high radiation doses while the plant was in operation. [57]. In 1968, General Electric began construction of a reprocessing facility near Morris, Illinois, based upon a newly developed, but unproven, process to eliminate liquid effluents [58]. In 1974, it was announced that the \$64 million facility did not work and that its redesign was not feasible.

Subsequently, a third commercial reprocessing plant was constructed in Barnwell, South Carolina, by Allied Nuclear Services. The design of the Barnwell facility was based upon the Purex separation process used for recovering plutonium when producing weapons. As of 1980, owing to an earlier national policy decision, the reprocessing facility had not begun operation [59]. The U.S. decision of the late 1970s to defer the reprocessing of spent reactor fuel was partly based upon the fact that large quantities of separated plutonium-239, suitable for producing weapons, would be recovered when fuels are reprocessed [60]. As little as a few kilograms obtained by a sophisticated terrorist group would be sufficient to build a crude atomic bomb. Furthermore, if other nations also develop a capability for reprocessing fuel, the number of nations with a nuclear weapons capability is also likely to increase. The decision to defer reprocessing was also influenced by economic considerations. While the recovered plutonium-239 reduces the amount of uranium-235 required for new fuel rods (and hence enrichment costs), the costs of fabricating fuel rods from plutonium, owing to its toxicity, are considerably higher than if only uranium is used. A Ford Foundation study concluded that the economic benefit of reprocessing fuel from light-water reactors is not very great [61]. The benefit of a large-scale breeder reactor program, however, depends upon the ability to reprocess fuel. Even if the technical problems are solved, the immense quantities of plutonium involved give rise to an extremely complex set of moral dilemmas and social questions that will need to be resolved [62, 63].

Liquid storage of wastes (primarily from weapons production) has always been considered an interim measure. The wastes produced by commercial power reactors are presently (1980) being stored as spent fuel rods in cooling ponds at the reactors. In terms of fission products, Krugman and von Hippel have concluded that waste inventories accrued as a result of weapons production and power generation are comparable [64]. Isolating these wastes from the biosphere represents an unprecedented engineering challenge [65] and may also require complementary social institutions [6]. The liquid wastes will need to be solidified and encapsulated in sealed containers of manageable size. The ultimate disposal of the waste products of the spent fuel rods depends upon the reprocessing that may be employed. If they

are not reprocessed, the fuel rods, after being stored sufficiently long for much of their radioactivity to decay, could be cut up and encapsulated.

Once the nuclear wastes are suitably encapsulated, some type of permanent burial in a suitable geological formation is envisioned [66-76]. In 1970, a salt mine near Lyons, Kansas, was tentatively selected for storing wastes [77, 78]. A disadvantage of salt storage is that the wastes would not be readily retrievable, that is, the waste container, due to the heat generated by the decay process, would not remain intact. The proposal, at least for the particular salt mine chosen, was discarded, partially as the result of objections raised by Hambleton, the Director of the Kansas Geological Survey [79]. Salt deposits, at first, appeared an ideal storage sanctuary since their formation could not have occurred had water been present. While the integrity of the deposits was assured in the past, it is now in question due to the drilling that has occurred in the area. Disposal schemes utilizing something other than salt formations have also been proposed. These include burial in granite, polar ice, and Antarctic rock. Sending the waste products into space has been suggested, although this is not considered practical for the near future. At the conclusion of the 1970s, the nation's policy for handling radioactive wastes was described by L. J. Carter in *Science* as being in "disarray" [80].

## 9. ACCIDENTS

Prior to the Three Mile Island accident on March 28, 1979, the main safety concern was that of an accident due to a sudden catastrophic failure of a major reactor component. (Such an accident, for example, a large core rupture, could, if not contained, result in a disaster of unprecedented proportions.) This did not occur at Three Mile Island. The accident was a result of the failure of a relatively minor component and the misinterpretation of the abnormal conditions that ensued. While it is generally accepted that the effects of the radioactive releases of the accident were negligible, this was not the case for the mental stress created. The uncertainty of what might occur during the first days of the accident resulted in an almost total disruption of the normal activities in the surrounding communities. In addition, the economic cost to the operating utility was (and will continue to be) immense. Not only is it unlikely that the damaged reactor can be repaired, but the cost of removing the contaminated core and radioactive wastes may exceed the original cost of the reactor [81].

The radioactive fission products, the fission fragments, plutonium-239 and other actinides, are entrapped by the fuel rods during the normal operation of a reactor. Only a portion of the fuel rods is normally replaced when a reactor is refueled. Hence, except for a new reactor, the core contains a very large inventory of these radioactive products. A theoretical study of the possible consequences of a major release of these products was completed by the Atomic Energy Commission in 1957 [82].

For this study, a 500-MW (thermal) reactor that operated 180 days was assumed to rupture. Experts who did express an opinion (many refused) on the probability

of such a major accident placed the probability in the range of one chance in  $10^5$  to  $10^9$  per year. All agreed that the probability was exceedingly small. The activity of the core waste products 24 hours after an accident was assumed to be  $4 \times 10^8$  Ci (a quantity in approximate agreement with that of Figure 6.17, which is for a reactor twice the size that operated for five years). The reactor was assumed to be located 30 miles from a major city.

Three types of accidents were considered. The first was a "contained case" in which only the core ruptured and the fission products were wholly contained by the containment vessel. Direct gamma radiation was the only hazard of this type of failure. The second type of failure was the "volatile release case" in which a rupture of the core resulted in a release by the containment vessel of all volatile fission products and 1% of the strontium-90. The most serious accident was the "50% release case" in which 50% of all fission products were released due to a catastrophic rupture of both the reactor core and the containment vessel. Damage was the result of the subsequent dispersal of the waste products. While the last type of accident was considered the least likely, it resulted in the greatest damage.

Several alternative combinations of conditions were investigated for the 50% release case. Both hot ( $149^\circ\text{C}$ ) and cold or ambient ( $21^\circ\text{C}$ ) temperatures were

**TABLE 6.7 Catastrophic Accident 500-MW (Thermal) Reactor After 180 Days**

**PERSONAL DAMAGE**

	<i>Persons</i>	<i>Conditions at release</i>
Lethal exposure		
Minimum	0	Hot release at any time
Maximum	3,400	Cold release, $1\ \mu$ particle size, temperature inversion
Injury likely		
Minimum	0	Hot release at any time
Maximum	43,000	Cold release, $1\ \mu$ particle size, temperature inversion, dry

**PROPERTY DAMAGE**

	<i>Persons</i>	<i>Area</i> <i>(sq. mi.)</i>	<i>\$ Millions</i>	<i>Conditions</i>
Evacuation				
Minimum	0	0	0	Hot temperature inversion
Maximum	460,000	760	2,300	Cold, $1\ \mu$ , temperature inversion, rain
General restrictions				
Minimum	0	0	0	Hot, $1\ \mu$ , dry
Maximum	3,800,000	8,200	2,800	$1\ \mu$ , rain
Agricultural restrictions				
Minimum		18	.5	Hot, $1\ \mu$ , day, dry
Maximum		150,000	4,000	Hot, $1\ \mu$ , day, rain

SOURCE: Reference 82.

**TABLE 6.8 Radiation Exposures Used for Theoretical Study**

Lethal exposure	Over 450 R
Injury likely	100-450 R
Injury unlikely (some expense may be incurred; observation required)	25-100 R
No injury or expense	Less than 25 R (one exposure) 50 R (in three months)

assumed. Median particle sizes of both one micron and seven microns for the releases were considered. Resultant effects were also dependent upon local conditions, that is, rainy or dry weather. The time of day was likewise important, since nighttime temperature inversions are common. The expected damages are summarized in Table 6.7. For the worst case, 3400 persons would be killed and 43,000 injured. Property damage would be nearly \$7 billion. In the study, very high radiation levels, compared to present accepted standards, were used in estimating human effects. These are summarized in Table 6.8. Assuming the dose (expressed in rem) is comparable to the exposure (expressed in roentgen), the upper radiation level for the "injury unlikely" category of 100 R is questionable. A downward revision of harmful radiation levels would result in a marked increase in the theoretical injuries. Furthermore, many commercial reactors now operate at power levels of approximately five times that considered in the WASH-740 study. Their fission product inventory is correspondingly larger and releases would also be expected to be much greater. Also, the estimates of dollar damages of Table 6.7, owing to the inflation that has occurred since 1957, should be nearly tripled (and further increased if a larger reactor is considered).

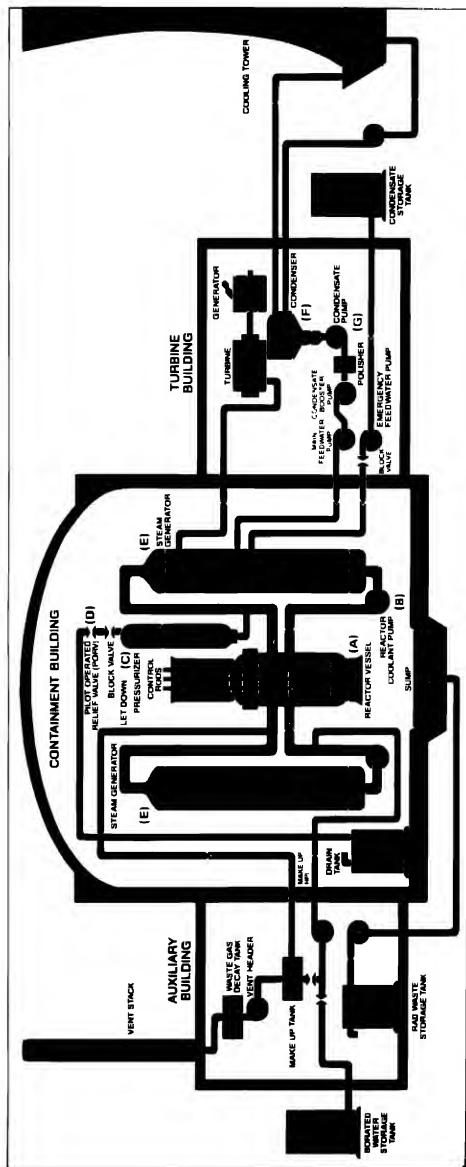
While the probability of a catastrophic accident is very small, the damage, should an accident occur, could be exceedingly great. The probable yearly cost per reactor is the product of small probability multiplied by a very high accident cost. For several reactors the following is obtained.

$$\text{Probable yearly cost of an accident} = \left( \begin{array}{c} \text{yearly probability} \\ \text{of a failure} \end{array} \right) \left( \begin{array}{c} \text{number of} \\ \text{reactors} \end{array} \right) \left( \begin{array}{c} \text{expected cost} \\ \text{of a failure} \end{array} \right)$$

The probable cost is not unlike a mathematical indeterminacy of the product of a small quantity (approaching zero) and a near infinite quantity. Proponents of reactors typically emphasize the small probability of an accident, while opponents emphasize the expected cost of a failure. Of importance, however, is the product of these quantities.

As a result of numerous questions related to the safety of nuclear reactors, the former AEC prepared two reports. The preliminary version of the first report, WASH-1250 [83], contained no reference to the 1957 WASH-740 report. As a result of reviewers' comments, a chapter was added (Chapter 8) which discussed the early report. In essence, the WASH-1250 report concluded that a catastrophic





**FIGURE 6.18 Schematic Drawing of The Three Mile Island Unit 2 Facility. Major Components Are Labeled as Follows:**

- (A) **REACTOR VESSEL:** A cylindrical vessel made of steel—40 feet high and 8½ inches thick—which contains the reactor (core and control rods) and through which the reactor coolant flows, carrying heat away from the core to the steam generators. The TMI-2 reactor contains 177 fuel assemblies with 208 fuel rods in each assembly.
- (B) **REACTOR COOLANT PUMP:** One of four pumps which move the reactor coolant through the core to the steam generators and back to the core in a closed system (the primary system) of what is normally only slightly radioactive water. About one hour into the TMI accident, the operators shut down two of these pumps because they were vibrating severely, the result of the steam in the primary system. Half an hour later they shut down the other two pumps for the same reason. At that point, damage to the fuel in the core began, causing releases of radioactive material into the coolant.
- (C) **PRESSURIZER:** A large vessel connected to the primary system between the reactor and the steam generators which is normally a little more than half full of water, with a steam bubble in the upper portion of the vessel. It is designed to keep the pressure in the reactor coolant relatively constant.
- (D) **PILOT OPERATED RELIEF VALVE:** The pressurizer relief valve located at the top of the pressurizer and designed to open automatically when primary system pressure rises to a preset level and it becomes desirable to let off steam. When pressure is back to normal, the relief valve is supposed to close by itself. At TMI-2 it failed to do so, and reactor coolant flowed through the relief valve and down to a drain tank on the floor of the containment building. This valve remained open for more than two hours.
- (E) **STEAM GENERATOR:** The large vessel in which the transfer of heat from the reactor coolant to the feedwater takes place. The transfer results in the conversion of the feedwater into steam, as it flows around tubes carrying the pressurized, core-heated coolant from the reactor. This steam is conveyed to the turbine which powers the electrical generator.
- (F) **CONDENSER:** The vessel in which the steam which has passed through the turbine is condensed to a liquid state again. The heat is removed by pipes carrying condenser water which flows to the cooling towers and back to the condenser.
- (G) **CONDENSATE PUMP:** The pump which moves the feedwater (the condensate) from the condenser to the polisher or demineralizer which cleanses the water before it flows back to the steam generator. The TMI accident began at this point in the feedwater system when plant personnel were trying to clear a line associated with the polisher and the condensate pump automatically shut down, followed by a similar “tripping” of the feedwater pump and subsequently of the turbine and the reactor.
- (Reference 95.)
- (Reprinted with Permission of the U.S. Nuclear Regulatory Commission, Courtesy of the NRC.)

accident of the type considered in WASH-740 (summarized in Table 6.7 was too unlikely to be of relevance. A review by *Science* [84] was not very reassuring.

A second report, the result of a study chaired by Rasmussen of MIT, attempted to obtain a quantitative estimate of the probability of a major accident [85]. This study was based upon an event-tree methodology in which the multitude of sequences by which an accident might occur were considered. A fault probability was assigned to each component comprising the various branches of the event tree and each branch of the tree was chosen so as to result in a radioactive release affecting the surrounding population. The health effect of each release was assessed, that is, the early fatalities and delayed radiological effects were determined. Then, for each failure mode, a branch of the event tree, the probability of the occurrence was calculated. The overall probability of an accident resulting in a given set of fatalities was then obtained by a consideration of all relevant branches. Finally, a probability profile, the probability of an accident as a function of its seriousness (expressed in fatalities) was obtained.

According to the executive summary of the report, the probability of an individual being killed by a nuclear power plant was exceedingly remote—less than that of being struck by a meteorite. The data of the main part of the report placed the likelihood considerably higher but still low (the executive summary was misleading). Numerous critics disputed the claims of this accident report [86–91]. Another accident study was done by the American Physical Society (APS). While it urged a continued effort to improve the safety of reactors, it concluded that light-water reactors were probably safe [92]. The APS authors, however, expressed the opinion that the Rasmussen report greatly overstated the certainty of its probabilistic results.

Responding to the criticisms of the WASH-1400 report, the Nuclear Regulatory Commission established an *ad hoc* review group to perform a critical peer review. The *ad hoc* group concluded that the absolute probability values of the WASH-1400 report should not be used uncritically either in the regulatory process or for public policy purposes [93]. The executive summary, in particular, had been widely used for these purposes. Subsequently, the Nuclear Regulatory Commission officially withdrew its endorsement of the WASH-1400 executive summary and stated that the numerical estimates of the overall risks of reactor accidents are unreliable [94].

The March 28, 1979, accident at the Three Mile Island Nuclear Station provides another perspective on reactor accidents [95–98]. A schematic diagram of the 890-MW (electric) unit is indicated in Figure 6.18 and the sequence of the early events of the accident are summarized in Table 6.9 [95]. Instead of a catastrophic failure of a major component, the accident developed in a relatively slow manner, exacerbated by the inappropriate actions taken by the operators. The equipment failure that triggered the accident was a pressure relief valve that failed to close. Not only did large quantities of the reactor coolant escape from the reactor but the stuck valve also resulted in a set of instrument readings that confused the operators. That the pressurizer relief valve was stuck open was not discovered until over two hours into the accident. By then, a portion of the core was severely damaged and radioactive products of the damaged fuel rods had been carried into the containment building by the leaking coolant. Had the operators realized at the beginning

of the accident that the relief valve had failed to close, the pressurizer block valve (closed two hours too late) could have been closed and the reactor brought under control. Conversely, had the valve remained open for another 30 to 60 min, a core meltdown, an event which based on previous studies had been considered exceedingly unlikely, could have occurred [99]. A precautionary evacuation of thousands would have been required. The accident progressed sufficiently far in its first few hours that those attempting to control the reactor were unsure of what was happening. Since well-understood principles no longer applied, an experimental mode of operation was required.

A special commission was appointed by President Carter to conduct a comprehensive study and investigation of the accident [100–103]. The commission report (often referred to as the Kemeny Report, after its chairman) concluded that the fundamental problems associated with operating a nuclear reactor are “people-related problems.” Equipment problems, it stressed, were secondary. The report concluded that an accident such as that of Three Mile Island was “eventually inevitable” and that if such accidents are to be prevented, fundamental changes must be made in the organizations and in the attitudes of the people involved. The Nuclear Regulatory Commission report on the accident was even more critical. It stated that similar accidents, perhaps with the potentially serious consequences to public health and safety that were only narrowly averted at Three Mile Island were likely to recur unless there are fundamental changes [99].

As a result of the Kemeny Report, several changes have occurred within the Nuclear Regulatory Commission [104]. In addition, an Oversight Committee was appointed by President Carter to monitor the progress of governmental authorities and the nuclear industry in improving safety and implementing recommendations of the Report. However, changes in attitudes, one of the more important recommendations of the Kemeny Report, do not come quickly. Hence, one can conclude only that the role of nuclear power (fission) as a major energy source is presently uncertain.

**TABLE 6.9 Sequence of Early Events of the Three Mile Island Accident (March 28, 1979)**

0 sec	Just after 4:00 a.m., a condensate pump and the main feedwater pumps (Figure 6.18) connected to one of the steam generators shut down resulting in an automatic shutdown of the turbine. This not uncommon occurrence may have been the result of a polisher maintenance procedure being carried out at the time.
3 to 8 sec	With the feedwater flow halted, a buildup of thermal energy in the primary reactor system caused the pressure of the coolant to increase. The pressurizer relief valve opened. With the overpressure, the control rods were automatically inserted into the core causing the fission reaction to cease. The heat produced by the decay of the fission products, about 7% of reactor operating power, continued.
12 sec	The primary coolant pressure dropped and the current of the solenoid which activated the relief valve was automatically turned off. The relief valve,

**Table 6.9—cont.**

	however, did not close. This was not known by the operators since the indicator on the control panel responded to the solenoid's position, not to that of the stuck valve.
38 to 40 sec	The emergency feedwater pumps were automatically activated. Since the block valves in these water lines were inadvertently left in a closed position (following an earlier maintenance procedure), the pumps had no effect. The indicator lamps for the valves were obscured by a maintenance tag.
2 min 2 sec	As a result of the leaking relief valve, the coolant pressure of the reactor core continued to decrease. When it fell to 75% of its normal value, the emergency core cooling system (ECCS) automatically injected cold water from the borated water storage tank into the reactor. The water level of the pressurizer increased. Under normal conditions (a closed relief valve), the pressurizer coolant level provides a reliable indication of the coolant level in the reactor. Hence, the operators concluded that the reactor core was full when the pressurizer level became normal. They turned off one of the ECCS pumps and throttled the flow of the other. With the ensuing drop in pressure, the reactor coolant boiled producing steam voids.
8 min	It was discovered that the block valves of the emergency feedwater pumps were closed. These valves were opened.
8 min to 2 h 20 min	Many other actions were taken over the next two hours in an attempt to gain "control" of the reactor. With the low coolant pressure, the result of the leaking relief valve, boiling occurred and a portion of the fuel rods were exposed. Many overheated, burst, and released radioactive fission products which were carried by the coolant into the containment building. Coolant water accumulated on the floor of the containment building when an overpressure seal of the drain tank ruptured. Some of the radioactive coolant water was pumped by the sump pump of the containment building into a tank in an auxiliary building. With the buildup of steam voids in the reactor coolant system, the coolant pumps malfunctioned and were shut down to avoid possible damage to them. If it were not for the steam voids, convective circulation of the coolant would have been sufficient to cool the core. Because of the high coolant level in the pressurizer the operators assumed this would occur. Hydrogen was formed by a reaction of the steam with the zirconium cladding of the overheated fuel rods. It escaped through the leaking relief valve and resulted in a hydrogen explosion in the containment building 9½ hours after the accident started.
2 h 20 min	It was finally realized that the relief valve might be stuck. The block valve on the pressurizer was closed, thus ending the coolant loss through the stuck relief valve. The reactor core, however, was already seriously damaged.
2 h 50 min	A "site emergency" was declared and local and state authorities were notified. This emergency status implied that an uncontrolled release of radiation might occur.
3 h	A "general emergency" was declared indicating a situation which could result in radioactive releases affecting public health and safety.

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## PROBLEMS

1. The energy equivalent of matter is given by the well-known equation  $E = mc^2$ .
  - (a) Calculate the quantity of matter that must be converted to energy to supply the 1979 U.S. electrical energy requirement (thermal) for one day. ( $P = 6.6 \times 10^{11}$  W).
  - (b) If  $^{235}\text{U}$  is used for a fission process, 200 MeV of energy are obtained for each atom. Determine the mass converted to energy by one  $^{235}\text{U}$  atom. What is the fraction of the atom's initial mass that is converted to energy?
  - (c) Determine the number of metric tons of  $^{235}\text{U}$  needed for the energy of part (a).

- (d) Uranium ore consists of an oxide of uranium ( $\text{U}_3\text{O}_8$ ). Only .711% of the uranium atoms are fissionable isotopes,  $^{235}\text{U}$ . The remainder are primarily  $^{238}\text{U}$  isotopes. Rich uranium ore contains by mass approximately .25% uranium oxide. Determine the number of metric tons of ore necessary for part (a).
- (e) What is the number of metric tons of coal necessary for the same energy requirement?
2. Uranium-238 is known to decay with a half-life of  $4.51 \times 10^9$  years.
- (a) Calculate the number of disintegrations per second for one kilogram of uranium-238.
- (b) In the chain of decay, one of the intermediate products is radon gas. Assuming radon gas (mass number 222) is produced at the same rate at which  $^{238}\text{U}$  decays, calculate the volume rate of production from one kilogram of  $^{238}\text{U}$  for a pressure of one atmosphere and a temperature of  $20^\circ\text{C}$ .
3. The isotope  $^{235}_{92}\text{U}$ , as  $^{238}_{92}\text{U}$ , is the first naturally occurring isotope of a unique decay chain that also terminates with lead. Plot the principal decay reactions of this chain in the fashion of Figure 6.4.
4. The naturally occurring concentration ratio of  $^{235}\text{U}$  to  $^{238}\text{U}$  is approximately 1 : 140. Assume that the concentrations of  $^{235}\text{U}$  and  $^{238}\text{U}$  were equal at an early time in the evolution of the earth. Calculate the time elapsed since this equality.
5. Assume that the production of  $^{219}_{86}\text{Rn}$  is determined primarily by the decay rate of  $^{235}_{92}\text{U}$ , while that of  $^{222}_{86}\text{Rn}$  is determined by the decay rate of  $^{238}_{92}\text{U}$ . Using a ratio of 1 : 140 for the isotopes of uranium, determine the ratio of the generation rate of the two radon isotopes.
6. Naturally occurring thorium,  $^{232}_{90}\text{Th}$ , radioactively decays in much the same manner as the uranium isotopes. Plot its decay chain for its principal decay mode (Figure 6.4).
7. Another unique radioactive decay series is that of the nonnaturally occurring isotope of neptunium,  $^{237}_{93}\text{Np}$ . Plot its decay chain.
8. A nonspreading flux of neutrons is known to have exponential relation for flux density,  $I = I_0 e^{-\sigma N x}$ . Show that the average distance that a neutron which starts at  $x = 0$  travels is equal to  $1/\sigma N$ .
9. Consider a fission event in which  $^{235}_{92}\text{U}$  splits into identical isotopes and five neutrons. What would the element and isotope be? How many more neutrons does this isotope have than the most massive naturally occurring isotope of the element? Assuming  $\beta^-$  decay for the isotope, what is the first stable isotope obtained? What is the approximate half-life of the decay?
10. A likely set of  $^{235}_{92}\text{U}$  fission products would be elements with atomic numbers of 38 and 54. What are these elements? Assuming that three neutrons are given off during the fission process and that the neutrons divide in proportion to the "natural" atomic weight of the elements, what would be the resultant isotope numbers? (Neutron quantities must obviously be rounded off to whole numbers.)

11. The product yield for the fission of uranium-235 by thermal neutrons is given in Figure 6.5. Determine the mass numbers of the four fission products that have yields of 2%. Determine the pairs of fragments that occurred from single fission events. Does the sum of the mass numbers, for the pairs, agree with what would be expected? In sufficient time, these products will probably decay by a succession of beta emissions to a stable isotope. What are the elements that are likely to result? (*Hint:* Find the elements with the lowest atomic number that have stable isotopes with the appropriate mass numbers.) Repeat for fission products that have yields of .2% and .02%.
12. The Watt equation given in the text is an approximation for the energy spectrum of fission-produced neutrons.
  - (a) Show that the likelihood of neutrons of all energies is unity.

$$\int_0^{\infty} \pi(E) dE = 1$$

- (b) Determine the portion of the fission-produced neutrons that have energies of less than 1 eV. What is the portion that have energies less than 100 eV?
  - (c) What is the portion of the neutrons that have energies in excess of 1 MeV?
  - (d) What is the average energy of the fission-produced neutrons?
13. A high-energy neutron is scattered by an elastic collision with an atom with a mass number  $A$ . The atom is initially at rest.
  - (a) The neutron is scattered at an angle of  $45^\circ$  ( $\psi$ ) from its initial line of travel. What is the ratio of the neutron's final energy,  $E$ , to its initial energy,  $E_0$ ?
  - (b) The neutron is scattered at an angle of  $90^\circ$ . What is  $E/E_0$  for this condition?
  - (c) Find  $E/E_0$  for the scattering angle of  $135^\circ$ .
  - (d) Suppose the atom is normal hydrogen. What is the numerical value of  $E/E_0$  for the angles of parts (a), (b), and (c)? What are the values of  $E/E_0$  for scattering by a carbon atom ( $A = 12$ )?
14. Assume that a particular radioactive isotope is the result of the decay of another radioactive isotope. The initial quantity of the daughter is zero and the isotopes have equal half-lives. Obtain an expression for the quantity of the daughter product,  $N_2$ , in terms of the initial quantity,  $N_{01}$ , of the first isotope. What is the decay rate,  $\lambda N_2$ , for the second isotope? At what time is this a maximum?
15. Radium,  $^{226}_{88}\text{Ra}$ , decays by emitting an alpha particle and a gamma ray with an energy of 186 keV. What is the power of the gamma-ray emission for a sample with a one-curie activity? What is the gamma-ray power density ( $\text{W/m}^2$ ) at one-meter distance from the sample? Assume the radiation is isotropic, that is, uniform in all directions.
16. The linear energy absorption coefficient for air corresponding to the photon energies of Problem 15 is approximately .0036 per meter. That is, an incident radiation decreases at the rate of .36% per meter ( $dP/dx = .0036 P$ ). Determine the power absorbed per unit mass of air at a distance of one meter from the radium source of Problem 15. What is the radiation intensity in roentgens per second?

- 17.** Table 6.6 summarizes the radioactivity and thermal power associated with decay of the longer half-life reactor waste products. What is the total activity and power of these isotopes when removed from the reactor? What are the corresponding values for 5, 10, 15, and 20 years after having been removed? Determine the fraction of the activity and thermal power due to the strontium-90 and cesium-137 for these years.
- 18.** Radioactive strontium-90 can be used as a source of thermal power for an electric power producing thermocouple converter. Assume a thermal power of 100 W is required. Determine the disintegration rate assuming the average kinetic energies of the beta emissions of the strontium and yttrium are recovered. What is the strontium-90 mass required? What will be the thermal power after one year of operation? What will it be after 10 years and after 100 years?
- 19.** During the normal operation of a light-water reactor, plutonium-239 is bred as a result of neutron absorption by uranium-238. Assume a constant conversion ratio of  $C$  and a constant fission rate of  $R$  for the reactor.
- Determine the time, expressed in units of  $N_{u0}/R$ , for which the number of plutonium-239 atoms is a maximum.
  - Evaluate these times for  $C = \frac{1}{3}, \frac{1}{2},$  and  $\frac{2}{3}$ .
  - What are, in terms of  $N_{u0}$ , the plutonium-239 quantities for the times of part (b)?
  - Assume the fuel is removed from the reactor at the times of part (b). What is the fraction of the fission-produced energy that can be attributed to the fission of plutonium-239 for each of the conversion ratios?
- 20.** Consider the 1-GW (thermal) reactor of Figure 6.17. Assume a constant conversion ratio,  $C$ , for forming plutonium-239. The initial fission inventory of uranium-235 atoms is  $N_{u0}$  while that of plutonium-239 is zero. When the fuel is removed (at five years) the total fission inventory is  $.5 N_{u0}$ . Ignore the decay of plutonium-239 within the reactor.
- Determine, for a conversion ratio,  $C$ , of .5, the number of atoms of plutonium-239 removed from the reactor. What is the mass of the plutonium? What is the activity of the plutonium ( $T_{1/2} = 24,400$  y)?
  - Repeat the problem for conversion ratios .4 and .6.
  - The decay of plutonium-239 results in a 5.1 MeV alpha particle. What is the thermal power that can be associated with the decay of the plutonium-239 of the previous parts?
- 21.** The fission products in a reactor consist of several radioactive isotopes of iodine. Iodine is gaseous at core temperatures and will, if a rupture occurs, be released. If not confined in an accident, iodine poses a significant health hazard. Consider the iodine-131 isotope with a half-life of 8.05 days and an average  $\beta$  decay energy of .19 MeV.
- What is the yield for the thermal neutron fission of uranium-235 for a mass number of 131? Assume this yield is entirely iodine-131.
  - What is the activity of iodine-131, expressed in Curies, during the operation

of a 1-GW (thermal) reactor? What is the equilibrium mass of iodine-131 during operation?

- (c) Iodine tends to be absorbed by the thyroid gland (approximate mass of 16g) of humans. Assume that a quantity of iodine-131 absorbed by a thyroid gland totally decays within the thyroid gland. Determine the activity and mass of the iodine-131 that results in a radiation dose of 170 mrem. (*Note:* The thyroid gland can be protected from the effect of radioactive iodine by a prior intake of potassium iodide. This drug serves as a blocking agent in that the thyroid is, in effect, saturated with normal iodine.)

# CHAPTER 7

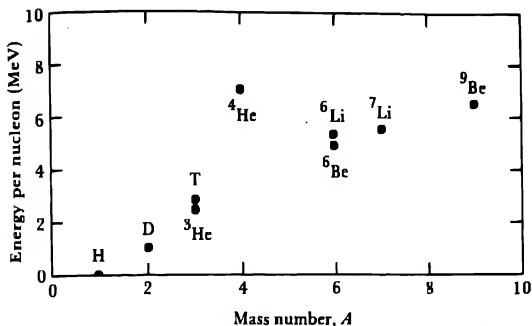
## *Fusion*

### 1. INTRODUCTION

As pointed out in the previous chapter, both the very light and the very heavy elements tend to be excessively massive. In terms of nucleon binding energies, elements at each end of the atomic spectrum have average energies less than those more centrally positioned with mass numbers between 50 and 200. Binding energies for a few of the very light elements are given in Figure 7.1 and Table 7.1. Since the common isotope of helium,  ${}^4_2\text{He}$ , has a relatively high binding energy, fusion reactions involving isotopes of hydrogen, which result in the formation of  ${}^4_2\text{He}$ , are accompanied by a considerable energy release.

Efforts in the United States to construct a controlled fusion reactor were initiated by the Atomic Energy Commission in 1951. An excellent description of the then classified research, "Project Sherwood," is provided by Bishop [1]. The early developmental work can be characterized by the discovery of numerous types of plasma instabilities. Neither the very high temperatures nor the required confinement times necessary to initiate a fusion reaction were even remotely approached, since the behavior of high temperature plasmas proved to be considerably more complex than initially anticipated.

The tokamak, developed in the U.S.S.R. in the 1960s, is considerably more stable than earlier plasma devices. Development efforts in both the Soviet Union and the United States have been directed toward improving this device. As a result, a plasma temperature approaching that required for fusion was achieved in the late 1970s using a beam of energetic neutral particles to heat the plasma of a tokamak.



**FIGURE 7.1** Average Binding Energy of the Light Isotopes.

Many technical hurdles, however, must yet be overcome to achieve an energy-producing fusion reaction. Reports by the media, unfortunately, often misinterpret technical results and give the impression that fusion energy is imminent [2]. In addition to the magnetic fusion program, there is an effort to initiate a fusion reaction with high-power lasers.

While controlled fusion reaction is yet to be achieved, it now appears that its feasibility may soon be demonstrated.

The hydrogen bomb is an uncontrolled fusion reaction, triggered by a fission device (an atomic bomb). For the fast reacting bomb, the inertia of the hydrogen isotopes holds them together. Even though the fusion reactions of the stars and the sun occur on a very immense scale, the rate per unit of mass at which these reactions occur is exceedingly slow. Stellar gravitational forces suffice to hold the reactants together. To obtain useful quantities of energy for conventional energy needs, a reaction rate intermediate between that of a bomb and that of stellar reactions is necessary.

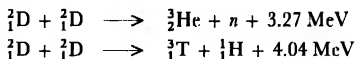
**TABLE 7.1** Binding Energies of Isotopes Involved in Fusion Reactions

	Energy per nucleon (MeV)	Total energy (MeV)
Hydrogen <sup>1</sup> H	0	0
<sup>2</sup> D	1.112	2.225
<sup>3</sup> T	2.827	8.482
Helium <sup>3</sup> He	2.573	7.718
<sup>4</sup> He	7.074	28.296
Lithium <sup>6</sup> Li	5.332	31.993
<sup>7</sup> Li	5.061	39.245

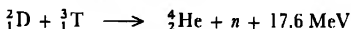


Predictions as to when fusion energy will be available vary. Thirty years elapsed between the operation of the first fission reactor in 1942 and the production of even moderate quantities of nuclear fission-derived electricity by utilities. Projections for the United States do not foresee any installed capacity before the end of the century [3]. While controlled fusion may be an ultimate source of energy, its development will very likely have no effect on short-term twentieth-century problems.

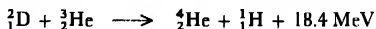
The interest in fusion stems from the high abundance and low separation costs of the isotopes intended for fuel. Reactions of interest involve isotopes of hydrogen. The common isotope of hydrogen,  ${}^1_1\text{H}$ , has a single proton nucleus. Deuterium, a hydrogen isotope with one nuclear neutron, has a natural occurrence of .015%, whereas tritium, a nonnaturally occurring isotope, has two neutrons. Even though these are isotopes of hydrogen, special symbols are commonly used for the isotopes:  ${}^2_1\text{D}$  (same as  ${}^2_1\text{H}$ ) for deuterium and  ${}^3_1\text{T}$  ( ${}^3_1\text{H}$ ) for tritium. The nuclides themselves are often referred to as deuterons and tritons. Two deuterium-deuterium (D-D) reactions with approximately equal probabilities for anticipated conditions for controlled fusion are important.



For both reactions, the binding energy of the two deuterons is 4.445 MeV (Table 7.1), whereas the energy of the products,  ${}^3_2\text{He}$  (7.718 MeV) and  ${}^3_1\text{T}$  (8.42 MeV), is considerably greater. The energy yield, which is in the form of kinetic energy of the resultant particles, is the difference of the respective binding energies. Both the reaction products, helium and tritium, can also react with deuterium, but the reaction involving  ${}^3_1\text{T}$  is the one with the greatest likelihood of occurring.

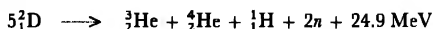


Again, the kinetic energy of the resultant particles can be predicted from the change in binding energies. The fusion reaction of  ${}^2_1\text{D}$  and  ${}^3_2\text{He}$  (a rare isotope of helium) is much less probable, owing to the two nuclear charges of helium.



For this reaction to occur, a coulomb repulsive force twice that for a reaction involving hydrogen isotopes must be overcome.

The results of the two D-D reactions and the D-T reaction may be combined in a single equation.

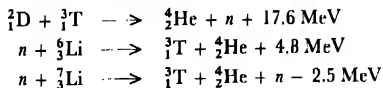


On the average, each deuterium isotope accounts for one-fifth of the resultant energy; that is, 4.98 MeV. Compared with the 200-MeV fission energy, this energy may seem rather small, but on a per unit mass basis the fusion energy of deuterium is nearly three times the fission energy of uranium or plutonium. Since water is the most abundant source of hydrogen, it is also the most abundant source of deuterium. Each kilogram of water contains .112 kg or .111 kg-moles of hydrogen. Multiplying by Avogadro's number, this yields  $6.7 \times 10^{25}$  atoms of hydrogen for each kilogram of

water: .015%, or  $10^{22}$  of these atoms are deuterium. The energy equivalent of these atoms (4.98 MeV per atom) is  $8 \times 10^9$  J or 254 Wy. A fusion reactor using the deuterium atoms of one kilogram of water will produce an energy equivalent of 270 kg of coal or 61 gal of gasoline. Fusion can thus be figuratively spoken of as "burning" water. Inasmuch as the  ${}^2_1\text{D}$  that is removed from one kilogram of water is a nearly negligible quantity (.03 g), very little of the water is actually "burned." One cubic kilometer or  $10^9$  m<sup>3</sup> of water contains  $10^{12}$  kg. Based upon the above fusion reaction, this volume of water contains sufficient  ${}^2_1\text{D}$  to produce an energy of  $8 \times 10^{21}$  J ( $2.54 \times 10^{14}$  Wy). For a world consumption rate of  $8.1 \times 10^{12}$  W (1978), this amounts to 31 years' supply. Since the oceans of the world contain approximately  $1.5 \times 10^9$  km<sup>3</sup> of water, they represent a deuterium energy reserve, based upon the present rate, of 47 billion years. Obviously, only an extremely minute fraction of the ocean's deuterium need be removed to provide any foreseeable energy needs.

The separation of  ${}^2_1\text{D}$  from  ${}^1_1\text{H}$  is considerably easier and less costly than the enrichment of uranium. The deuterium isotope is twice as massive as normal hydrogen, whereas the ratio for the uranium isotopes ( ${}^{235}_{92}\text{U}$  and  ${}^{238}_{92}\text{U}$ ) is only 1.013. Large quantities of  $\text{D}_2\text{O}$ , heavy water, are currently being produced to moderate fission reactors. Separation is based upon a water-hydrogen exchange reaction which relies on a mass-dependent exchange rate. Even for a heavy water cost of \$90/lb, the separation cost of a quantity of deuterium equal to that contained in one kilogram of ordinary water is only three cents.

While from a resource perspective the deuterium-deuterium reaction is highly desirable, early reactors will probably rely on a more readily achievable reaction of tritium and deuterium. Since the naturally occurring quantities of tritium are negligible, tritium must, as part of the overall process, be produced from other materials. Most fusion proposals envision utilizing the high-energy neutron of the D-T reaction to breed tritium from lithium.



Both naturally occurring isotopes of lithium are included in the above reactions:  ${}^6_3\text{Li}$  has a natural isotopic occurrence of 7.4%, while that of  ${}^7_3\text{Li}$  is 92.6%.

The energy-producing reaction of lithium-6 has a particularly large cross section for thermal neutrons, on the order of 950 barns. If each neutron of the D-T reaction reacts with one lithium-6 atom, not only will a tritium atom be produced to replace the initial reactant but an additional energy of 4.8 MeV will also be released. The reaction with the more abundant isotope, lithium-7, due to its small cross section, is much less likely to occur. In addition, since the reaction energy is negative, only neutrons with an energy in excess of 2.5 MeV (the neutron produced by the D-T reaction has an energy of 14.1 MeV) can result in this reaction. The process does, however, result in an additional neutron which may, if it is not lost, react with lithium-6.

The viability of a D-T reaction in which lithium is used to breed tritium is

dependent upon the availability of lithium. Naturally occurring lithium will probably be used for a "breeding blanket" to capture the high-energy fusion neutrons. Since the lithium-6 reaction has the greatest likelihood of occurring, the reaction will be considered to estimate lithium requirements. Based upon an energy of 22.4 MeV per lithium-6 atom (17.6 MeV for the D-T reaction plus 4.8 MeV for the  $n$ - ${}^6\text{Li}$  reaction), one kilogram of  ${}^6\text{Li}$  will result in an energy of  $3.6 \times 10^{14}$  J.

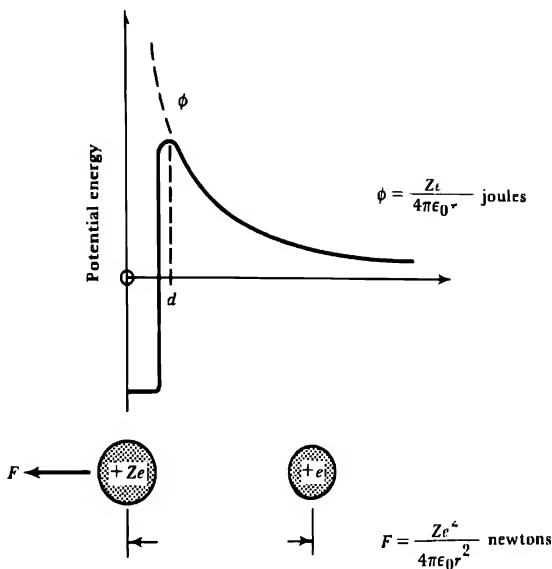
The total 1979 energy rate for the United States ( $2.6 \times 10^{12}$  W) therefore would correspond to a lithium consumption rate of  $3.1 \times 10^3$  metric tons per year. Lithium is obtained from rare rock deposits, pegmatites, from salts of saline lakes, and can, if the need is sufficiently great, be obtained from sea water. The rock and salt reserves of the United States are estimated to contain approximately  $9 \times 10^6$  metric tons of lithium metal [4, 5]. These reserves are sufficient, assuming a D-T- ${}^6\text{Li}$  reaction, to supply all the energy for the United States, based upon the 1979 consumption rate, for 2900 years. The reserve is increased to the extent that lithium-7 breeding increases the tritium yield. World reserves based upon the lithium content of sea water, however, increase the availability of lithium by a factor of 10,000. Lithium reserves, even excluding sea water extraction, are therefore more than sufficient for the foreseeable future.

Once viable fusion reactors utilizing the D-T reaction are developed, it would seem likely that the development of a D-D reactor would soon follow. Fuel costs for both lithium and deuterium will probably be negligible. Unfortunately, controlled fusion reactors, because of the difficulty of obtaining conditions necessary for a reaction, will be extremely complex and hence expensive. Electricity produced by fusion may, as a result, be no less costly than that provided by fossil fuels.

## 2. REQUIREMENTS FOR A THERMONUCLEAR FUSION REACTION

Fusion reactions require bringing together positively charged nuclides. In order to accomplish this, the coulomb repulsive force, as illustrated in Figure 7.2, must be overcome. The potential energy,  $\phi$ , is the energy required to bring a charge of  $+e$  from infinity to within a distance  $r$  of a nucleus with a charge  $+Ze$ . For conditions corresponding to those necessary for a fusion reaction, the orbital electrons have been removed from the nucleus.

The forces that hold nucleons together are extremely short-range forces. Only when nucleons are, in a classical sense, "touching," do the attractive forces exceed the coulomb repulsive forces. A potential profile of Figure 7.2 in which  $d$  corresponds to the approximate center to center separation of the nuclei when touching is obtained. Based upon this model, a fusion reaction requires nuclides with a kinetic energy sufficient to overcome the coulomb potential barrier. At Rutherford's urging in 1930, Cockcroft and Walton constructed the first proton accelerator in order to initiate a fusion reaction. Their accelerator consisted of a battery of condensers which were initially charged while connected in parallel by a high-potential source and then reconnected in series to achieve accelerating potentials as high as one



**FIGURE 7.2** *The Coulomb Repulsive Force and Potential Energy of a Nucleus.*

million volts. Protons and deuterons with kinetic energies of 1 MeV were then available for experiments.

When a lithium target was bombarded by a high-energy proton, two alpha particles, that is, two helium nuclei, were produced.



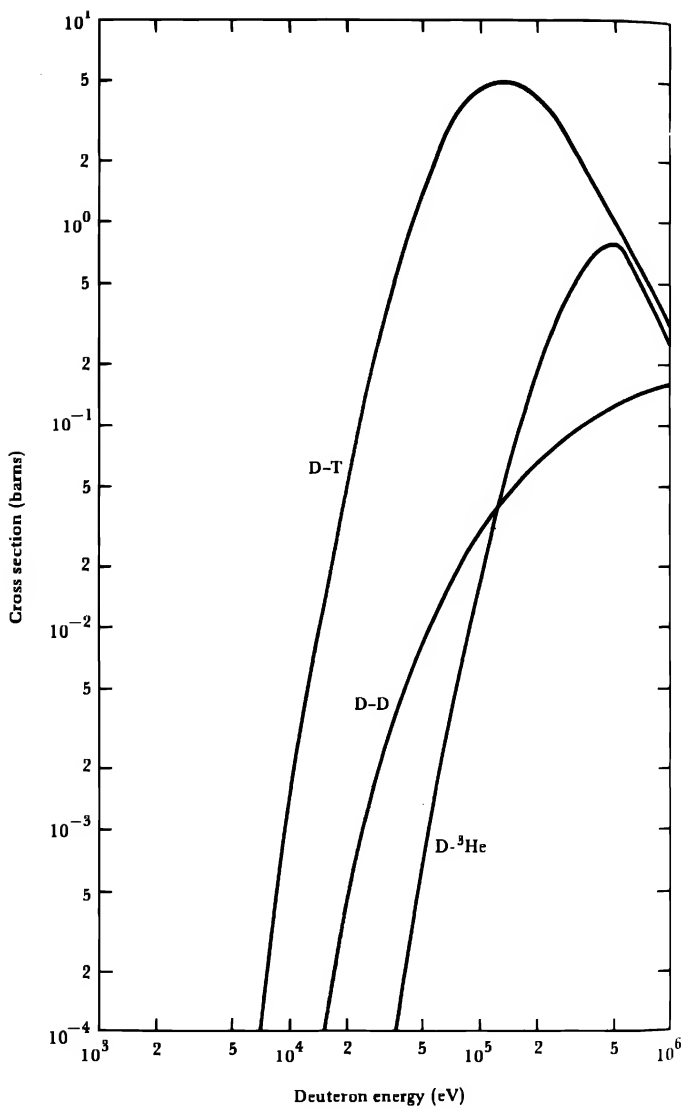
Since the average nuclide binding energy of  ${}^7_3\text{Li}$  is considerably less than that of  ${}^4_2\text{He}$ , very energetic and readily detectable alpha particles resulted. Based upon purely classical considerations, an energy corresponding to the coulomb potential for touching nuclei should be needed by the bombarding proton. Assuming a uniform nuclear density, the radius of a nucleus is proportional to the cube root of the mass number  $A$ .

$$R = R_0 A^{1/3}$$

The proportionality constant,  $R_0$ , is the approximate radius of a proton ( $A = 1$ ).

$$R_0 \simeq 1.5 \times 10^{-15} \text{ m}$$

Since the mass number of  ${}^7_3\text{Li}$  is 7, its radius is approximately  $3 \times 10^{-15} \text{ m}$ . The center-to-center separation for a touching proton is therefore about  $4.5 \times 10^{-15} \text{ m}$



**FIGURE 7.3** Cross Section for Important Fusion Reactions (Reference 19).

which corresponds to a potential energy of  $1.53 \times 10^{-13}$  J, or, .96 MeV. Hence it was not surprising that fusion reaction occurred for 1-MeV protons. It was surprising, however, that the reaction occurred for protons with energies considerably less than 1 MeV. A quantum effect, barrier penetration, accounted for the interaction. A proton, with an energy considerably less than the peak coulomb barrier potential, has a finite probability of crossing the barrier (a tunneling effect).

Shortly after observing the lithium reaction, both the D-D and D-T fusion reactions were observed by bombarding a target containing deuterium or tritium isotopes with high-energy deuterons. Even less energy was required for these reactions, since the target atoms had only a single nuclear charge. Unfortunately, very few of the bombarding nuclei (typically less than one in  $10^4$  or  $10^5$ ) resulted in an energy releasing fusion reaction. Most of the incident particles lost their energy through the ionization of the target atoms. Owing to the low fusion probability of a high-energy deuteron, considerably more energy is expended in accelerating deuterons than is released through the few fusion reactions achieved.

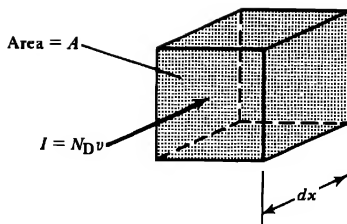
Efforts to produce useful quantities of fusion power have, as a result, been directed toward developing thermally produced reactions—thermonuclear reactors. At the temperatures necessary to achieve particle kinetic energies sufficient for fusion, the constituents will be in the form of a fully ionized plasma consisting of positively charged nuclei and free electrons. While energy exchanges occur due to exchange interactions (collisions) between nuclei, energy, except for the radiative effects of high-energy electron-nuclei encounters, is conserved. Several texts [6-11] as well as many excellent review articles [12-18] provide a general discussion of thermonuclear fusion.

For a mixture of fusion reactants, the reaction rate depends upon the fusion cross section [19]. The D-T reaction of Figure 7.3 may be seen to have the largest cross section for the minimum deuteron energy. A deuteron energy of 40 keV is necessary for a cross section of one barn, while the maximum cross section for the reaction of 5 barns occurs at an energy of just over 100 keV. Equating the kinetic energy of 40 keV to  $\frac{3}{2}kT$  corresponds to a temperature of  $3 \times 10^8$  K. At lower energies, the cross section of the D-T reaction is approximately one hundred times greater than that of the D-D or D- $^3\text{He}$  reaction. Therefore, for a given temperature, the likelihood of achieving a D-T reaction is one hundred times greater than that of the other two reactions. Conversely, a much higher plasma temperature is necessary to achieve a D-D or D- $^3\text{He}$  reaction. Since, at least at present, the conditions to achieve a D-T reaction appear sufficiently challenging, the ensuing discussion will be primarily concerned with aspects of the D-T reaction.

The concept of cross sections has already been discussed relative to neutron interactions in Chapter 6. Consider the D-T interactions in which a flux of deuterons bombards a tritium target of  $N_T$  nuclei per cubic meter (Figure 7.4). If  $\sigma$  is the cross section for the reaction, the flux density of the bombarding deuterons,  $I$ , has the following exponential behavior.

$$I = I_0 e^{-N_T \sigma x} \quad \text{deuterons/m}^2\text{-s}$$

$$dI = -N_T \sigma I_0 e^{-N_T \sigma x} dx = -N_T \sigma I dx$$



**FIGURE 7.4** Volume for a Fusion Reaction of a Flux of Deuterons.

where  $x$  = distance,  $m$   
 $\sigma$  = cross section,  $m^2$   
 $N_T$  = density of tritons,  $m^{-3}$

The number of fusion reactions occurring in a differential distance  $dx$  is thus  $N_T \sigma I dx$ . If the incident beam has a density of  $N_D$  deuterons per cubic meter, the flux crossing an area of  $A$  square meters is  $N_D v A$ , in which  $v$  is the velocity of the deuterons. The rate of the fusion reaction is thus the following.

$$\text{Fusion rate} = N_T \sigma N_D v A dx$$

Dividing by the volume,  $A dx$ , gives the fusion rate per unit volume.

$$\text{Volume fusion rate} = N_T N_D \sigma v \quad \text{number}/m^3 \cdot s$$

Similar relations can be written for the D-D and D- $^3\text{He}$  reactions by substituting the appropriate subscripts and using the appropriate value of  $\sigma$ .

Thermal equilibrium of the particles implies a Maxwellian velocity distribution in which the relative velocity of the reacting nuclei is determined by the temperature. Since the cross section,  $\sigma$ , is also velocity-dependent, an average value of the product,  $\sigma v$ , determines the fusion rate. For a discussion of the mathematical approximations utilized and the result,  $\langle \sigma v \rangle_{av}$ , the reader is referred to available materials [9, 20, 21]. Figure 7.5 is a plot of  $\langle \sigma v \rangle_{av}$ . Knowing this temperature-dependent quantity, the volume fusion rate for a thermonuclear reaction is obtained.

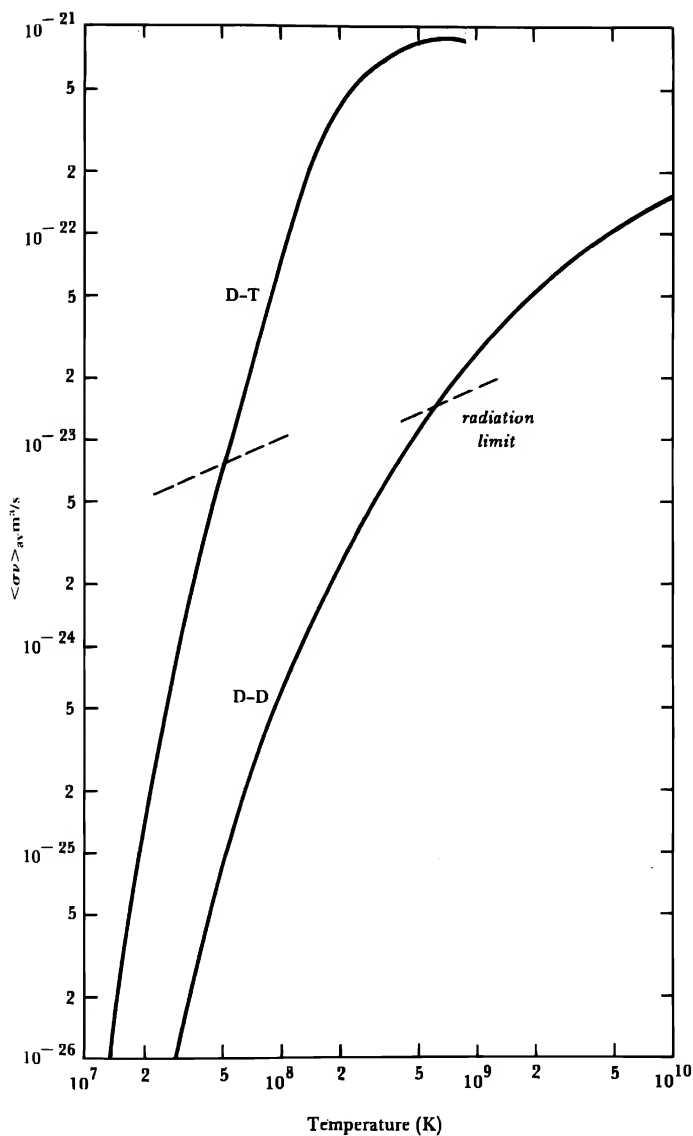
$$\text{Volume fusion rate} = N_T N_D \langle \sigma v \rangle_{av}$$

The volume rate of energy release or power,  $P_F$ , is the rate multiplied by the energy per fusion event,  $U_{DT}$  ( $2.8 \times 10^{-12}$  J, which corresponds to 17.6 MeV for a D-T reaction).

$$P_F = N_T N_D \langle \sigma v \rangle_{av} U_{DT} \quad W/m^3$$

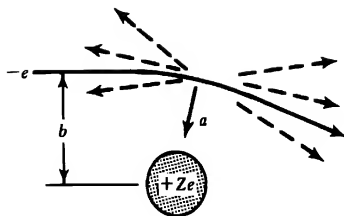
A suitable modification of the subscripts and a substitution of the appropriate value of  $\langle \sigma v \rangle_{av}$  yield the power for other fusion reactions.

An unavoidable plasma energy loss is that of bremsstrahlung radiation (decelera-



**FIGURE 7.5** Thermonuclear Reaction Rates,  $\langle \sigma v \rangle_{av}$  (Reference 21).





**FIGURE 7.6** *Radiation of an Accelerated Electron.*

tion radiation). The most significant contribution to this loss is due to electrons being deflected by close encounters with nuclei (Figure 7.6). Owing to the attractive force between an electron and a nucleus, the electron is accelerated toward the nucleus. The radiated power depends upon the acceleration,  $a$ , and is given by the following expression [22].

$$p_r = \frac{Ze^2 a^2}{6\pi\epsilon_0 c^2} \text{ W}$$

where  $\epsilon_0$  = permittivity of free space,  $8.854 \times 10^{-12}$  F/m  
 $c$  = velocity of light,  $3 \times 10^8$  m/s  
 $a$  = acceleration, m/s

The acceleration (a function of time) depends upon the electron's velocity as well as upon its impact parameter,  $b$ .

For an ionized gas, all electron-ion encounters must be considered. To obtain the total electron radiated power, an integration over all velocities and impact parameters must be carried out. The result given by Spitzer [23] is dependent upon the square root of the electron's temperature ( $Z = 1$ ).

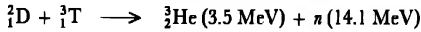
$$P_B = bn^2 \sqrt{T_e} \text{ W/m}^3$$

where  $b = 1.4 \times 10^{-40} \text{ Wm}^3/\text{K}^{1/2}$   
 $n$  = electron density (= ion density),  $\text{m}^{-3}$   
 $T_e$  = electron temperature, K

Even though ions experience an acceleration in electron-ion and ion-ion encounters, their acceleration, due to their larger mass, is much less than that of electrons. Also, for nonrelativistic velocities, the radiation due to individual ions involved in an ion-ion encounter tends to cancel. Bremsstrahlung radiation is in the form of x rays; the greater the temperature, the shorter the wavelength of the radiation. For a temperature of  $10^8$  K, the maximum spectral density of the radiation occurs at a wavelength of approximately one angstrom ( $10^{-10}$  m).

While the design of fusion reactors anticipates the recovery of the bremsstrahlung radiation power, the radiation represents an important plasma power loss. For a

steady-state operating reactor, the fusion power must be greater than this loss if the plasma is to maintain itself. Unfortunately, not all the energy of a fusion reaction,  $U_{DT}$  for a D-T reaction, is available to sustain the plasma. A substantial portion of it, 14.1 MeV, is in the form of the neutron's kinetic energy. The kinetic energy of  ${}^3_2\text{He}$  is only 3.5 MeV. Only  ${}^3_2\text{He}$ , a charged particle, will interact with the plasma.



The power available for maintaining the plasma,  $P_F^*$ , is thus only that corresponding to the kinetic energy of the  ${}^3_2\text{He}$ , namely,  $U_{DT}^*$ .

$$P_F^* = n_D n_T \langle \sigma v \rangle_{av} U_{DT}^*$$

For the plasma to be self-maintaining, the generated power,  $P_F^*$ , must be at least equal to the radiation loss,  $P_B$ . Since  $\langle \sigma v \rangle_{av}$  increases with temperature more rapidly than  $P_B$ , a minimum plasma temperature is implied that corresponds to  $P_B = P_F^*$ . Assuming an equal mixture of deuterons and tritons, the following, in which the electron temperature,  $T_e$ , is assumed to be equal to the ion temperature,  $T$ , is obtained.

$$n_D = \frac{1}{2}n$$

$$P_F^* = P_B$$

$$\frac{1}{4}n^2 \langle \sigma v \rangle_{av} U_{DT}^* = bn^2 \sqrt{T}$$

$$\langle \sigma v \rangle_{av} = \frac{4b\sqrt{T}}{U_{DT}^*}$$

Since both  $P_F^*$  and  $P_B$  are proportional to  $n^2$ , a temperature independent of density is obtained. The right-hand side of the above relation which is proportional to  $\sqrt{T}$  is indicated in Figure 7.5 (radiation limit). The intersection with  $\langle \sigma v \rangle_{av}$  occurs at the minimum temperature, a quantity often referred to as the *ignition temperature*. For a D-T reaction, this is approximately  $4.6 \times 10^7$  K, while the corresponding temperature for a D-D reaction is  $4 \times 10^8$  K.

Frequently plasma temperatures are expressed in electronvolts. An energy is defined as equal to  $kT$ .

$$eV = kT$$

$$V = \frac{k}{e}T = 8.62 \times 10^{-5} T$$

where  $V$  = temperature in electronvolts

$T$  = temperature in Kelvin

A temperature of  $4.6 \times 10^7$  K, corresponding to the ignition temperature of a D-T mixture, is thus approximately 4 keV. That required for the D-D reaction is approximately 35 keV.

While a steady-state reactor would be desirable for energy conversion applications, the difficulties associated with maintaining a stable high-temperature plasma have led to pulsed experimental systems. For such systems, minimum conditions based upon an overall energy balance (often referred to as the *Lawson criteria*), are important. While this criteria is based upon an ideal system, it sets a minimum goal for thermonuclear reactors and hence provides a comparison for judging experimental results. Once this goal is achieved, additional improvements will be needed to produce a useful output power.

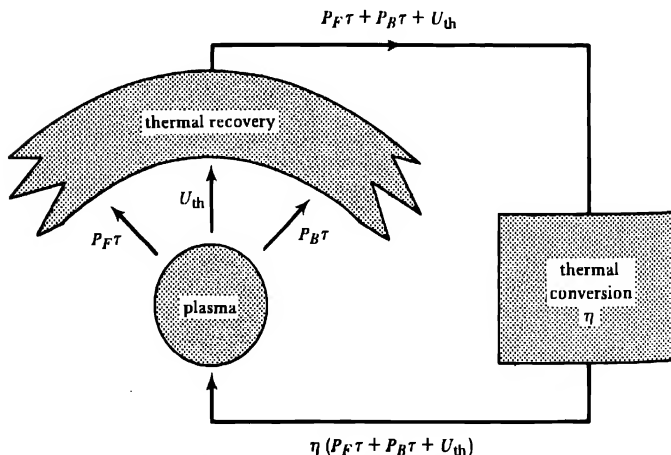
A D-T fusion reaction which lasts for a duration of  $\tau$  seconds is assumed to give up energy to a thermal recovery system with an overall efficiency of  $\eta$ . To simplify the analysis, it is assumed that only a small quantity of the reactants undergo a fusion reaction and that the temperature remains constant for the duration of the reaction. The fusion energy per unit volume is thus  $P_F\tau$ . Assuming equal quantities of deuterons and tritons, the following is obtained.

$$P_F\tau = \frac{1}{4}n^2\langle\sigma v\rangle_{av}U_{DT}\tau \quad \text{J/m}^3$$

The bremsstrahlung energy loss is  $P_B\tau$ .

$$P_B\tau = bn^2\sqrt{T}\tau \quad \text{J/m}^3$$

Thermal equilibrium of electrons with ions is assumed to exist. Individual electrons and ions have an average kinetic energy equal to  $\frac{3}{2}kT$ . The total energy per unit volume is thus the sum of the energies of the electrons and ions



**FIGURE 7.7** *Energy Balance for a Finite Duration Thermonuclear Reaction.*

$$U_e = \frac{3}{2}nkT \quad (\text{electrons})$$

$$U_i = \frac{3}{2}nkT \quad (\text{ions})$$

$$U_{th} = 3nkT \quad (\text{electrons and ions}) \quad J/m^3$$

Energy must be supplied to both heat the plasma,  $U_{th}$ , and provide for the bremsstrahlung radiation. To establish a break-even condition, the input energy is assumed to be derived from the plasma output, as indicated in Figure 7.7. An overall energy balance may be written.

$$\eta(P_F\tau + P_B\tau + U_{th}) = U_{th} + P_B\tau$$

$$\eta \left[ \frac{1}{4}n^2 \langle \sigma v \rangle_{av} U_{DT}\tau + bn^2 \sqrt{T}\tau + 3nkT \right] = 3nkT + bn^2 \sqrt{T}\tau$$

The left-hand side is the thermally recoverable plasma energy, and the right-hand side is the energy which needs to be supplied to the plasma to initiate and maintain the reaction.

Dividing the energy balance by  $n$  and rearranging the terms results in an expression for  $n\tau$ .

$$n\tau \left[ \frac{1}{4} \langle \sigma v \rangle_{av} U_{DT}\eta - b\sqrt{T}(1 - \eta) \right] = 3kT(1 - \eta)$$

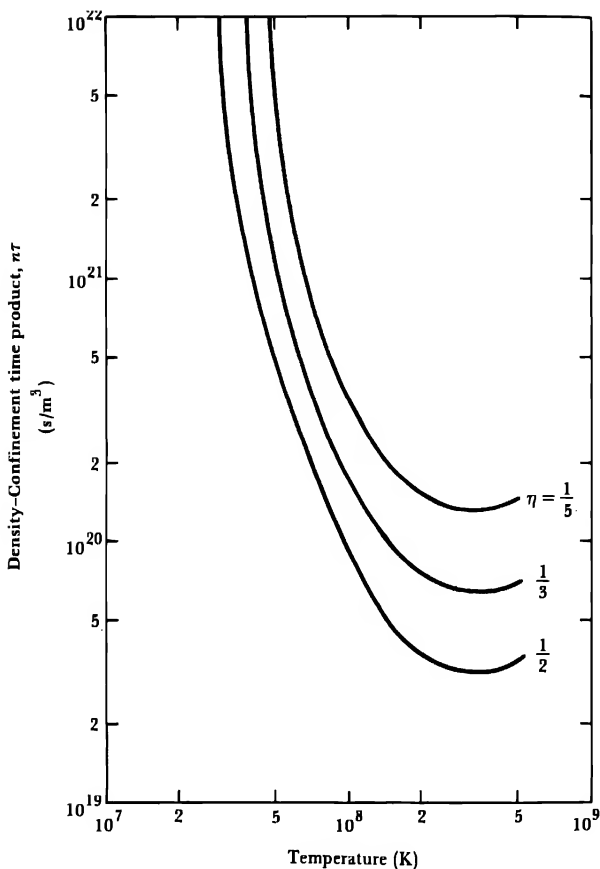
$$n\tau = \frac{3kT(1 - \eta)}{\frac{1}{4} \langle \sigma v \rangle_{av} U_{DT}\eta - b\sqrt{T}(1 - \eta)}$$

The expression for  $n\tau$ , as may be seen from Figure 7.8, is a function of both the temperature and the efficiency of the thermal recovery system. A value of  $n\tau$  on the order of  $10^{20}$  sec/cm<sup>3</sup> or  $10^{14}$  sec/cm<sup>3</sup> ( $\eta = \frac{1}{2}$ ) is required for a temperature on the order of 100 million Kelvin. The often quoted Lawson criteria is that  $n\tau = 10^{14}$  sec/cm<sup>3</sup> [24]. A more general result, in which additional fuel is provided and heated during the confinement period, is given by Mills [25]. For this condition, the curves of Figure 7.8 form closed contours.

Experimental plasma devices have been characterized by small values of  $n$  and yet to be achieved confinement times. For  $n = 10^{15}$  cm<sup>-3</sup>, a confinement time of .1 sec is necessary. High-temperature plasmas with densities much greater than  $10^{15}$  cm<sup>-3</sup> appear unlikely, owing to the excessively high pressures necessary. Laser experiments which rely on inertial confinement involve confinement times on the order of one nanosecond ( $10^{-9}$  sec). Densities of  $10^{23}$  cm<sup>-3</sup> corresponding to that of liquid hydrogen are thus required. These two schemes are the subject of the following two sections.

### 3. MAGNETICALLY CONFINED PLASMA FUSION DEVICES

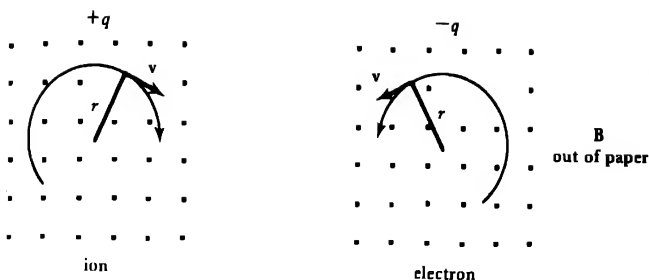
The extremely high temperatures ( $\approx 10^8$  K) necessary for a thermonuclear deuterium-tritium reaction negate the usage of a conventional confinement vessel.



**FIGURE 7.8** The Variation of  $n\tau$  with Temperature.

With such a container, the energy loss of the ions and electrons at the walls would be excessively large and so prevent the achievement of required temperatures. Electric fields are also unsuited for confinement, since electrons and ions, having opposite polarity charges, will tend to move in opposing directions.

Forces that arise from magnetic fields are suitable for containing a plasma. For



**FIGURE 7.9** *Circular Motion of a Charge in a Uniform Magnetic Field.*

a charge,  $q$ , the force in newtons is given by the cross product relationship already discussed (in Section 4 of Chapter 5).

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

If an electron or ion has a velocity parallel to the magnetic field,  $\mathbf{v} \times \mathbf{B} = 0$  and the resultant force is zero. A velocity component perpendicular to the field results in a force which is perpendicular to both the velocity component and the field. For a uniform magnetic field, circular motion results (Figure 7.9). The radial centrifugal force necessary for circular motion is provided by the magnetic field.

$$F_r = qvB = \frac{mv^2}{r}$$

Solving for the radius of gyration,  $r$ , the following is obtained.

$$r = \frac{mv}{qB}$$

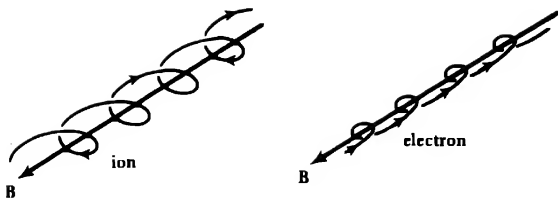
A quantity often of interest is the angular rotational rate,  $\omega$ .

$$\begin{aligned} v &= r\omega \\ \omega &= \frac{v}{r} = \frac{qB}{m} \quad \text{rad/sec} \end{aligned}$$

Converting this to revolutions per second,  $f$ , results in a quantity referred to as the *cyclotron frequency*.

$$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m} \quad \text{cycles/sec (or Hz)}$$

For a field of one tesla ( $1 \text{ Wb/m}^2$  or  $10^4$  gauss), electrons have a cyclotron frequency of  $2.8 \times 10^{10}$  Hz, a frequency corresponding to that of microwave radiation (a wavelength of approximately 1 cm). Since a deuteron is about 3700 times more



**FIGURE 7.10 Helical Motion of an Ion and Electron in a Magnetic Field.**

massive than an electron, its cyclotron frequency for the same field is only  $7.6 \times 10^6$  Hz and that of a triton is  $5.1 \times 10^6$  Hz.

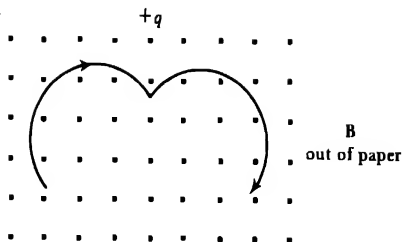
The radius of gyration depends upon the particle's velocity and hence upon its kinetic energy. A velocity corresponding to the average energy of an electron or ion may be obtained by equating its kinetic energy,  $\frac{1}{2}mv^2$ , to  $\frac{3}{2}kT$ . The resultant velocities and the corresponding radii of gyration for a temperature of  $10^8$  K and a magnetic field of 1 T are tabulated in Table 7.2. Even for very intense fields, the radius of gyration for the ions is on the order of one centimeter.

In general, an ion or electron will have a velocity vector which can be resolved into two components: one parallel and the other perpendicular to the magnetic field. Since the motion parallel to the field line will be unaffected by the field, the resultant trajectory is a helical motion about the field line (Figure 7.10). It is this motion about the field line that tends to confine ions and electrons to a trajectory determined by the magnetic field.

Electrostatic interactions with other plasma constituents tend to modify the behavior of individual particles. An electron or ion is deflected by close encounters with electrons and ions, a phenomenon corresponding to that of collisions between uncharged particles. As indicated in Figure 7.11, this results in a migration of ions and electrons across field lines, the rate of migration depending upon the ratio of the collision and cyclotron frequencies.

**TABLE 7.2 Magnetic Effects in a Deuterium-Tritium Plasma ( $T = 10^8$  K;  $B = 1$  T)**

	<i>Electron</i>	<i>Deuteron</i>	<i>Triton</i>
Cyclotron frequency	$2.8 \times 10^{10}$ Hz	$7.6 \times 10^6$ Hz	$5.1 \times 10^6$ Hz
Velocity (energy of $\frac{3}{2}kT$ )	$6.6 \times 10^7$ m/sec ( $v/c = .22$ )	$1.1 \times 10^6$ m/sec	$.9 \times 10^6$ m/sec
Radius of gyration	.38 mm	2.4 cm	2.9 cm



**FIGURE 7.11** *The Effect of a Collision on the Motion of an Ion in a Magnetic Field.*

An ordered motion or drift of plasma ions and electrons results in a current. Using Maxwell's equations, the interaction of the plasma current and the magnetic field may be obtained.\* If  $\mathbf{v}_i$  and  $\mathbf{v}_e$  are the average drift velocities of ions and electrons respectively (the thermal velocities have a zero vector average), the following forces per unit volume are obtained.

$$\mathbf{F}_i = ne[\mathbf{E} + \mathbf{v}_i \times \mathbf{B}] \quad (\text{ions})$$

$$\mathbf{F}_e = -ne[\mathbf{E} + \mathbf{v}_e \times \mathbf{B}] \quad (\text{electrons}) \quad n/m^3$$

The densities of the ions and electrons, assuming charge neutrality, are equal ( $n$ ). Adding the above forces yields an expression for the total force per unit volume.

$$\mathbf{F}_i + \mathbf{F}_e = ne(\mathbf{v}_i - \mathbf{v}_e) \times \mathbf{B}$$

The force on the ions and electrons due to the electric field cancels. If an equilibrium condition exists, the above force must equal that which arises from a pressure differential or gradient,  $\nabla p$ .

$$ne(\mathbf{v}_i - \mathbf{v}_e) \times \mathbf{B} = \nabla p$$

A one-dimensional pressure gradient for a differential volume is illustrated in Figure 7.12. The current density,  $\mathbf{J}$ , is related to the velocity terms.

$$\mathbf{J} = ne(\mathbf{v}_i - \mathbf{v}_e)$$

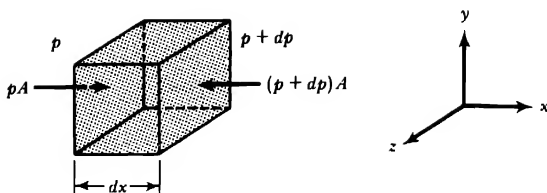
$$\mathbf{J} \times \mathbf{B} = \nabla p$$

Using Maxwell's curl relationship for the magnetic field, the left-hand side of the above equation may be reduced to an expression involving only the magnetic field.

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad (\text{assuming } \partial \mathbf{D} / \partial t = 0)$$

\*For readers unfamiliar with electromagnetic theory, it is advised that they proceed directly to the results of this treatment, namely, the expression for the maximum pressure,  $p_{\max}$ , which is equal to  $(1/2\mu_0)B_0^2$ .





$$(F_{ix} + F_{rx}) A dx = (p + dp)A - pA$$

$$F_{ix} + F_{rx} = \frac{dp}{dx} \quad (\text{x component of } \nabla p)$$

**FIGURE 7.12 One-Dimensional Pressure Gradient.**

where  $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$  (permeability of free space)

$$\frac{1}{\mu_0}(\nabla \times \mathbf{B}) \times \mathbf{B} = \nabla p$$

Employing a vector identity for  $(\nabla \times \mathbf{B}) \times \mathbf{B}$ , a more convenient relationship is obtained.

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = -\frac{1}{2}\nabla B^2 + (\mathbf{B} \cdot \nabla)\mathbf{B}$$

$$-\frac{1}{2\mu_0}\nabla B^2 + \frac{1}{\mu_0}(\mathbf{B} \cdot \nabla)\mathbf{B} = \nabla p$$

For a uniform plasma, the pressure will be uniform ( $\nabla p = 0$ ). At the edge of a confined plasma, however, a pressure difference exists which must, if the plasma is to be contained, be provided by a spatial variation of the magnetic field. Electrons and ions tend to move in helical paths along field lines. A field variation perpendicular to the field would, therefore, tend to minimize particle losses. For this condition, the operator  $\mathbf{B} \cdot \nabla$  is zero.

$$-\frac{1}{2\mu_0}\nabla B^2 = \nabla p \quad (\text{perpendicular variations of magnetic field})$$

$$\nabla \left( p + \frac{1}{2\mu_0} B^2 \right) = 0$$

$$p + \frac{1}{2\mu_0} B^2 = \text{a constant}$$

Outside the plasma the pressure is zero. Hence, the constant is that corresponding to the external magnetic field,  $B_0$ .

$$p + \frac{1}{2\mu_0} B^2 = \frac{1}{2\mu_0} B_0^2$$

$$p \leq \frac{1}{2\mu_0} B_0^2$$

The maximum pressure that can exist within the plasma occurs when the internal magnetic field is zero ( $B = 0$ ).

$$p_{\max} = \frac{1}{2\mu_0} B_0^2 \quad \text{n/m}^2$$

The right-hand side of the above equation is also the energy density (joules per cubic meter) of the magnetic field. Both pressure (newtons per square meter) and energy density have the same dimensions, since energy or work is equal to force times distance (N-m). A magnetic field of one tesla results in a pressure of  $4 \times 10^5$  newtons per square meter (four times atmospheric pressure).

The plasma pressure may be determined from the ideal gas law.

$$pV = RT$$

The volume of one kilogram-mole of molecules,  $V$ , contains Avogadro's number,  $N_0$ , of molecules. Dividing  $N_0$  by  $V$  yields the number of molecules in one cubic meter, that is, the number density,  $n$ .

$$n = N_0/V$$

Since  $R = N_0 k$ , the following is obtained.

$$p(N_0/n) = RT = N_0 k T$$

$$p = nkT$$

Since a fully ionized deuterium-tritium plasma contains an equal quantity of electrons and ions, the pressure is, if  $n$  is the ion or electron density, twice as great as that for the unionized gas.

$$p = 2nkT$$

It is this pressure that must be provided by a magnetic field to contain the plasma. As the pressure and density are linearly related, the magnetically achievable pressure tends to set an upper limit for the density.

The elevated temperature necessary for a fusion reaction can result in exceedingly high pressures. At atmospheric pressure and a temperature of  $20^\circ\text{C}$ , the number of molecules is  $2.5 \times 10^{25} \text{ m}^{-3}$  ( $V = 24.1 \text{ m}^3$ ). A plasma of this density and a temperature of  $10^8 \text{ K}$  would have a pressure of  $7 \times 10^{10} \text{ n/m}^2$ , or nearly 700,000 atmospheres. An inordinate magnetic field would be necessary to achieve this pressure. Hence, densities considerably less than  $2.5 \times 10^{25} \text{ m}^{-3}$  are necessary. For a density of  $10^{21} \text{ m}^{-3}$ , the pressure required is still  $2.76 \times 10^6 \text{ n/m}^2$  or 27.3 atmospheres. At  $20^\circ\text{C}$ , a molecular density of  $10^{21} \text{ m}^{-3}$  corresponds to that of a moderate vacuum ( $4 \times 10^{-5}$  atmospheres). The minimum required magnetic field to confine a plasma of this density is 2.6 T (26 kgauss). Based upon the Lawson criteria of  $\pi\tau = 10^{20} \text{ sec/m}^3$ , a confinement time,  $\tau$ , of .1 sec would be necessary.

To provide a quantitative perspective of the conditions necessary for a plasma

**TABLE 7.3 Lawson Criteria Break-Even Conditions for a Deuterium-Tritium Plasma Reaction**

Density $n$	$10^{21}/\text{m}^3$	
$(n_D = n_T = \frac{1}{2}n)$		
$\tau$	.1 sec	
Temperature $T$	$10^8$ K	
Pressure $p$	$2.76 \times 10^6$ n/m <sup>2</sup>	(27.3 atmospheres)
$P_F$	$49 \times 10^6$ W/m <sup>3</sup>	49 W/cm <sup>3</sup>
$P_F \tau$	$4.9 \times 10^6$ J/m <sup>3</sup>	4.9 J/cm <sup>3</sup>
$P_B$	$1.4 \times 10^6$ W/m <sup>3</sup>	1.4 W/cm <sup>3</sup>
Number of fusion reactions	$1.75 \times 10^{18}/\text{m}^3$	
Fraction of initial quantity of deuterons or tritons reacting	.0035 or .35%	
Deuteron energy of $\frac{3}{2}kT$		
D-T cross section	$8 \times 10^{-3}$ b	
Mean free path	$2.5 \times 10^9$ m	
Mean free time	2300 sec	

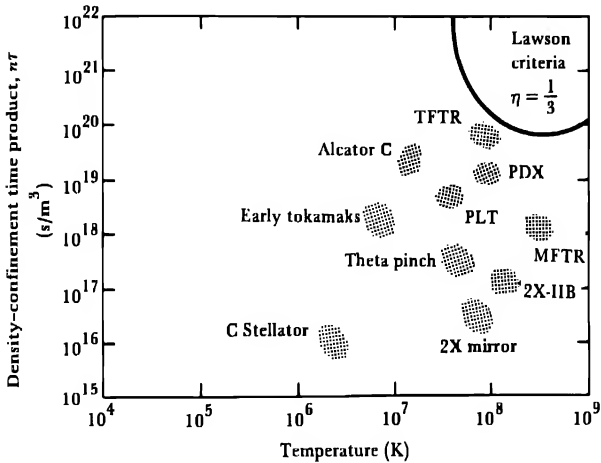
fusion reaction based upon the Lawson criteria, a plasma density of  $10^{21} \text{ m}^{-3}$ , a confinement time of .1 sec, and a temperature of  $10^8$  K was assumed. The resultant plasma parameters are indicated in Table 7.3. The fusion power density of 49 W/cm<sup>3</sup> is not excessive and is in fact typical of conventional energy conversion devices such as the internal combustion engine. Only a very small quantity of the deuterons or tritons interact for the assumed conditions. For an energy-producing reactor, either a higher temperature or a longer confinement time would be necessary.

The mean free path of a deuteron depends upon the plasma density and reaction cross section ( $l = 1/(N_T \sigma)$ ). For a deuteron with a velocity corresponding to an average energy of  $\frac{3}{2}kT$ , the mean free path is  $2.5 \times 10^9$  m. Its mean free time, that is, the approximate time required for a fission reaction to occur, is 2300 sec. A deuteron of this energy has a probability of only  $4.5 \times 10^{-5}$  (.1/2300) to interact in the one-tenth second fusion reaction duration. Fortunately, a higher overall reaction rate occurs. The relative velocity between the reacting deuterons and tritons can be greater, due to the thermal velocities of the tritons. Also, many of the deuterons and tritons have energies in excess of the average energy. For a deuteron with twice the velocity corresponding to  $\frac{3}{2}kT$ , the respective fusion cross section is 1.5 b, the mean free path is  $1.3 \times 10^7$  m, and the mean free time is only 6 sec. This deuteron has a 1.67% probability of reacting. For a thermonuclear fusion reaction, it is therefore, primarily, the ions with energies in excess of the average values that react.

Two magnetic plasma confinement schemes, tokamaks and magnetic mirrors, have produced plasma conditions that have been considered very encouraging (Figure 7.13) [3]. Three tokamaks, the Alcator C, the Princeton Large Torus (PLT), and the Poloidal Divertor Experiment (PDX), have come the closest to achieving conditions necessary for a fusion reaction, albeit that both densities and confinement times

are too small. Mirror machines have produced the highest temperatures and tokamaks have had the highest density-confinement-time products. The anticipated performance of toroidal and a mirror fusion test reactor (TFTR and MFTR) scheduled to be built and tested in the 1980s is also indicated on Figure 7.13. The TFTR is expected to come close to achieving the Lawson criteria break-even condition.

The confinement of the tokamak plasma depends upon the magnetic field produced by the plasma current. A plasma column with an axial current produces an azimuthal magnetic field as indicated in Figure 7.14. The cross product of the magnetic field component,  $B_\theta$ , and the axial current density,  $J$ , results in an inward directed constraining force on the plasma. This mechanism is frequently referred to as a *z-pinch* since the current is along what is normally designated as the *z* axis



PLT: Princeton Large Torus

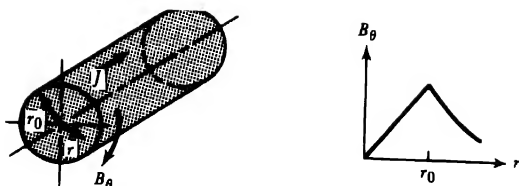
PDX: Poloidal Divertor Experiment (Princeton)

Alcator C: High Magnetic Field Experiment  
(Massachusetts Institute of Technology)

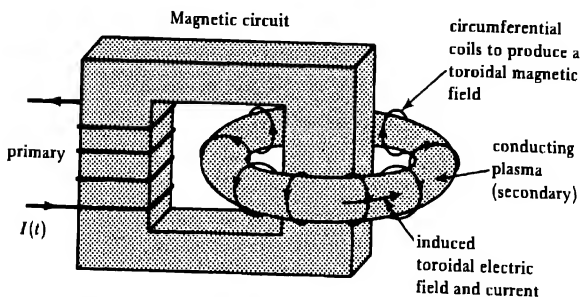
2X mirror, 2X-IIB: Magnetic Mirror Machine  
(Lawrence Livermore Laboratory)

TFTR and MFTR: Toroidal and Mirror Fusion Test  
Reactor, respectively. Anticipated  
performance since both are to be built  
and tested in the early 1980s.

**FIGURE 7.13 Status of Plasma Fusion Experiments, 1980 (Reference 3).**



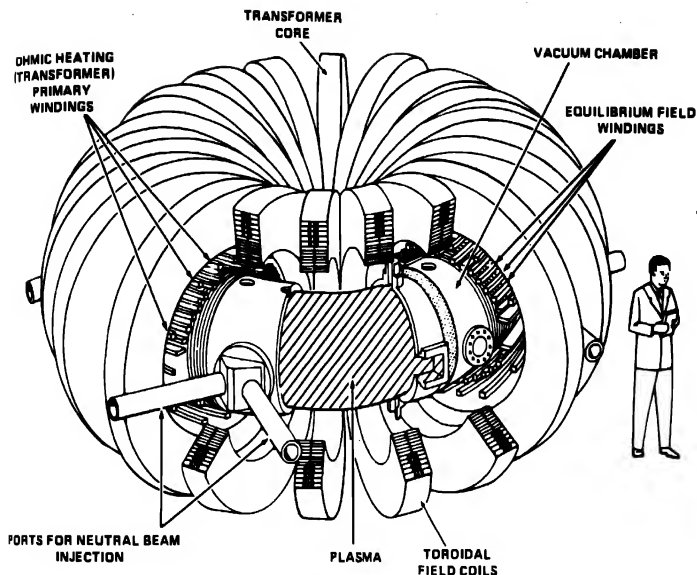
**FIGURE 7.14** An Azimuthal Magnetic Field Produced by an Axial Plasma Current.



**FIGURE 7.15** Schematic of a Tokamak Plasma Device.

in cylindrical coordinates. The maximum pressure corresponds to the maximum magnetic field, which occurs at the edge of the plasma ( $r = r_0$ ). Ohmic heating of the plasma is also achieved by the axial current. The simplest configuration for obtaining a z-pinch is by a discharge between two parallel plates. Losses due to plasma particles streaming out the ends of such a discharge, however, are excessive.

To eliminate end losses, the plasma column is bent into a torus, as indicated in Figure 7.15 [26–28]. In addition to the azimuthal field produced by the plasma current (the poloidal direction for a torus), an intense toroidal field produced by a circumferential winding is used to achieve the toroidal-shaped plasma. The conducting plasma forms the secondary winding of a transformer. An increasing primary current,  $I(t)$ , results in an increasing transformer flux which links the plasma. The change in flux induces an electric field which lies along the axis of the plasma and produces the plasma current. Not shown in Figure 7.15 are the external currents which produce compensating fields (1) to inhibit the outward motion of the plasma due to the curvature of the toroidal magnetic field and (2) to compensate for plasma instabilities. For example, external coils are used to twist the toroidal magnetic field lines causing an ion or electron to return to a new position upon completing one toroidal revolution.



**FIGURE 7.16** *Schematic Diagram of the Princeton Large Torus (Reference 3). (Courtesy of the Department of Energy.)*

Figure 7.16 is a schematic diagram of the Princeton Large Torus [3, 29]. The primary transformer winding is adjacent to the toroidal vacuum chamber (forming an air-core transformer which couples to the plasma). An intense toroidal magnetic field, 3.8 T, is produced by the toroidal field coils which surround the vacuum chamber and the equilibrium field windings are used to twist the toroidal magnetic field lines. Ohmic heating, the result of an increasing transformer primary current, has been insufficient to achieve the plasma temperature required for fusion. Further heating of the plasma has been accomplished with high-velocity neutral particle beams of deuterium. Neutral particles, unlike ions, can readily cross the magnetic field lines confining the plasma. Upon entering the plasma, the energetic particles become ionized and through interactions with the less energetic plasma ions the ionized particles are slowed down, thus heating the plasma. Owing to the very high temperatures achieved with neutral beam heating, this type of heating is expected to be used in early plasma fusion reactors.

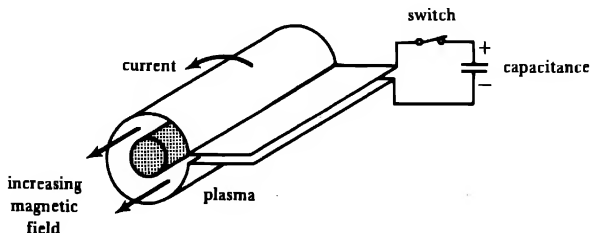
The plasma of the PLT has a minor toroidal radius of 45 cm and a major radius of 130 cm. It is anticipated that a power-producing fusion reactor three to five times this size, with an electrical power output of less than 1 GW, will be feasible. In the early 1970s it was thought that a fusion reactor would need to be

very large, having an electrical output power on the order of 10 GW. Not only would reactors of this size be extremely expensive but they would also be of limited value to electric utilities. Sufficient information has been obtained from the PLT and other experiments for doing the preliminary engineering design of a power-producing fusion reactor with a much smaller output [30, 31].

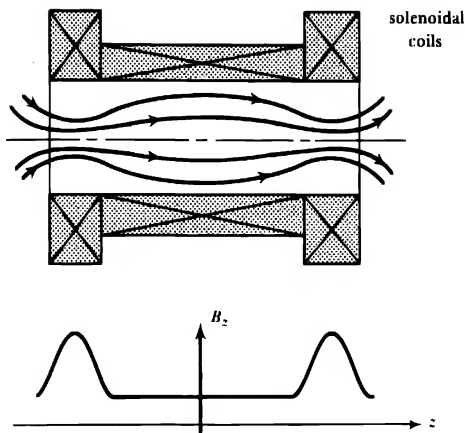
A major problem associated with fusion devices is the escape of high-energy ions from the plasma which strike the wall of the vacuum chamber and cause sputtering. Impurities sputtered from the vacuum chamber walls are primarily heavy atoms which tend to cool the plasma discharge. To minimize wall sputtering, the PLT used a material limiter of graphite to remove ions at the edge of the plasma which would have struck the walls. In the Poloidal Divertor Experiment (PDX), poloidal magnetic fields are being used to divert the edge of the plasma to an isolated collecting chamber, eliminating the impurities introduced by the unavoidable sputtering of material limiters. The Alcator C (as well as its predecessor, the Alcator A) is a toroidal device that is smaller than the PLT and PDX tokamaks. To reduce plasma losses which tend to increase as the size of a reactor is reduced, an even more intense magnetic field is used. Smaller, and possibly more versatile, fusion reactors may be possible based upon the Alcator design.

Various stellerators were designed and constructed during the 1950s. These machines, while similar to the tokamak, did not employ a pinch-producing plasma current. It was hoped that a steady-state reactor would be achieved by confining the plasma with only a toroidal field. Various stellerator configurations were tried, such as that of a torus twisted into a figure eight to minimize plasma curvature losses. As can be seen from Figure 7.13, the conditions achieved were far short of those needed for fusion.

Another promising plasma configuration used for early experiments was a theta pinch in which the plasma was compressed by an external magnetic field. An increasing axial magnetic field was produced by a capacitive discharge through a one-turn coil surrounding the plasma column, as shown in Figure 7.17. The rapidly increasing magnetic field is equivalent to an inward motion of the magnetic field lines. Consequently, ions and electrons spiraling about the field lines were compressed. Concurrent with the compression was a heating of the plasma. End losses



**FIGURE 7.17** *A Theta Pinch Plasma Device.*



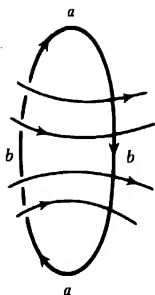
**FIGURE 7.18** *Axial Magnetic Field Mirror System.*

were reduced by increasing the field strength at the ends of the column, or by bending the plasma into a torus. The results of Figure 7.13 were limited by end losses.

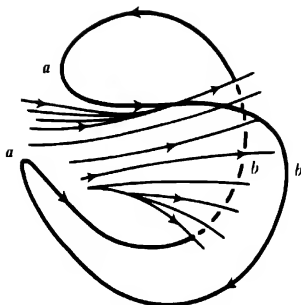
The simplest mirror-type plasma device is that of Figure 7.18, with a simple axial magnetic field. While the intense fields produced by the end coils tend to reduce end losses (the gradient of  $B^2$  results in a pressure gradient or inward force on the plasma), ions and electrons with velocity components nearly aligned to the field lines stream out the ends. A permutation of the axial mirror confinement scheme is a magnetic field produced from a current winding with a configuration resembling that of a baseball seam, yin-yang magnets, as shown in Figure 7.19. The coil may, for conceptualization, be viewed as a distorted single-turn coil in which opposite segments (*a-a* and *b-b*) are bent together. This has the effect of squeezing the field lines between the segments while resulting in a weaker field or well in the central region. Moving outward from the well results in an increase in field intensity and hence an inward-directed magnetic force which can counteract the plasma pressure.

The yin-yang magnet is being used in the Beta II mirror experiment at Lawrence Livermore Laboratory (Figure 7.20). This experimental device is a modification of the 2X-IIB mirror, the performance of which is indicated in Figure 7.13. Confinement will be enhanced by a plasma current that is sufficient to produce a reversal of the magnetic field. A pair of plasma guns, surrounded by DC magnets (coils) producing an axial field, will be used to form the initial plasma. Heating will be achieved by 12 energetic neutral beams injected perpendicular to the plasma. The tandem mirror experiment (TMX) utilizes two sets of yin-yang magnets forming



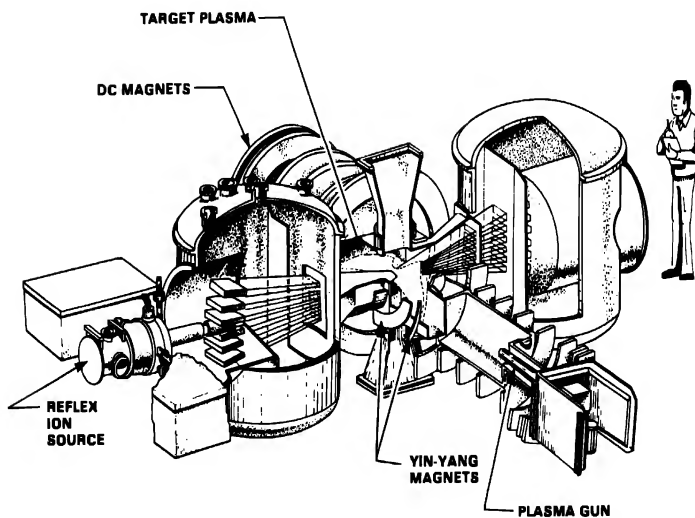


single-turn coil

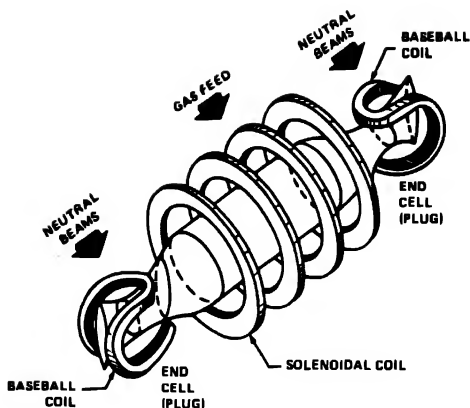


baseball current

**FIGURE 7.19 Baseball Magnetic Well Configuration, Yin-Yang Magnets.**



**FIGURE 7.20 Schematic Diagram of the Beta II (Reference 3).**  
(Courtesy of the Department of Energy.)



**FIGURE 7.21** *A Conceptual Diagram of a Tandem Mirror Device (Reference 3). (Courtesy of the Department of Energy.)*

"plugs" at the end of a plasma formed in a solenoidal magnetic field (Figure 7.21). The mirror fusion test reactor is expected to resemble this device.

Forms of plasma heating other than that of complex neutral beam sources are also being studied. The Phaedrus is a tandem mirror device in which radio-frequency electric fields will be used for heating. If successful, the amount of neutral beam heating required will be reduced. Finally, the Elmo Bumpy Torus (EBT) consists of a series of magnetic mirror sections arranged in a circle. Each section is designed to function as a microwave cavity, microwave energy being used for heating the plasma. It is hoped that steady-state operation, as opposed to pulsed operation that characterizes all other high-performance devices, will be achieved by the EBT.

The favorable results of magnetic confinement fusion experiments in the 1970s prompted the passage of the Magnetic Fusion Engineering Act of 1980 (PL 96-386). The Act established national goals of demonstrating the engineering feasibility of a magnetically confined fusion reaction by the early 1990s and of achieving an operating demonstration fusion plant at the turn of the century. This ambitious program is expected to cost about \$20 billion.

#### **4. INERTIAL CONFINEMENT**

In an inertially confined fusion reaction, the inertia of the fuel constituents alone is relied upon to hold the reactants together. Not only is an extremely rapid heating of the fuel required but the rapid fusion reaction will also have an explosive character. An extremely intense, highly focused source of energy is required. While most inertial

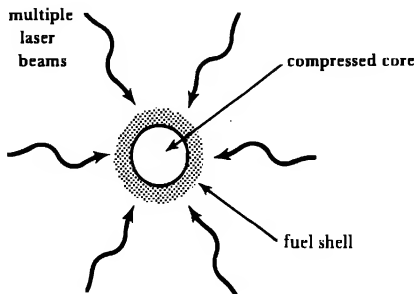
fusion experiments have used laser energy sources [32–35], particle-beam energy sources are also being considered [36].

The speed of a deuteron or triton with an average kinetic energy corresponding to a temperature of  $10^8$  K is on the order of  $10^6$  m/s (Table 7.2). Therefore, in a time interval of one nanosecond ( $10^{-9}$  sec), the nuclei will travel a distance of one millimeter. Higher energy neutrons, those which are much more likely to take part in a fusion reaction, travel considerably farther. For fuel pellets smaller than a millimeter in diameter, heating and fusion must occur in a time interval that is considerably less than a nanosecond.

The Lawson criteria for a break-even fusion reaction requires a product of density and confinement time of  $10^{20}$  sec/m<sup>3</sup> and a temperature of  $10^8$  K. Liquid hydrogen ( $-252^\circ\text{C}$  boiling point), which has a density of approximately  $4 \times 10^{28}$  atoms/m<sup>3</sup>, results in a product of only  $4 \times 10^{18}$  sec/m<sup>3</sup> for  $\tau = 10^{-10}$  sec. To obtain a higher product of density and confinement time, the fuel will need to be compressed. Fuel densities of one hundred to one thousand times that of liquid hydrogen are expected to be necessary for an energy-producing fusion reaction.

Heating and compression of the small fuel pellet will very likely be by multiple laser beams as indicated in Figure 7.22. Both neodymium-glass lasers and carbon dioxide gaseous lasers are being used for fusion experiments. The former laser has a wavelength of  $1.06\mu$  which results in a relatively efficient absorption of energy by the fuel pellet, while the  $10.6\mu$  wavelength of the carbon dioxide laser is much less efficiently absorbed. Carbon dioxide lasers, however, have not only produced higher average powers but they are also considerably more efficient at converting electrical energy to coherent radiation (5% vs. less than 1%). Multiple laser systems with energies of up to 1 MJ, considerably greater than those presently (1980) available, must be developed before laser-initiated fusion is achieved.

Initially, it was proposed that simple deuterium-tritium solid fuel pellets, formed from a falling droplet and cooled by surface evaporation, be used for fuel. The laser energy initially vaporized the outer portion of the pellet and then the electrons

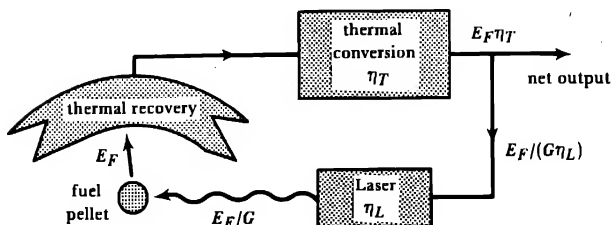


**FIGURE 7.22 A Laser-Irradiated Fuel Pellet.**

of the vapor, heated by electron-photon interaction, heated the ions through a coulomb interaction. An inward traveling shock wave was generated by the suddenly heated ions. The implosive force of the shock wave, however, did not provide sufficient compression of the fuel. Recent experiments (end of the 1970s) have used fuel targets referred to as *microballoons* [37]. A glass shell, .05 to .5 mm in diameter, is used to enclose a high-pressure mixture of deuterium and tritium gas. The glass shell is surrounded by a thick "pusher" layer which is in turn surrounded by an outer "absorber/ablator" layer. Laser energy is absorbed by the outer layer which vaporizes. The rapid vaporization results in an inward reaction force on the pusher layer which compresses the fuel. A high-density material with a high atomic number is used for the pusher, thus shielding the fuel from the laser. Since this minimizes the preheating of the fuel, high compressions can be achieved.

Of importance in determining the feasibility of a laser-induced fusion reaction is the energy gain, that is, the ratio of the output fusion energy to that of the laser necessary to produce the reaction. For nuclei to react, an average nuclide energy corresponding to  $\frac{3}{2}kT$  or approximately 13 keV is necessary for a temperature of  $10^8$  K. Since the D-T fusion reaction results in an output energy of 17.6 MeV, the energy gain for the two fusing nuclides is 677. However, if only 10% of the available fuel fuses, the energy gain is only 68. The  $^4_2\text{He}$  fusion product can further react with the fuel nuclides and result in additional heating. A quantitative perspective, in which  $G$  is the fusion energy gain,  $\eta_T$  is the thermal to electric energy conversion efficiency, and  $\eta_L$  is the laser efficiency, is provided by Figure 7.23. If  $E_F$  is the fusion-generated energy (including the recoverable thermal and bremsstrahlung energy), the electric output will be  $\eta_T E_F$ . A portion of this energy will be needed for the laser. The fusion energy gain of  $G$  requires a laser output of  $E_F/G$  and hence a laser input energy of  $E_F/(G\eta_L)$ . The net electrical energy output is that not used for the laser.

$$\begin{aligned}\text{Net electrical output energy} &= \eta_T E_F - E_F/(G\eta_L) \\ &= E_F[\eta_T - 1/(G\eta_L)]\end{aligned}$$



**FIGURE 7.23 A Laser-Induced Fusion Reaction.**

For an energy-producing reactor, the following inequality must be true.

$$\eta_T > 1/(G\eta_L)$$

$$\eta_T\eta_L > 1/G$$

An energy gain of 100 and a thermal conversion efficiency of 40% require a laser efficiency of 2.5%. A laser efficiency of twice this value, 5%, would require that half of the electrical output be used for the laser. Even for a laser efficiency of 10%, a substantial portion of the electrical output will be used to energize the laser "ignition" system.

Particle beams, in particular high-energy electron beams, are also being considered for inertially confined fusion reactors. Not only have electron beams with much higher energies than lasers been developed but the efficiency of electron sources is also much higher than that of lasers.

Only the most tentative speculations on the configuration of an inertially confined fusion reactor are justified. The surrounding vessel, in addition to containing lithium to breed tritium, will need to be able to withstand the blast effect of the reaction [38].

As mentioned in the introduction, an energy-producing fusion reactor has not yet been built. An alternative approach taken up in the next chapter is solar energy, that is, the utilization of an already operating fusion reactor, namely, the sun.

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## PROBLEMS

1. In the fusion of a deuteron and triton, momentum is conserved. The initial velocities and momenta of the reactants may be assumed to be negligible compared to those of the reaction products. Show that for a D-T reaction the energy of the resultant neutron must be four times that of the helium nucleus.
2. A potential fusion reaction is that of lithium-6 and a proton. The fusion products are helium-3 and helium-4. Based upon the data of Table 7.1, determine the fusion energy. Using the assumption of Problem 1, determine the division of energy between the two helium nuclides.
3. Calculate the "classical" energy (in megaelectronvolts) based upon the coulomb repulsive force for the following nuclei when they are "touching": D-D, D-T, D- $\frac{3}{2}\text{He}$ . Assume the radius of the nuclides is proportional to the cube root of their mass numbers ( $R_0 = 1.5 \times 10^{-15} \text{ m}$ ).
4. Based upon a heavy water ( $\text{D}_2\text{O}$ ) cost of \$100/lb, determine the fuel cost of one kilowatt-hour of electricity produced by a D-D reaction. Assume a 40% thermal to electric conversion efficiency.
5. An internal combustion engine with a displacement of  $1600 \text{ cm}^3$  has an output power of 50 hp. What is the average power density in watts per cubic meter based upon the displacement of the engine?
6. The power generated by the fusion reaction of the sun may be determined from an integration of the solar constant over a sphere with the earth's radius (see Chapter 3). Determine the average volume power density of the sun's fusion reaction. Assuming the sun is composed primarily of normal hydrogen, determine the number density of the hydrogen atoms ( $M_{\text{sun}} = 1.97 \times 10^{30} \text{ kg}$ ).
7. The data of Table 7.3 is based upon the minimum plasma parameters necessary to achieve a break-even condition. Suppose the temperature of the plasma was increased to  $2 \times 10^8 \text{ K}$  while other conditions,  $n$  and  $\tau$ , remained the same.

What would be the corresponding pressure and the fusion and bremsstrahlung power densities? Determine the fraction of the fuel which would undergo fusion for this condition.

8. The Princeton Large Torus of Figure 7.16 has a minor toroidal radius of 45 cm and a major radius of 130 cm.
  - (a) What is the volume of the torus?
  - (b) What is the magnetic energy stored in this volume for a uniform field of 3.8 T?
  - (c) Suppose a power source of 10 kW is available to supply the magnetic field energy. How long, ignoring losses, does it take to provide the energy stored in the toroidal volume?
  - (d) Assume that a power-producing reactor is five times as large as the PLT. What is the volume of the torus and the stored magnetic energy for a field of 3.8 T? What would be the power required if the magnetic energy of the torus is to be supplied in one hour?
9. Liquid hydrogen (ordinary) has a specific gravity of .070 at a temperature of  $-252^{\circ}\text{C}$ . What is its number density expressed in atoms per cubic meter? Why would a mixture of deuterium and tritium be expected to have the same number density?
10. Determine the mass of TNT necessary to produce a blast with an energy release of 100 MJ. Since the neutrons of a D-T fusion reaction readily escape the reaction volume, the shock of the blast will be due primarily to the  ${}^4_2\text{He}$  fusion products. Determine the TNT equivalent of the  ${}^4_2\text{He}$  energy. What is the rate (blasts/second) that would need to occur for a reactor with an average output thermal power of 1 GW? Assume the energy of both the neutron and helium is recovered.
11. Suppose that a laser fusion device results in a fusion energy of 100 MJ. Assume the thermal conversion efficiency is 40%, the laser efficiency is 10%, and the fusion energy gain is 50. What is the net electrical output energy? What is the value of the electrical energy if it sells for 5¢/kWh?



## CHAPTER 8

# *Solar Energy*

### 1. THE POTENTIAL OF SOLAR ENERGY

The quantity of solar energy incident upon the earth is very large, but it is also diffuse. Based upon the solar energy balance (Chapter 3), the average absorbed and radiated power by the earth and the atmosphere is  $220 \text{ W/m}^2$ . Approximately 20% of the incident power is absorbed by the atmosphere. The average worldwide incident power at the earth's surface is therefore  $176 \text{ W/m}^2$ , that is, a daily average energy of  $4.2 \text{ kWh/m}^2$ .

Many natural sources of energy depend ultimately upon solar energy. Hydro power depends upon solar energy for evaporation from large bodies of water. Atmospheric absorption of heat results in winds. Heat absorbed by the oceans not only produces currents but also vertical temperature gradients which, at least in theory, could be used for a source of energy. A discussion of these indirect solar energy sources will be deferred to the next chapter; only the direct use of solar energy will be considered here.

As of 1980, solar collector-type systems were used primarily for space and water heating in residential and commercial structures. Based upon energy prices anticipated for the 1980s (projections of the prices of Figures 1.12 through 1.14) and applicable tax credits, solar energy systems are becoming more economically attractive. Prior to 1973, residential fuel prices were only about  $\$1/\text{GJ}$ . Therefore, a typical moderate-sized residence which might have used  $1.5 \times 10^{11} \text{ J}$  for space and water heating had a yearly fuel bill of only  $\$150$ . Although not negligible, fuel costs comprised only a minor portion of overall household expenditures. For a fuel cost of  $\$1/\text{GJ}$ , not only were solar energy systems uneconomic but even the most

modest of energy-saving investments, such as attic insulation, were only marginally beneficial. Heating costs are no longer small and if widely accepted projections of fuel costs should indeed occur, the yearly heating cost for the residence considered will exceed \$1000.

Since solar-heated structures require fuel only for their "back-up" system, they, in effect, save fuel. A substantial portion of the fuel saved by a solar-heated structure can be attributed to its being better built and insulated than structures built prior to the mid-1970s. To be economical, the heating needs of a solar-heated building must be first reduced to a relatively small value. A solar heating system, if designed to supply the heating needs of a poorly insulated structure, would be inordinately expensive. (In addition, the roof area of the structure would probably be insufficient for the collector area required.) Conversely, a very well insulated structure can be kept warm with a passive solar energy system, that is, a system consisting primarily of optimally oriented windows that function as solar collectors.

Solar heating and energy conservation must be considered jointly if overall building costs are to be minimized. This is also the case not only for other solar energy systems but for most alternative energy supplying systems as well. Energy obtained from these systems is, and is expected to remain, considerably more expensive than that derived from the abundant supplies of fossil fuels of the past. For a solar energy system to be economic, the supplying system has to be matched to the energy using need or task. It is the overall cost of carrying out a desired task, such as keeping a structure warm or cool, that is important, not simply the cost of supplying a given quantity of energy. Since the design of both the energy supplying and using system must be given comparable consideration, this represents a unique engineering opportunity.

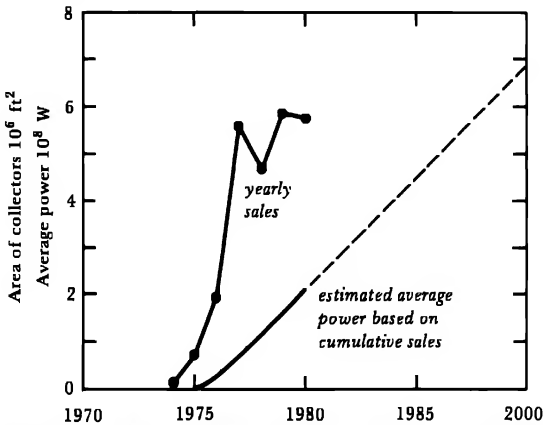
Solar energy systems and energy conservation are both capital intensive, that is, they tend to require large initial investments. The pay-off consists of reduced future expenditures for conventional sources of energy. While an investment decision ultimately depends upon one's perception of likely future events, an important economic factor that is usually considered is the interest rate for borrowing money.\*

To pay back a solar energy or energy-conserving investment from future energy savings, not only must the cumulative fuel savings be greater than the initial investment but the savings must also be sufficient to provide for interest payments. Hence, an economic justification for an energy-saving investment is usually based upon anticipated energy prices since it is energy costs which are avoided that determine the value of the initial investment. Both interest rates and fuel costs depend upon market forces and constraints and are affected by public policies. Subsidies, price controls, and various tax credits have tended to keep energy prices lower than they might be otherwise, whereas monetary policies, such as restricting the money supply to control inflation, have resulted in abnormally high interest rates. Furthermore,

\* Interest rates, in effect, reflect a time preference for having money, the present being weighted more heavily than some time in the future. If one has a given quantity of money, its present use will be deferred if at some later date the initial quantity of money will have increased, accrued interest. Conversely, if money is borrowed to satisfy a present desire, it must be returned along with a payment of interest.

if the conservation of fossil fuels is considered to be in the national interest, investment incentives such as special low-interest loans and tax credits are granted to increase the attractiveness of solar energy and energy-conserving investments. The deregulation of natural gas and petroleum prices has also increased the attractiveness of energy-saving investments. It is the anticipated effect of these factors, over the useful life of the investment, that must be considered. Since many systems will last for 20 or more years, investment decisions, no matter how quantitative and detailed an analysis is utilized, depend upon a subjective projection of future events.

Commercially produced solar collectors are classified into two categories: low-temperature collectors designed primarily for the heating of swimming pools and medium-temperature collectors used primarily for space and water heating. Many other solar collecting systems are being developed, but it is primarily the aforementioned types of collectors that have been commercially produced. Yearly sales of the medium-temperature collectors are indicated in Figure 8.1 [1]. Approximately 50% of these collectors have been used for water heating and 30% have been used for space heating systems which usually also provide hot water. The remainder of the collectors are used for special applications, cooling, and industrial and agricultural processing. Few solar energy systems are sufficiently well instrumented to provide an accurate indication of the energy they provide. The average power estimate of Figure 8.1 is based upon what might be considered typical for a residential collector system,  $1.8 \text{ kWh/m}^2\text{day}$ . Less than .01% (1980) of the nation's total energy usage can be attributed to these collectors. However, to the extent that energy conservation measures are used in conjunction with solar collection systems, the energy saved by these systems is greater than that calculated based upon the performance of the



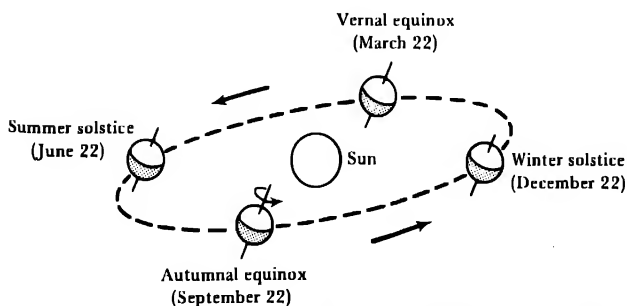
**FIGURE 8.1** Yearly Sales and Estimated Average Power Produced by Solar Collectors (Low-Temperature Collectors Excluded) (Reference 1).

collectors. A projection of the power that may be produced, based upon a continuation of the average yearly sales from 1977 through 1980, is also included. For solar energy systems to have a significant impact on the overall national energy system, there must be a substantial increase in the production of collectors. In terms of collector area, the sales of low-temperature collectors used primarily for pool heating exceed the sales of medium-temperature collectors. However, not only is the collection efficiency of the low-temperature collectors less but their use tends to be seasonal. As a consequence, the yearly averaged power they supply is probably considerably less than that of the medium-temperature collectors.

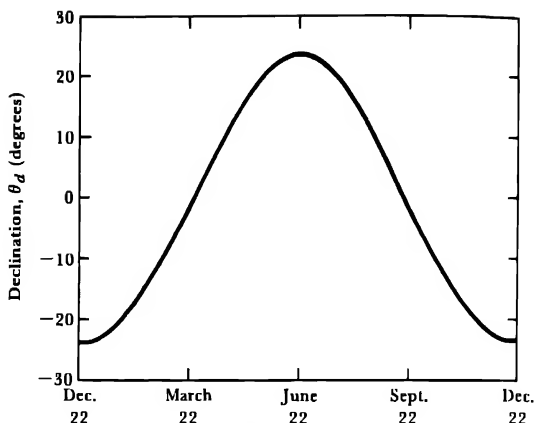
The energy crisis of the 1970s has resulted in a renewed interest in solar energy. Many of the systems now being used or developed date from earlier periods, such as the mid-1950s when interest was also high. The "classic" work on solar energy is by Daniels [2] (now available in paperback), but many other, more recently published, books provide excellent comprehensive technical treatment of solar energy systems [3-17].

## 2. INCIDENT RADIATION

The average solar radiation power density outside the earth's atmosphere, the solar constant, is approximately  $1353 \text{ W/m}^2$  for a plane normal to the rays of the sun [18]. A small variation in intensity,  $\pm 3.3\%$  or  $\pm 45 \text{ W/m}^2$ , results from the slightly elliptical orbit of the earth. The significant seasonal variation in solar flux per unit area of the earth's surface, however, is due to the  $23.5^\circ$  tilt of the earth's axis relative to the plane of its orbit about the sun. For both the vernal and autumnal equinox (Figure 8.2), the sun is in the earth's equatorial plane. During the daily rotation of the earth, the sun's rays remain perpendicular to the earth's surface at the equator and illuminate both the northern and southern hemispheres equally. Summer in



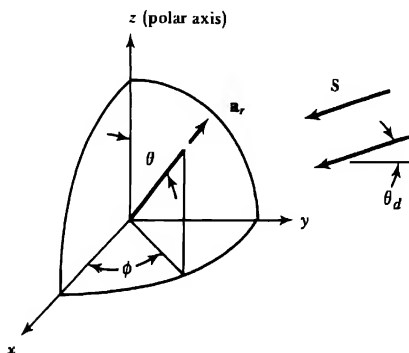
**FIGURE 8.2** *The Earth's Solar Orbit (Northern Hemisphere Seasons).*



**FIGURE 8.3 The Sun's Declination.**

the northern hemisphere corresponds to that hemisphere's being tilted toward the sun; winter, the converse occurs.

The declination of the sun,  $\theta_d$ , is the angle between the sun's rays and the earth's equatorial plane. Its maximum value corresponds to the tilt of the earth's axis ( $23.5^\circ$ ), as indicated in Figure 8.3. For convenience, the spherical coordinate system of Figure 8.4, with its origin corresponding to the center of the earth and the  $z$  axis along the polar axis, will be used. The seasonal variation of the sun's incident rays can be accounted for by a variation in  $\theta_d$ . The hourly rotation of the earth is



**FIGURE 8.4 Polar Coordinate System for Solar Radiation.**

equivalent to a change in the azimuthal angle,  $\phi$ , while latitude is determined by the polar angle,  $\theta$ . Assuming the sun's rays,  $\mathbf{S}$ , are in the  $y$ - $z$  plane, noon coincides with  $\phi = \pi/2$ . The following is thus obtained for the latitude angle (measured from the equator) and the hour angle (measured from the midnight position).

$$\text{North latitude} = \pi/2 - \theta \quad \theta \leq \pi/2$$

$$\text{South latitude} = \theta - \pi/2 \quad \theta \geq \pi/2$$

$$\text{Hour angle} = \pi/2 + \phi$$

$$\text{Hour} = (12/\pi)(\pi/2 + \phi)$$

By means of the appropriate vector manipulations, the hourly and daily variation of incident flux for a given latitude may be obtained.

A quantity often of interest is the flux normally incident to the earth. Since a perpendicular inward pointing unit vector is  $-\mathbf{a}_r$ , the scalar product,  $-\mathbf{a}_r \cdot \mathbf{S}$ , is that desired,  $S_r$ .

$$S_r = -\mathbf{a}_r \cdot \mathbf{S}$$

If both  $\mathbf{S}$  and  $\mathbf{a}_r$  are resolved into rectangular components, the product may readily be obtained.

$$\mathbf{S} = -\mathbf{a}_y S \cos \theta_d - \mathbf{a}_z S \sin \theta_d$$

$$\mathbf{a}_r = \mathbf{a}_x \sin \theta \cos \phi + \mathbf{a}_y \sin \theta \sin \phi + \mathbf{a}_z \cos \theta$$

$$S_r = S \cos \theta_d \sin \theta \sin \phi + S \sin \theta_d \cos \theta$$

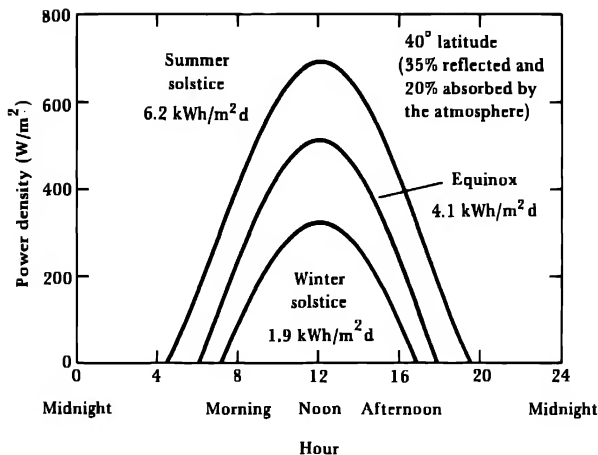
The above expression is obviously only valid for  $S_r > 0$ . Negative values correspond to a surface facing away from the sun. If  $S = 1353 \text{ W/m}^2$ ,  $S_r$  corresponds to the incident radiation normal to the atmosphere of the earth.

Radiation reaching the surface of the earth differs from that at the outer atmosphere, owing to two important effects: atmospheric reflection and absorption. In Chapter 3 an average reflectivity of 35% was assumed. Atmospheric absorption reduces the incident flux, on the average, by approximately 20%.\* The average radiation reaching the surface of the earth is thus  $(1 - .35)(1 - .2)$  or .52 of that incident at the outer atmosphere. An average value of  $S$  at the surface of the earth is thus  $704 \text{ W/m}^2$  (52% of  $1353 \text{ W/m}^2$ ). In general, both reflection and absorption depend upon the angle of the incident radiation as well as local climatic conditions. Since absorption tends to depend upon the length of the atmospheric path, calculations based upon an average value tend to underestimate a noontime radiation (short atmospheric path length) and overestimate early morning and late afternoon values (a long path length). The data of Figure 8.5 for a latitude of  $40^\circ$  ( $\theta = 50^\circ$ ) was obtained using the average value of  $S$ .

It is also of interest to consider the total daily energy density incident on a surface, that is, the time integrated power density (the area under the curves of

---

\* While this quantity is somewhat difficult to justify, the calculated data tends to agree with observed results.



**FIGURE 8.5** *Power and Daily Energy Incident on a Horizontal Surface (Energy Density Is Daily Integrated Power Density).*

Figure 8.5). If first the average power density is found (watts per square meter), it can be multiplied by the 24 hours of a day to yield an expression for the daily energy density (watt-hours per square meter).

$$S_{rav} = \frac{S}{2\pi} \int_{\text{daylight}} (\cos \theta_d \sin \theta \sin \phi + \sin \theta_d \cos \theta) d\phi$$

The angle corresponding to sunrise,  $\phi_m$ , is that for which the integrand is zero.

$$\cos \theta_d \sin \theta \sin \phi_m + \sin \theta_d \cos \theta = 0$$

$$\sin \phi_m + \frac{\tan \theta_d}{\tan \theta} = 0$$

$$\sin \phi_m = -\frac{\tan \theta_d}{\tan \theta}$$

The angle of sunset corresponds to  $\pi - \phi_m$ . In the northern hemisphere ( $0 \leq \theta \leq \pi/2$ )  $\tan \theta$  is positive. When the sun is above the equatorial plane,  $\theta_d$  is positive and  $\phi_m$  is negative. This results in an angular variation of  $\phi$  which is greater than  $\pi$  and hence more than 12 hours of daylight. Since at an equinox  $\theta_d = 0$  and  $\phi_m = 0$ , all latitudes have 12 hours of daylight. For this condition a simple expression results for  $S_r$  and its average value.

$$i_r = S \sin \theta \sin \phi$$

$$S_{rav} = \frac{S}{2\pi} \sin \theta \int_0^\pi \sin \phi \, d\phi = \frac{S}{\pi} \sin \theta$$

At 40° latitude ( $\theta = 50^\circ$  and  $\sin \theta = .766$ ) an average value of 172 W/m<sup>2</sup> is obtained. Multiplying by 24 hours results in a daily energy of 4.1 kWh. When  $\theta_d$  is negative,  $\phi_m$  is positive, and fewer than 12 hours of daylight are available in the northern hemisphere. For the general case the following average incident radiation density is obtained.

$$\begin{aligned} S_{rav} &= \frac{S}{2\pi} \int_{\phi_m}^{\pi - \phi_m} (\cos \theta_d \sin \theta \sin \phi + \sin \theta_d \cos \theta) \, d\phi \\ &= \frac{S}{\pi} \int_{\phi_m}^{\pi/2} (\cos \theta_d \sin \theta \sin \phi + \sin \theta_d \cos \theta) \, d\phi \\ &= \frac{S}{\pi} [\cos \theta_d \sin \theta \cos \phi_m + \sin \theta_d \cos \theta (\pi/2 - \phi_m)] \end{aligned}$$

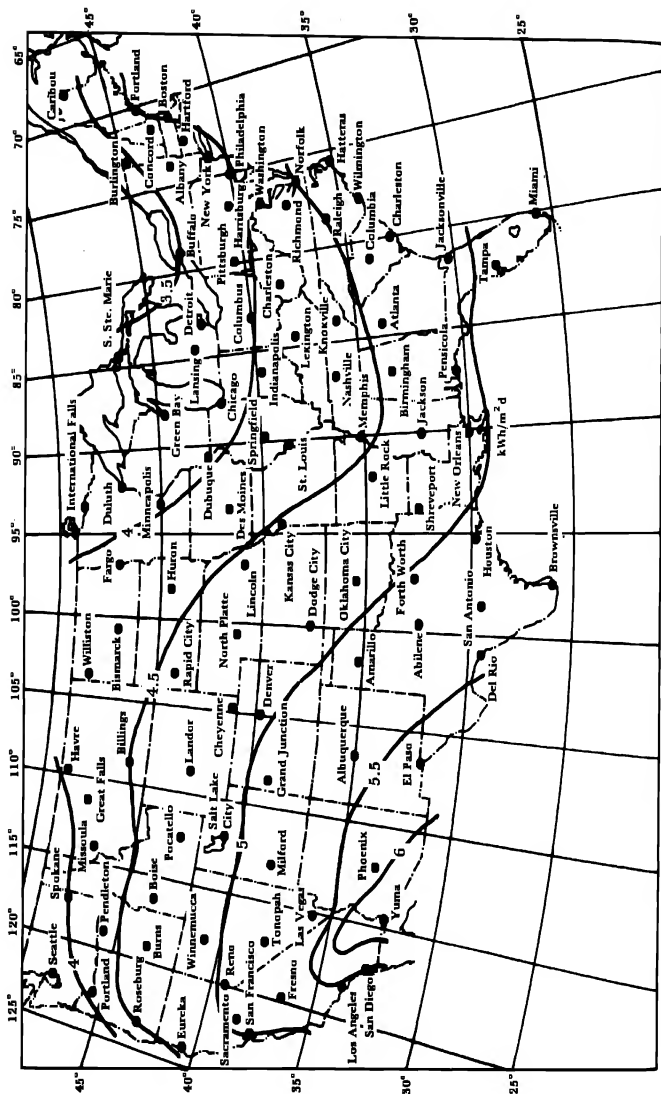
At the winter solstice the incident energy is 1.9 kWh/m<sup>2</sup> (40° latitude), while at the summer solstice it is 6.2 kWh/m<sup>2</sup>.

A yearly averaged energy density for a horizontal surface may be obtained by taking into account the yearly variation of  $\theta_d$  (Figure 8.3). An average daily energy density approximately equal to that obtained for the equinoxes is obtained (4.1 kWh/m<sup>2</sup> for the conditions of Figure 8.5). The yearly averaged daily energy density contours for the United States, shown in Figure 8.6, are based upon data from 117 observing stations that were recorded over a period of several years [19]. Energy values based upon the approximations of this section tend to agree with those for the Eastern United States, while actual energy values for the West are about 25% higher. The seasonal energy variation is also available [19-21].

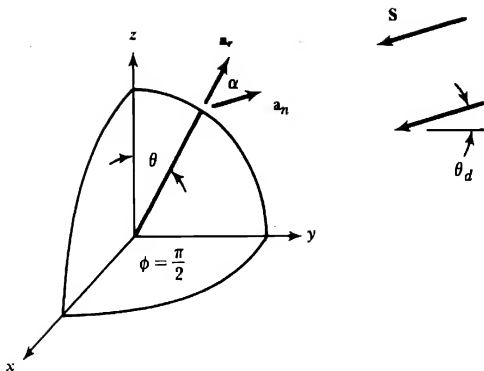
Tilting the solar collector to face the sun increases the incident flux per unit area of the collector. If the collector tracks the sun, that is, if its orientation is continually changed throughout the day so as to face the sun, the incident radiation remains constant (ignoring absorptive changes) throughout the daylight hours. The daily energy collected per unit of collector area is thus 8.4 kWh at an equinox (704 W × 12 h), somewhat larger in the summer (10.5 kWh at 40° latitude), and smaller in the winter (6.4 kWh). For a tracking collector, the energy variation, ignoring atmospheric effects, is due only to the variation in the daily collection time.

A frequently used collection scheme is to tilt the collector so as to receive maximum radiation at noon and leave it fixed in this position during the day. This necessitates only a daily change of the elevation angle. A still further simplification useful for comfort heating is a fixed collector, say a portion of the roof, oriented to optimize the energy collection in the winter, when the greatest input is needed. Figure 8.7 illustrates the geometrical relations for a collector tilted such that its normal,  $\mathbf{a}_n$ , is at an angle  $\alpha$  relative to the vertical,  $\mathbf{a}_v$ . The radiation incident on the collector,  $-\mathbf{a}_n \cdot \mathbf{S}$ , may be obtained by replacing  $\theta$  in the previously derived expressions by  $\theta + \alpha$ .





**FIGURE 8.6** Annual Average of Energy Incident on a Horizontal Surface, United States (Reference 19).



**FIGURE 8.7 Tilted Collector.**

$$S_n = -\mathbf{a}_n \cdot \mathbf{S}$$

$$= S \cos \theta_d \sin (\theta + \alpha) \sin \phi_d \cos (\theta + \alpha)$$

If the collector is oriented for maximum noontime radiation, the following must be valid.

$$\theta + \alpha + \theta_d = \pi/2$$

$$\theta + \alpha = \pi/2 - \theta_d$$

Introducing the above relation for  $\theta + \alpha$ , an expression for  $S_n$  is obtained in terms of  $\theta_d$  and  $\phi$ .

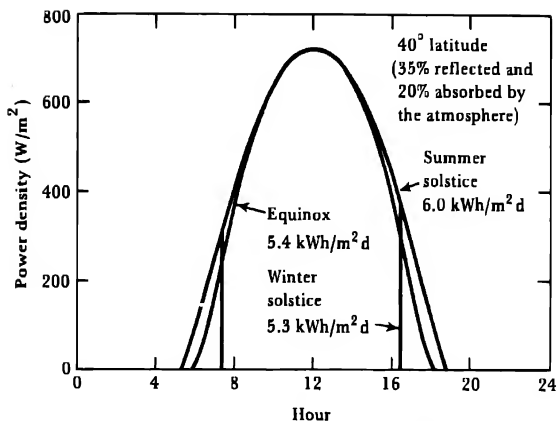
$$\sin (\theta + \alpha) = \sin (\pi/2 - \theta_d) = \cos \theta_d$$

$$\cos (\theta + \alpha) = \cos (\pi/2 - \theta_d) = \sin \theta_d$$

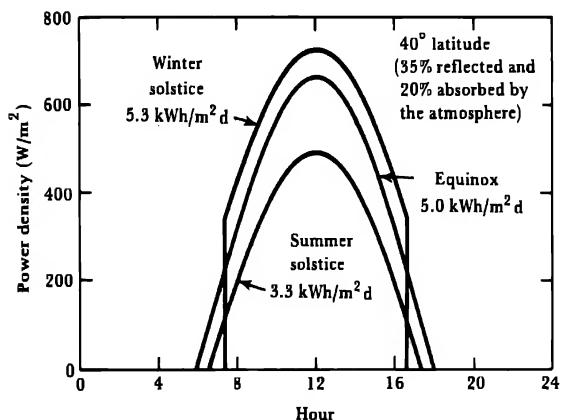
$$S_n = S \cos^2 \theta_d \sin \phi + S \sin^2 \theta_d$$

Since the above expression is only valid for daylight hours, it is, as a result, an indirect function of  $\theta$ . Figure 8.8 is a plot of  $S_n$  for  $40^\circ$  latitude and that of Figure 8.9 is for a fixed collector optimized for winter collection. The energy values were obtained by first finding the average values of  $S_n$ . The incident energy of the tilted collector for the winter solstice is more than three times that of a horizontal collector for  $40^\circ$  latitude. To achieve this, the normal of the collector was tilted  $63.5^\circ$  ( $\pi/2 - \theta_d - \theta$ ) from the vertical. For home heating, the collector could be the south-facing slope of a rather steep roof.

For some applications, a more accurate value of the time-dependent solar radiation incident on a collector is desired. Both the scattering and absorption of



**FIGURE 8.8** Power and Daily Energy Incident on a Collector Facing the Sun at Noon.



**FIGURE 8.9** Power and Daily Energy Incident on a Fixed Collector Optimized for Winter Collection.

the radiation by the earth's atmosphere must be accounted for. These effects depend not only upon the composition of the earth's atmosphere but also upon the path of the radiation through the atmosphere. The length of the path depends upon the angle between the zenith,  $\psi$ , and the sun's rays.

$$\begin{aligned}\cos \psi &= (-\mathbf{a}_r \cdot \mathbf{S})/S \\ &= \cos \theta_d \sin \theta \sin \phi + \sin \theta_d \cos \theta\end{aligned}$$

Except for angles corresponding to the sun's being close to the horizon ( $\psi > 75^\circ$ ), an expression for the path length,  $L$ , assuming a flat earth/atmosphere model, is usually sufficiently accurate.

$$\begin{aligned}L &= h/\cos \psi = h \sec \psi \\ m &= L/h = \sec \psi\end{aligned}$$

An atmospheric height of  $h$  was used. The dimensionless quantity  $m$ , the path length normalized to when the sun is directly overhead, is known as the *air mass ratio*. This is the relative atmospheric mass that is traversed by incident solar radiation. Both scattering and absorption are a function of the air mass ratio. If  $I_0(\lambda)$  is the spectral radiation power density outside the atmosphere (at a wavelength  $\lambda$ ),  $I(\lambda)$ , the spectral radiation power density at the earth's surface, would be expected to depend upon an exponential function of the air mass ratio.

$$I(\lambda) = I_0(\lambda)e^{-k_\lambda m}$$

The coefficient,  $k_\lambda$ , depends upon the absorption and scattering properties of the atmosphere.

Neither the density nor the composition of the atmosphere is uniform with height. To account for these effects, a transmission factor,  $\tau(\lambda, m)$ , can be used.

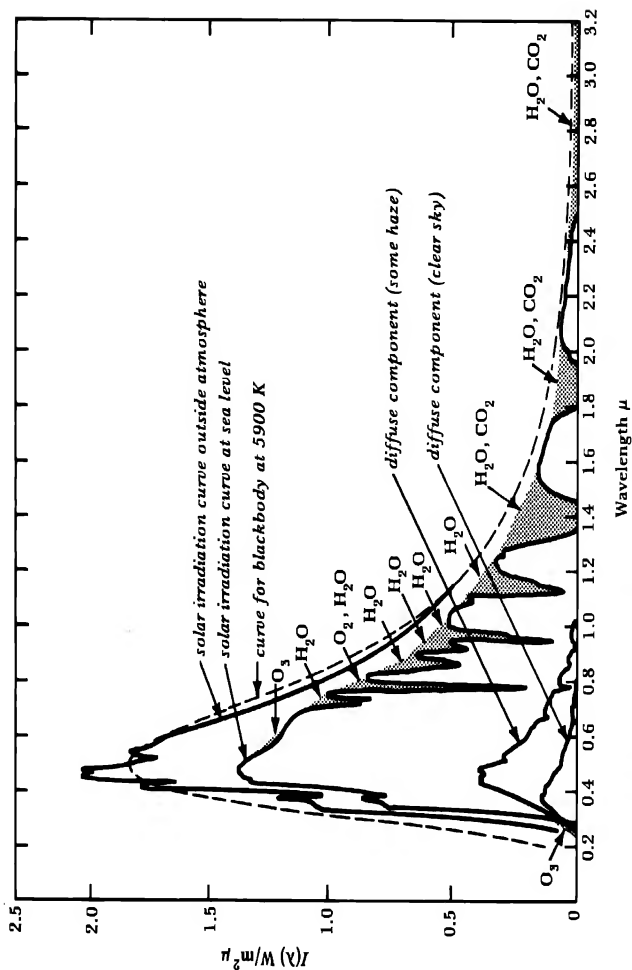
$$I(\lambda) = I_0(\lambda)\tau(\lambda, m)$$

Figure 8.10 illustrates the effect of the atmosphere on incident radiation for an air mass ratio of unity (vertically incident radiation). The absorption and scattering of incident radiation is strongly dependent upon its wavelength. Radiation is scattered by the main atmospheric constituents, oxygen and nitrogen molecules, as well as by water vapor and dust. It is absorbed by ozone, carbon dioxide, oxygen, water vapor, and dust. There are several absorption bands. With a knowledge of the transmission coefficient,  $\tau(\lambda, m)$ , an overall radiation power density at the earth's surface,  $S^*$ , can be obtained [22].

$$S^* = \int I(\lambda) d\lambda = \int I_0(\lambda)\tau(\lambda, m) d\lambda$$

The overall effect of the atmosphere may be described by an overall power transmission coefficient,  $\tau(m)$ , that is a function of the air mass ratio.

$$\begin{aligned}S^* &= \tau(m) \int I_0(\lambda) d\lambda = \tau(m)S \\ \tau(m) &= \frac{\int I_0(\lambda)\tau(\lambda, m) d\lambda}{\int I_0(\lambda) d\lambda}\end{aligned}$$



**FIGURE 8.10** The Effect of the Atmosphere on Incident Solar Radiation for an Air Mass Ratio of One (Reference 19).

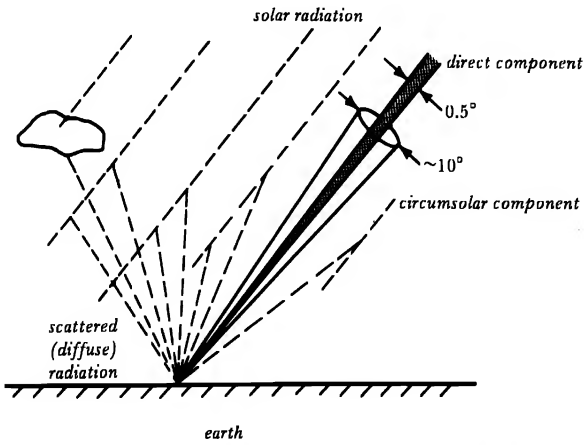
The radiation power density outside the atmosphere, the integral of  $I_0(\lambda)$ , is the solar constant,  $S$ .

Various approximate expressions for the power transmission coefficient have been introduced, but that by Hottel is very simple and yields particularly good results [23, 24].

$$\tau(m) = a_0 + a_1 e^{-km}$$

23-km visibility	(5-km visibility)
$a_0 = .1283$	(.0270)
$a_1 = .7559$	(.8101)
$k = 0.3878$	(.7552)

The first set of values of the dimensionless constants is based upon a model of a standard atmosphere with a 23-km visibility and the second set is for reduced visibility due to haze. The constants are strongly dependent upon altitude (the values given are for sea level) and weakly dependent upon climatic differences. The power transmission coefficient,  $\tau(m)$ , applies to radiation that appears to be coming directly from the sun (Figure 8.11). It includes the direct component, a cone with an angular width of about  $.5^\circ$ , and the circumsolar component, that due to forward scattering which subtends a disc with an angular width of about  $10^\circ$ . Scattering occurs for all rays of sunlight that enter the atmosphere. It is scattered radiation, usually referred to as *diffuse radiation*, that produces a bright daytime sky. On bright, clear days, the effect of the diffuse component tends to be minor, but on overcast days the



**FIGURE 8.11** Components of Incident Solar Radiation.

diffuse components is generally significant. The diffuse radiation component contributes to the output of nonfocused solar energy systems such as flat-plate collectors. Tabulated radiation values (for average conditions) often give both the direct radiation power density or daily energy density and that due to the sum of the direct and diffuse components [25].

### 3. COLLECTORS

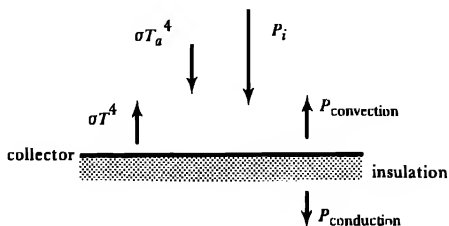
Collectors of solar energy often rely on a thermal process in which the temperature of a working fluid, typically water or air, is increased by the absorbed radiation. The efficiency of a collector depends upon temperature: the higher the temperature of a given collector, the greater its heat losses. Home and water heating, for example, requires only a moderate collector temperature. To achieve an indoor air temperature of 20 to 25°C, a temperature of 30 to 40°C for the heating air or water is required. If, however, the collected heat is to be stored in a thermal reservoir for use when the solar radiation is unavailable, a collector temperature of at least 50 to 60°C is necessary.

Both concentrating and nonconcentrating collectors can be used. The concentrating collector utilizes a mirror system to increase the intensity of the solar radiation at the point of collection and, as a result, considerably higher temperatures are achieved than with a nonconcentrating or flat-plate collector. A disadvantage of concentrating collectors is that a tracking mechanism to follow the sun may be required. The flat-plate collector can usually be left in a fixed position.

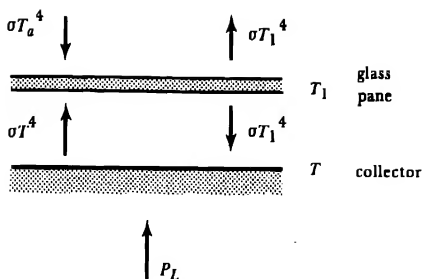
The simplest type of collector is a blackened flat-plate surface as indicated in Figure 8.12. Black-body radiation, convection, and conduction result in heat losses by the plate. Even at moderate temperatures, for example 100°C (373 K), black-body radiation is very significant.

$$P_{\text{black body}} = \sigma T^4 = (5.67 \times 10^{-8})(373)^4 = 1098 \text{ W/m}^2$$

The radiated power tends to be partially compensated for by the incident atmospheric black-body radiation. Radiation corresponding to an ambient temperature of 0°C



**FIGURE 8.12** *An Elementary Flat-Plate Collector.*



**FIGURE 8.13 Radiation Balance for Calculating Losses—Single-Pane Glass Cover.**

( $T_a = 273$  K), results in an input of  $315 \text{ W/m}^2$  leaving a net loss of  $783 \text{ W/m}^2$ . An input power density of nearly  $800 \text{ W/m}^2$ , a quantity comparable to the peak incident solar radiation, would thus be necessary to provide for the radiative losses of a  $100^\circ\text{C}$  collector. If convective losses are considered, an equilibrium temperature considerably less than  $100^\circ\text{C}$  would result for an incident power density of  $800 \text{ W/m}^2$ .

Radiative and convective losses may be reduced by means of a "heat trap" formed by a window of glass or other transparent material placed over the collector (Figure 8.13). Whereas high-quality glass is transparent to incident solar radiation with an energy peak corresponding to a wavelength of  $.5\mu$ , it absorbs long wavelength radiation corresponding to that emitted by the collector. For a  $100^\circ\text{C}$  collector temperature, the energy peak corresponds to a wavelength of  $8\mu$ . To achieve an equilibrium condition, an input power,  $P_L$  (watts per square meter), equal to the rate at which energy is lost from the collector has been introduced. Ignoring convective losses, the following conditions result when the system is in equilibrium.

$$\begin{aligned}\sigma T_a^4 + \sigma T^4 &= 2\sigma T_1^4 && \text{glass} \\ \sigma T_1^4 + P_L &= \sigma T^4 && \text{collector}\end{aligned}$$

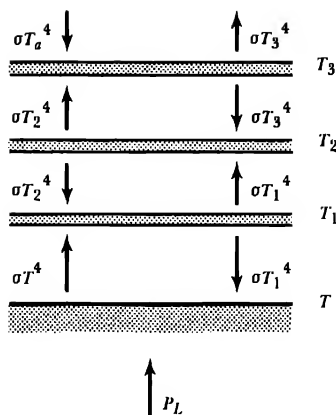
The glass is assumed to be at a temperature of  $T_1$ , the collector at a temperature of  $T$ , and both are assumed to behave as ideal black-body radiators. Eliminating  $T_1$ , the following is obtained for the collector loss,  $P_L$ .

$$\begin{aligned}\sigma T_1^4 &= \sigma T^4 - P_L \\ \sigma T_a^4 + \sigma T^4 &= 2(\sigma T^4 - P_L) \\ P_L &= \frac{1}{2}\sigma(T^4 - T_a^4)\end{aligned}$$

The collector behaves as if it had an emissivity for long wavelengths,  $\epsilon$ , of  $\frac{1}{2}$ .

$$P_L = \epsilon\sigma(T^4 - T_a^4)$$





**FIGURE 8.14 Radiation Balance—Several Glass Panes.**

Radiation losses are thus only one-half of those without a glass cover.

Additional layers of glass, as indicated in Figure 8.14, further reduce radiative heat losses. With  $n$  panes of glass, the following radiative heat loss results for a flat-plate collector.

$$P_L = \frac{\sigma}{n+1} (T^4 - T_a^4)$$

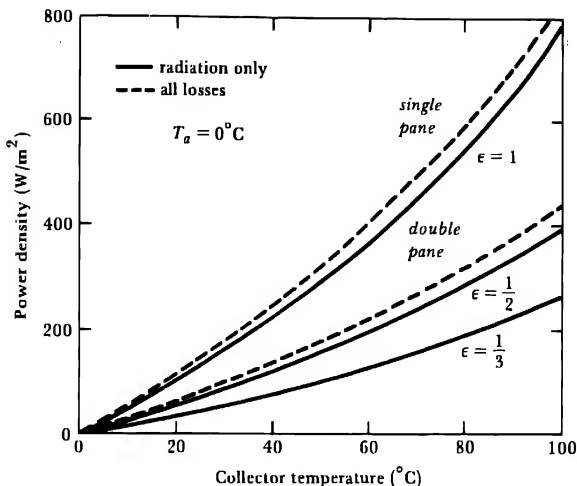
One pane of glass corresponds to an emissivity of  $\frac{1}{2}$ , two panes to  $\frac{1}{3}$ . Radiative losses for an ambient background temperature of  $0^\circ\text{C}$  in which convective effects are ignored are presented in Figure 8.15.

Convective heat transfers between the glass plates and from the outer plate to the surrounding air increase the heat losses of a collector. Along with the radiation losses, Figure 8.15 shows the total losses calculated from the data of Hottel and Woertz [26]. The data is for  $0^\circ\text{C}$  ambient temperature and a collector tilted  $30^\circ$  from the horizontal in a 10-mph wind. (Convective losses are dependent upon tilt and wind.) The overall heat loss is approximately twice the amount obtained from the above approximation based upon an elementary radiative model.

In designing a solar-energy system, the first-law collector efficiency,  $\eta$ , the ratio of the output power density,  $P_o$ , to the incident power density,  $P_i$ , is important.

$$\eta = P_o/P_i$$

The glass (or other transparent material) cover reflects as well as absorbs a portion of the incident radiation. If the transmission coefficient of one pane of glass is  $\tau$ , the radiation reaching the collector if the effect of multiple reflections is ignored is



**FIGURE 8.15 Radiation and Total Losses for a Flat-Plate Collector.**

$\tau^n P_i$  for  $n$  panes of glass. Assuming the collector plate behaves as an ideal black body, the absorbed power  $P_A$ , is equal to that transmitted through the glass.

$$P_A = \tau^n P_i$$

For equilibrium, the absorbed power is equal to the sum of the output and lost power. Hence, the following efficiency is obtained for a steady-state collector in which the effect of the slight temperature increase of the glass caused by the incident absorbed power is ignored.

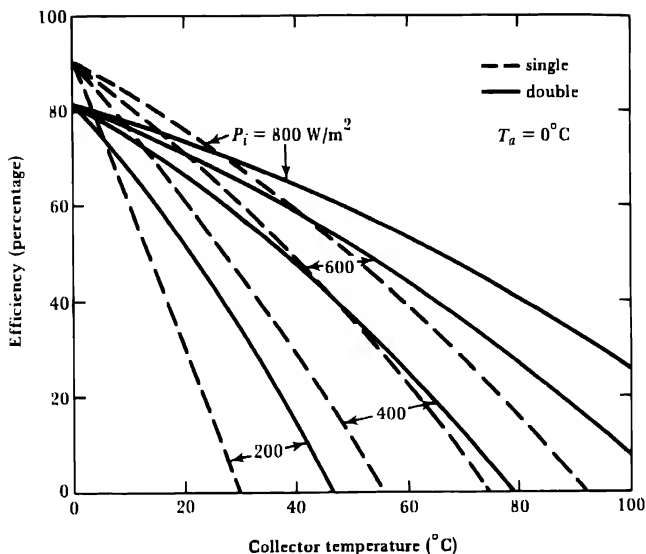
$$P_A = P_0 + P_L$$

$$P_0 = P_A - P_L = \tau^n P_i - P_L$$

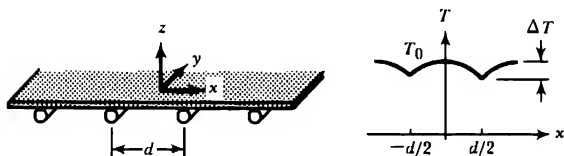
$$\eta = P_0/P_i = \tau^n - P_L/P_i$$

Figure 8.16 is a plot of efficiency assuming typical window glass ( $\tau = .9$ ) and based upon the power losses of Figure 8.15. Even for two panes of glass, collection efficiencies for temperatures above 50°C exceed 50% only for an incident power density greater than 600 W/m<sup>2</sup>. Just a small improvement is achieved with three glass panes since incident absorptive and reflective losses increase. Based upon this data, daily energy collection of a well-designed collector will tend to be less than 50% of the incident energy for a 0°C ambient temperature.

Another inefficiency arises in the transfer of heat from the collector to the working fluid. For example, a common collector system consists of a flat copper



**FIGURE 8.16** Efficiency of a Flat-Plate Collector.

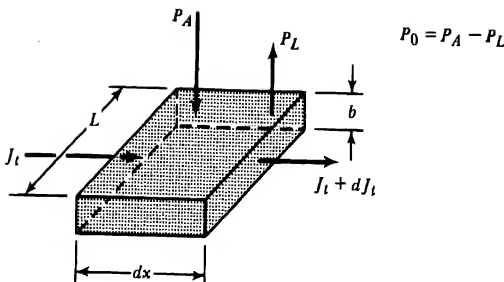


**FIGURE 8.17** A Flat-Plate Collector with Attached Water Tubes.

plate with water tubes attached to its backside. The temperature variation between the water tubes (Figure 8.17) depends upon the thermal conductivity,  $\sigma_t$ , of the collector plate. For a thin plate, a one-dimensional heat transfer equation is obtained in which  $J_t$  is the thermal power density.

$$J_t = -\sigma_t \frac{dT}{dx} \quad \text{W/m}^2$$

The dependence of  $J_t$  on the incident radiation may be found by considering a differential portion of the plate, Figure 8.18.



**FIGURE 8.18 Differential Portion of a Flat-Plate Collector.**

$$J_t L b + (P_A - P_L) L dx = (J_t + dJ_t) L b$$

$$P_o = P_A - P_L$$

$$P_o dx = b dJ_t$$

$$\frac{dJ_t}{dx} = \frac{P_o}{b}$$

Differentiating the one-dimensional heat transfer relationship and combining it with that above, a second-order differential equation results.

$$\frac{dJ_t}{dx} = -\sigma_1 \frac{d^2 T}{dx^2}$$

$$\frac{d^2 T}{dx^2} = -\frac{P_o}{b\sigma_1}$$

In general,  $P_o$  is a function of temperature. To minimize heat losses, a nearly uniform temperature is desired. If  $P_o$  is assumed to be constant, a rather simple expression results for the temperature.

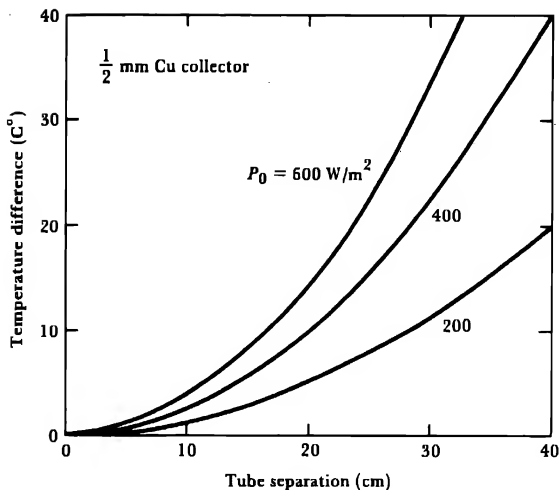
$$T = T_0 - \frac{P_o x^2}{2b\sigma_1}$$

A symmetrical solution was assumed in which the temperature midway between the tubes ( $x = 0$ ) was set equal to  $T_0$ . Evaluating the above expression for  $x = d/2$ , the temperature at the tube and the resultant temperature difference,  $\Delta T$ , can be found.

$$T(x = d/2) = T_0 - \frac{P_o d^2}{8b\sigma_1}$$

$$\Delta T = T_0 - T(x = d/2)$$

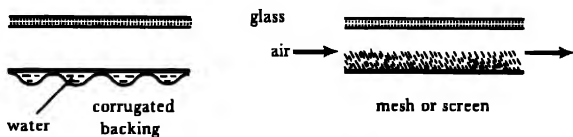
$$= \frac{P_o d^2}{8b\sigma_1}$$



**FIGURE 8.19** *Temperature Difference as a Function of Tube Spacing.*

Figure 8.19 is a plot of the temperature difference as a function of tube spacing for a  $\frac{1}{2}$  mm-thick copper sheet ( $\sigma_t = 400 \text{ W/m K}$ ). For a  $600 \text{ W/m}^2$  output power density, a tube separation of only 16 cm results in a  $10\text{C}^\circ$  temperature difference. Small tube spacings are thus required. If in order to reduce the collector cost,  $\frac{1}{2}$  mm-thick galvanized steel were used,  $\Delta T$  for a given spacing would be four times as great ( $\sigma_t \approx 100 \text{ W/m K}$ ); hence, even closer spaced tubes would be required. Nearly continuous contact of the heat transfer fluid with the collector is desirable. Figure 8.20 illustrates two improved collector configurations.

An approximate expression, initially suggested by Hottel and Whiller [26], has been found to be reasonably accurate in describing the behavior of flat-plate collectors. Both convective and conductive heat losses, for a given collector design,



**FIGURE 8.20** *Improved Collector Configuration for Minimizing Temperature Variations.*

tend to be proportional to the difference in temperature of the collector and the ambient surroundings. Radiative losses, however, depend upon the fourth power of the temperatures involved. The radiative heat transfer between two similar planar surfaces of temperatures  $T_1$  and  $T_2$  may be expressed as follows.

$$\begin{aligned} P &= \varepsilon\sigma(T_2^4 - T_1^4) \\ &= \varepsilon\sigma(T_2^2 + T_1^2)(T_2 + T_1)(T_2 - T_1) \\ &= h_r(T_2 - T_1) \end{aligned}$$

Even though the transfer coefficient,  $h_r$ , is a function of the two temperatures, it is not critically dependent upon the actual temperatures if their difference is small.

$$\begin{aligned} h_r &= \varepsilon\sigma(T_2^2 + T_1^2)(T_2 + T_1) \\ &\approx 4\varepsilon\sigma T_1^3 \quad \text{or} \quad 4\varepsilon\sigma T_2^2 \end{aligned}$$

If all radiative heat transfers are approximated as a linear function of the applicable temperature differences, the overall heat loss due to radiation of a collector will be a linear function of the overall temperature difference. All losses are then linearly related to the difference in temperature between the collector and the ambient surroundings.

$$P_L = U_L(T - T_a)$$

where  $U_L$  = overall loss coefficient,  $\text{W/m}^2 \text{K}$

A particularly simple expression is then obtained for the collector output power density and its efficiency.

$$\begin{aligned} P_o &= \tau^n P_i - U_L(T - T_a) \\ \eta &= P_o/P_i = \tau^n - U_L(T - T_a)/P_i \end{aligned}$$

The above expressions are for a unit area of collector ( $1 \text{ m}^2$ ), but a similar expression can be obtained for a complete collector in which edge effects are included. In experimentally evaluating a collector, the thermal efficiency is often plotted as a function of the temperature difference divided by the total incident power. The slope of the efficiency curve is thus the negative of the overall loss coefficient for the collector.

Numerous articles deal with aspects of flat-plate collectors [27–35]. Flat-plate collectors are relatively insensitive (for angles within  $45^\circ$  of normal incidence) to the direction from which the incident radiation originates. Both diffused sunlight, which tends to originate somewhat uniformly throughout the sky, and direct sunlight contribute to its output power. This can be important for frequently overcast regions where a substantial portion of the incident solar flux is in the form of diffuse radiation.

Tabulations of average expected daily energy densities which include the effect of diffuse or scattered radiation are generally used in designing solar energy systems [3–10, 36]. These tables are compiled for numerous cities and are based upon several years of recorded weather data.

The first-law efficiency of a solar collector depends upon the temperature at which the thermal energy is extracted from the collector. The efficiency is maximum when the collector temperature is equal to the ambient temperature, but the energy extracted from the collector for this condition is of no value since the rate at which available work is supplied is zero. In general, the rate at which available work is supplied by a collector depends upon its temperature,  $T_c$ , its thermal output power,  $P_o$ , and the ambient temperature,  $T_a$ .

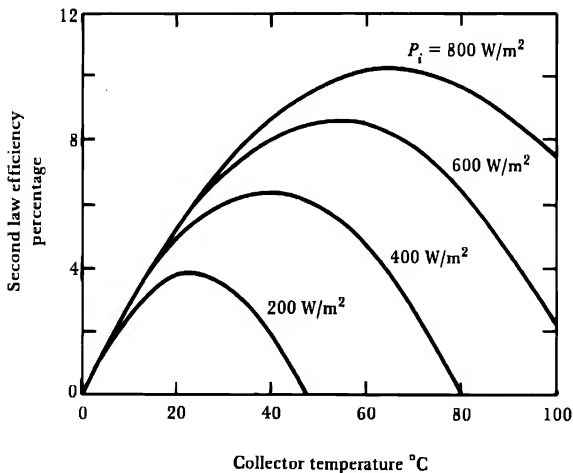
$$B(\text{actual}) = P_o(1 - T_a/T_c)$$

An ideal system, however, would not only recover the entire incident power,  $P_i$ , but would also recover it at a temperature corresponding to the observed surface temperature of the sun,  $T_s$ .

$$B(\text{ideal}) = P_i(1 - T_a/T_s)$$

The second-law efficiency for a collector may thus be obtained.

$$\begin{aligned} \epsilon &= \frac{B(\text{actual})}{B(\text{ideal})} = \frac{P_o(1 - T_a/T_c)}{P_i(1 - T_a/T_s)} \\ &= \eta \frac{(1 - T_a/T_c)}{(1 - T_a/T_s)} \end{aligned}$$



**FIGURE 8.21** *Second-Law Efficiency of a Flat-Plate Collector for an Ambient Temperature of 0° C (Derived from Data of Figure 8.16).*

The second-law efficiency of a flat-plate collector is indicated in Figure 8.21. The first-law efficiency of the collector of Figure 8.16 was utilized and the temperature of the extracted power was assumed to be equal to that of the collector surface.

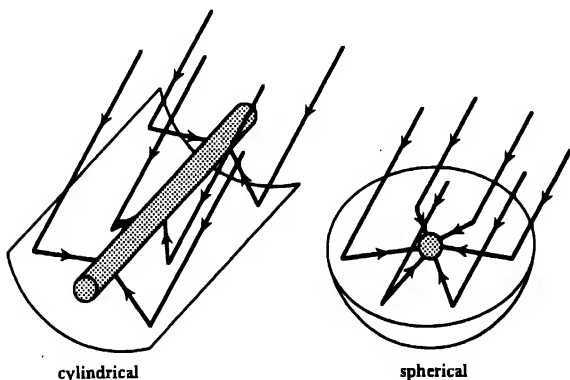
For a specified input power density, there is a collector temperature that results in a maximum second-law efficiency, the temperature of the maximum increasing with the input power level. At the condition corresponding to the maximum second-law efficiency, the rate at which entropy increases for the system is a minimum. At any other collector temperature, the entropy increase is larger. The small second-law efficiency of a flat-plate collector implies a large entropy increase and a corresponding large loss of available work. However, the entropy increase for a collector supplying an output power ( $P_o > 0$ ) is less than that occurring in the absence of a collector. Without the solar collector the incident radiation is simply absorbed and/or reflected by some surface (a roof top, for example) and is eventually degraded to low-temperature thermal energy at the ambient temperature, resulting in an even greater entropy increase. Therefore, a solar collector, regardless of how poorly it may function, results in smaller entropy increase than would otherwise occur.

Parabolic reflectors, Figure 8.22, are frequently used to concentrate solar energy. The active portion of the collector containing the heat transfer fluid can, if a high degree of concentration is utilized, be rather small compared to the total area from which the incident flux is intercepted. If  $P'_i$  is the average power density incident on the active or target region, a concentration ratio,  $X$ , may be defined as follows.

$$X = P'_i / P_i$$

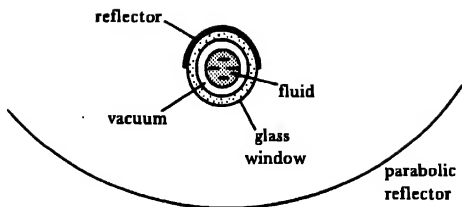
The efficiency may now be expressed in terms of  $P'_i$ .

$$\begin{aligned}\eta &= P_o / P'_i = \tau^n - P_L / P'_i \\ &= \tau^n - P_L / (X P_i)\end{aligned}$$



**FIGURE 8.22 Parabolic Reflectors.**





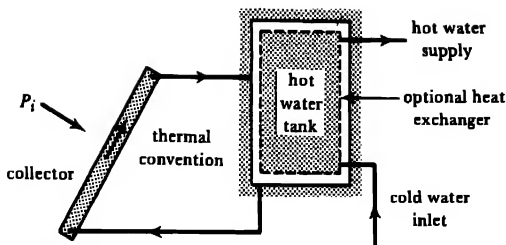
**FIGURE 8.23 A High-Temperature Parabolic Collector.**

Since large concentration ratios are possible, the loss factor,  $P_L/(XP_i)$ , can be significantly reduced. Conversely, higher temperatures which increase  $P_L$  can be achieved while maintaining a reasonable efficiency. Since only rays of light along or near the preferred direction of a concentrating system will be focused, a mechanical tracking system is necessary. Diffused radiation does not contribute to its output. An example of a collector using a parabolic cylinder for a reflector is that of Figure 8.23. The active collector area, and hence its total heat loss, can be small. In addition, heat losses can be further reduced by evacuating the portion adjacent to the tube containing the working fluid, so that only radiation losses are significant. The reflector surface should have a high reflectivity, but the dimensional tolerance of the surface need only be sufficient for all the reflected incident energy to be focused on the target (an optically precise surface is unnecessary).

#### 4. LOW-TEMPERATURE APPLICATIONS: WATER AND SPACE HEATING

Four percent of the total U.S. energy consumption in 1968 (Figure 1.8) was for residential and commercial hot water heating. Solar water heaters providing water with a temperature of 55 to 70°C are commercially available in several countries. With increasing fuel costs and scarcities, the heating of water by solar energy will very likely increase.

Based upon the data of Figure 8.9 (40° latitude and collector optimized for winter collection), an average winter energy density of 5.3 kWh/m<sup>2</sup> is incident on a collector. If an overall energy collection efficiency of one-third is achieved, an energy of 1.8 kWh/m<sup>2</sup> would be obtained. For locations closer to the equator, not only is the incident energy flux greater but higher ambient temperatures also tend to increase the collector efficiency, further increasing the output. The collector area for a daily hot water supply of 200 liters (the amount required for a small family) may readily be calculated. For an inlet water temperature of 10 to 20°C, a temperature rise of approximately 50°C is required. Since 10<sup>3</sup> cal are required to increase the temperature of one liter of water one Celsius degree, a total daily energy of 10<sup>7</sup> cal or 4.184 × 10<sup>7</sup> J is necessary. A collector system with an output of 11.6 kWh (1 kWh = 3.6 × 10<sup>6</sup> J) would thus be needed. For a winter output energy of 1.8 kWh/m<sup>2</sup>, a relatively small-sized collector of 6.5 m<sup>2</sup> will provide the 11.6 kWh. Collectors for warmer climates could even be substantially smaller.



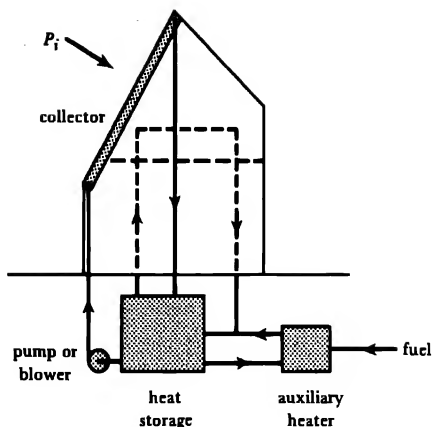
**FIGURE 8.24** *Solar Hot Water Heater.*

Figure 8.24 shows the basic elements of a hot water installation suitable for a flat-roofed building. With the hot water tank located above the collector, thermal convection circulates the water. This system, however, is suitable only for rooftop storage tanks. If the tank is below the collector, for example, in the basement of a residence, a small pump would be necessary to circulate the water. For the simplest water storage system, the collector system must withstand the full water main pressure. An alternative scheme is to use a heat exchanger formed by placing the pressurized hot water tank within a slightly larger tank which is connected to the collector. Not only would the collector not need to withstand the water main pressure but a water mixture of ethylene glycol (antifreeze) could be used for climates where below freezing ambient temperatures are common.

The collector size calculated for the above example was based upon providing sufficient hot water for one day's usage. For a system to be able to supply hot water during overcast periods, a larger storage tank and collector would be necessary. Alternatively, an auxiliary heater could be used to augment the solar input during inclement weather.

Space heating of residential and commercial structures, another usage of low-quality heat, accounted for nearly 18% of the 1968 U.S. energy consumption. A substantial portion of the energy required to heat buildings could be derived from the sun through solar collectors mounted on the roof. However, the greatest solar flux during a typical day usually corresponds to the interval during which the least heat is required. Energy storage is therefore necessary not only for nighttime heat but also for overcast or stormy periods. During extended periods of insufficient solar flux, an auxiliary heating source is usually needed, since a total reliance on solar energy would tend to result in an excessively costly system. Figure 8.25 shows a typical home heating system for a residential structure. Both air and water can be used for a heat transfer medium. Only for unique types of construction does one heat transfer fluid offer a significant advantage over the other.

Heat losses from buildings depend upon the thermal conductivity of the walls, ceiling, and floors, the type and size of windows and doors, and the air infiltration rate. An estimate of the thermal losses and gains of a single-story structure, with a floor area of  $150 \text{ m}^2$ , will be used to evaluate a solar heating system. Daily energy

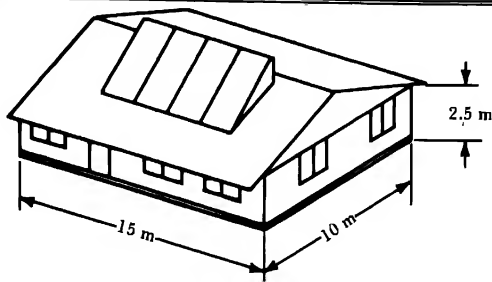


**FIGURE 8.25** *Typical Solar Home Heating System.*

losses and gains for two construction alternatives are listed in Table 8.1. Structure 1 has uninsulated  $2 \times 4$  framed walls, single-pane windows, and a relatively high infiltration rate. The thermal characteristics of this structure might be considered fairly typical of residences built between 1950 and 1975. Compared to today's construction practices, the building would be classified as poorly insulated. Structure 2 of Table 8.1 is well insulated, with insulated  $2 \times 6$  framed walls, double-pane windows, storm doors, a greater thickness of floor and ceiling insulation, and a low infiltration rate. The heat gains (for both structures) are due to the occupants (metabolism), electrical appliances, hot water usage, and solar gains of south-facing windows. The effect of average indoor/outdoor temperature differences of  $20^\circ\text{C}$  ( $20^\circ\text{C}$  indoor temperature and  $0^\circ\text{C}$  outdoor temperature, for example) and  $30^\circ\text{C}$  ( $\sim -10^\circ\text{C}$  outdoor temperature) are given in the table. Although this data is typical, a set of data can, and should, be compiled for the particular structure for which a solar-energy heating system might be considered.

For the poorly insulated building (structure 1 of Table 8.1), a collector area of  $93 \text{ m}^2$  would be required for an average outdoor temperature of approximately  $0^\circ\text{C}$  based upon a daily energy output from the collector of  $1.8 \text{ kWh/m}^2$ . For an outdoor temperature of  $-10^\circ\text{C}$ , a collector area of  $158 \text{ m}^2$  would be required (an area that is greater than the floor area of the building). For an overall system cost of  $\$300/\text{m}^2$  of collector, investments of  $\$28,000$  and  $\$47,000$  would be required to supply the heating needs for  $0^\circ\text{C}$  and  $-10^\circ\text{C}$ , respectively. The higher investment is comparable (and possibly greater) than the construction cost of the structure. The well-insulated structure requires a much smaller collector area,  $11 \text{ m}^2$  for an outdoor temperature of  $0^\circ\text{C}$  and  $34 \text{ m}^2$  for  $-10^\circ\text{C}$ . System costs would be  $\$3300$  and  $\$10,000$ , respectively. Although the construction cost of the well-insulated

**TABLE 8.1 Residential Heating Thermal Characteristics and Collector Requirements for Heating a Typical Residential Structure**



<i>Indoor/outdoor temperature difference</i>		20 °C (~ 0°C outdoor temperature)	30 °C ( ~ -10°C outdoor temperature)
Structure 1 poorly insulated <sup>a</sup>	Daily heat loss	236 kWh	354 kWh
	Daily heat gain <sup>c</sup>	69 kWh	69 kWh
	Daily heating requirement	167 kWh	285 kWh
	Collector area	93 m <sup>2</sup>	158 m <sup>2</sup>
Structure 2 well insulated <sup>b</sup>	Daily heat loss	85 kWh	127 kWh
	Daily heat gain <sup>c</sup>	65 kWh	65 kWh
	Daily heating requirement	20 kWh	62 kWh
	Collector area	11 m <sup>2</sup>	34 m <sup>2</sup>

<sup>a</sup> Uninsulated 2 × 4 framed walls, single-pane windows, relatively high infiltration rate, floor and ceiling insulation.

<sup>b</sup> Insulated 2 × 6 framed walls, double-pane windows, storm doors, low infiltration rate, extra floor and ceiling insulation.

<sup>c</sup> From occupants, electrical appliances, hot water usage, and solar gains of windows.

structure is greater than that of the noninsulated structure, the added cost is considerably less than the cost difference of the solar energy systems that would be required. A substantial investment in reducing the heating requirements of a building can be economically justified for solar heating.

Energy storage is required with an active solar collector system. To provide nighttime heating, the storage capacity must be sufficient to store the thermal energy required for one day's heating. A storage capacity of 62 kWh, the daily energy needed for the well-insulated structure (– 10°C outdoor temperature), can readily be achieved.

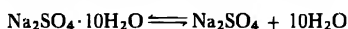
The lowest temperature from which heat can be extracted for heating the

residence to 20 to 25°C is approximately 30°C. The upper temperature is limited by the collector efficiency which diminishes at elevated temperatures. An upper storage temperature of 60°C results in a useful heat storage temperature differential of 30°C.

One kilogram of water has a specific heat of 1000 calories per Celsius degree. A 30-degree difference provides an energy storage of  $3 \times 10^4$  cal/kg ( $1.26 \times 10^5$  J/kg) or 34.9 kWh/kg. A total stored energy of 62 kWh therefore requires nearly 1800 kg of water (slightly less than 500 gallons). The corresponding water volume, 1.8 m<sup>3</sup>, is a cube only 1.2 m on a side. Tanks of twice this capacity, 1000 gal, are frequently used for water storage cisterns and hence are readily available. With two days' energy storage, auxiliary heating would be needed only for exceedingly severe weather or extended overcast periods. Water heat-storage systems are suitable for water-cooled collectors mounted on the roof. Since the storage tank would normally be below the house, a small pump, thermostatically controlled by the collector temperature, is needed to circulate the water. If the collector is designed to drain when the pump is off, freezing problems are avoided. The heat can be transferred to the house by a forced air system and a water-to-air heat exchanger. Included in the storage tank could be a heat exchanger for supplying hot water.

Rock storage is convenient for use with air-cooled collectors. On a per unit volume basis, however, the specific heat of loosely packed rocks (between which air can be circulated) is only about 40% that of water. A storage volume 2.5 times that of water is therefore necessary. Heat transfer is accomplished by means of blower systems connected between the collector and storage system and between the storage system and the area to be heated. Since no liquids are necessary, freezing is not a problem.

Alternative storage systems that have been proposed utilize chemical state transitions. One example is the hydrate transition of sodium sulfate.



This transition, which occurs at a temperature of 32.3°C, absorbs or releases an energy of  $5 \times 10^4$  cal/kg, that is, an energy equal to a 50 C° temperature change for an equal quantity of water. Since the chemical transition is at a temperature considerably less than that of the upper temperature of a water or rock storage system, higher collector efficiencies can be realized. Unfortunately, settlement of the sodium sulfate crystals and stratification of the mixture tend to negate the theoretical advantage of hydrate storage. While other chemical transitions have also been investigated water or rocks remain the presently accepted storage media.

As already mentioned, a monetary comparison between solar-energy and energy-conserving systems on the one hand and the use of conventional fuels or electricity on the other is difficult. Since costs and savings occur at different times, an economic analysis is generally based upon the interest rate for borrowing money. If a quantity of money,  $Y_0$ , accrues a yearly interest of  $i$ , the quantity after  $n$  yearly compoundings may readily be obtained.

$$Y_n = (1 + i)^n Y_0$$

Long-term loans, such as mortgages, are often based upon monthly transactions and hence a monthly compounding occurs. Therefore, a differential equation for monetary transactions, as used for the fuel and resource analysis of Chapters 1 and 2, is appropriate.

$$\frac{dY}{dt} = iY$$

$$Y = Y_0 e^{it}$$

The annual interest rate is  $i$  and the instantaneous value of an initial investment of  $Y_0$  is  $Y$  at a future time  $t$ . The factor determining the yearly increase, for exponential behavior, is  $e^i$  and the yearly increase is  $1 - e^i$ . For small values of interest, the increase is simply  $i$ ; for high values, it is larger. Fifteen percent per year (yearly rate of interest) when compounded continuously (or monthly) results in a yearly increase of approximately 16%.

Solar-energy systems generally require large initial investments. The investment,  $Y_0$ , might be borrowed and then paid back over the life of the system,  $T$  years. A simple but common procedure would be to pay off the debt at a constant yearly payment rate,  $C$ . If part of a mortgage, monthly payments would be approximately  $C/12$ . Yearly fuel savings could be compared with the repayment rate. It is therefore desirable to determine the repayment rate which depends upon the interest rate (the interest rate will be assumed constant). For a differential time period, the following applies for a differential change in the outstanding debt,  $Y$ .

$$dY = -(C - iY) dt$$

For a zero repayment rate ( $C = 0$ ), the debt continually increases, but if the parenthetical expression is positive, the debt decreases. Since the initial debt is  $Y_0$ , a minimum repayment rate is necessary if the debt is to be eventually retired.

$$C > iY_0$$

The differential equation may readily be solved assuming the initial value of the debt,  $Y_0$ , occurs at  $t = 0$ .

$$Y = 1/i[C - (C - iY_0)e^{it}]$$

If the debt is to be paid off in  $T$  years, the repayment rate is determined by setting the debt,  $Y$ , to zero at this time.

$$0 = 1/i[C - (C - iY_0)e^{iT}]$$

$$C = \frac{iY_0}{1 - e^{-iT}}$$

It will be noted that for a very long repayment time,  $T$ , a time that would be associated with an investment that is not expected to wear out, the yearly repayment rate is equal to the yearly interest on the initial investment.

$$C \longrightarrow iY_0 \text{ as } T \longrightarrow \infty$$

If only the repayment of an investment debt and saved energy costs need to be considered for a particular system, then a system is economically justifiable if the repayment rate is less than the yearly energy savings.

$$C = \frac{iY_0}{1 - e^{-iT}} < \text{yearly energy savings}$$

The above criteria is easy to apply if anticipated energy savings remain unchanged. If energy savings change (most likely increase), the repayment of the debt, at least in theory, might be arranged to follow in step with the change in energy prices. It is often assumed that monetary energy savings will increase at a constant yearly percentage, that is, exponentially.

$$S = S_0 e^{\alpha t}$$

For the above relation, the energy savings,  $S$ , are assumed to increase at a yearly rate of  $\alpha$  and have an initial value of  $S_0$ . A time-dependent repayment rate with the same time dependence may be used for an economic analysis.

$$C = C_0 e^{\alpha t}$$

It may be shown that for the debt to be repaid in  $T$  years, the initial repayment rate,  $C_0$ , is the following (Problem 8.9).

$$\begin{aligned} C_0 &= \frac{(\alpha - i) Y_0}{e^{(\alpha - i)T} - 1} \\ &= Y_0 / T \quad \text{for } \alpha = i \end{aligned}$$

The ratio of the yearly energy savings and the repayment rate is a constant.

$$S/C = S_0/C_0$$

The following thus applies for a money-saving investment.

$$C_0 = \frac{(\alpha - i) Y_0}{e^{(\alpha - i)T} - 1} < S_0$$

$$Y_0 / T < S_0 \quad \text{for } \alpha = i$$

While a fuel savings and repayment rate with any dependence on time might be considered, an analytical solution for the repayment rate may not exist.

An investment decision may involve yearly costs, such as maintenance and taxes, that have a different time dependence than anticipated fuel savings. For this situation, all benefits and costs can be translated to a given point in time through a process referred to as *future discounting*. A sum of money, invested at a yearly interest rate of  $i$ , increases exponentially in time.

$$x(t) = x_0 e^{it}$$

This may be viewed from another perspective.

$$x_0 = x(t) e^{-it}$$

To provide for a future expense,  $x(t)$ , a sum of money,  $x_0$ , could be set aside at  $t = 0$ , the "present" time. Similarly, a benefit at a future time, such as a fuel savings, is worth less at the present time than in the future since the present value of the savings, if suitably invested, would accrue sufficient interest, to be equal to the benefit that occurs at a later time. The higher the interest rate,  $i$ , the greater the discounting of future costs and savings.

If the yearly costs or benefits are  $x(t)$ , the present value,  $X_0$ , is simply the integral of the discounted value over the period of interest.

$$X_0 = \int_0^T x(t)e^{-it} dt$$

Regardless of the time dependence of both costs and benefits, their present value may be ascertained. Assuming the investment occurs at the present time, the total present cost for an energy-saving system is the investment cost plus the present values of all future costs. If the present value of future savings exceeds the present cost, the investment is economically justified. This type of economic analysis may readily be applied to solar-heating systems [37, 38].

**Example** A solar collector supplies, on the average, 80% of a daily thermal energy requirement of 11.6 kWh for heating water. The system costs \$2000 and it has an expected life of 20 years.

- Assume an interest rate of 6%/y (no inflation) and that the cost of electrical energy remains unchanged. Determine the price of electrical energy (cents per kilowatt-hour) that results in savings equalling costs (break-even condition) for the solar heating system. Without the collector, electricity would have been used to power a resistance-type water heater (100% first-law efficiency).
- As a result of a 10%/y inflation rate, the interest rate is 16%/y. Electricity prices increase at the inflation rate, that is, at 10%/y. Determine, for these conditions, the initial electricity price for a break-even investment.
- Assume that fuel prices increase at a yearly rate that is 5% greater than the interest rate. What is the initial electricity price for a break-even investment?

*Solution*

- First, the yearly repayment rate for the \$2000 is determined.

$$C = \frac{(.06)(2000)}{1 - e^{-(.06)20}} = \$171.72/y$$

Hence, a yearly energy cost of \$171.72 corresponds to a break-even condition. The electrical energy saved over a one-year period is found.

$$(.80)(11.6 \text{ kWh/day})(365 \text{ days/y}) = 3887 \text{ kWh}$$

Only for an electricity cost that is greater than 5.07¢/kWh can the investment for a solar system be justified.

- To match an exponentially increasing energy price, an exponentially increasing repayment is desired.



$$C = C_0 e^{\alpha t}$$

$$C_0 = \frac{(\alpha - i) Y_0}{e^{(\alpha - i)T} - 1} = \frac{(i - \alpha) Y_0}{1 - e^{-(i - \alpha)T}}$$

For an interest rate of 16%/y ( $i = .16$ ) and a 10%/y increase in the price of energy ( $\alpha = .10$ ), the initial yearly repayment,  $C_0$ , is the same as that of part (a) and hence the initial electricity price is also the same. For this case, it is only the difference between the interest rate and the inflation rate,  $i - \alpha$ , that is important.

- c. For the conditions of this part,  $\alpha - i = .05$ .

$$C_0 = \frac{.05(2000)}{e^{.05(20)} - 1} = \$58.20/\text{y}$$

An initial electricity price of 1.72 ¢/kWh results in a break-even investment. For an initial electricity price of 6 ¢/kWh, the first first year's electricity savings would be \$203.

**Example** For the system previously considered, a 30% tax credit is allowed for the initial investment. The yearly maintenance cost is \$100 and the prevailing interest rate is 6%/y. (Inflation is being ignored, that is, a solution in "fixed dollars" will be obtained.)

- Using a present-value type of analysis, determine the minimum electricity price that results in a break-even condition. The price of electricity is constant.
- Assume the price of electricity increases at a yearly rate of 5%. What is the initial electricity price that results in a break-even condition? What is the initial electricity price for a 10%/y increase?

*Solution*

- The effect of the 30% tax credit is to reduce the initial investment,  $Y_0$ , to an effective value of \$1400. The present value,  $X_0$ , of a yearly maintenance cost,  $K$ , of \$100 may be found.

$$X_0 = \int_0^T K e^{-it} dt = K/i(1 - e^{-iT}) = \$1165$$

A constant yearly fuel savings,  $S$ , has the following present value,  $S_0$ .

$$S_0 = \int_0^T S e^{-it} dt = S/i(1 - e^{-iT}) = 11.65S$$

The break-even condition may be applied to the present values.

$$S_0 = Y_0 + K$$

$$11.65S = 1400 + 1165$$

$$S = \$220/\text{y}$$

An electricity price greater than 6.5 ¢/kW is required to justify the investment.

- b. For an energy savings that increases at a yearly rate of 5% ( $\alpha = .05$ ), the following is obtained for the present value of the accumulated savings,  $S_0$ .

$$\begin{aligned} S &= R_0 e^{\alpha t} \\ S_0 &= \int_0^T R_0 e^{\alpha t} e^{-it} dt = \frac{R_0}{i - \alpha} [1 - e^{-(i-\alpha)T}] \\ &= 18.13 R_0 \end{aligned}$$

Since the present value of the costs remains unchanged, the initial yearly cost of \$141 (4.18 ¢/kWh) results in a break-even condition. For a 10%/y electricity price increase, an initial yearly electricity cost of \$83.71 (2.47 ¢/kWh) results in a break-even condition.

A determination of an economically optimal solar space- or water-heating system depends, among other factors, upon the day-to-day variation of available solar energy. For a specified daily variation in energy (over a period of a year), a computer simulation, for a given set of economic conditions, may be used to determine the optimum size of a solar collection and storage system. However, given the year-to-year variability of the weather, this optimization procedure is of little value. Actual designs of solar heating systems are therefore usually based upon a set of average conditions and empirically derived performance characteristics. A frequently used design criteria is known as the *f-chart method*, in which the fraction,  $f$ , of the heating need supplied by the solar heating system is determined [39]. This, as well as other design procedures, none of which may be considered as universally accepted, is presented in most texts on solar energy [3-10].

Heating structures with passive solar energy systems (no mechanically driven heat transfers) has also received considerable attention [40]. In this system, the collectors and thermal storage are an integral part of the structure of the building. South-facing windows, for example, might serve as the collector while a massive masonry structure (or containers of water) might be located adjacent to the windows (a Trombe wall). The wall is heated during the day and gives off its heat to the interior at night. A relatively large thermal storage mass is required to avoid excessive interior temperature fluctuations in a structure with passive solar heating. Due to the high costs of active systems using flat-plate collectors, passive systems, despite increased building expenses, may prove to be more economic for many climates.

## 5. HIGH-TEMPERATURE APPLICATIONS

While the viability of solar water heaters and space-heating systems have been demonstrated with commercially available units, the results of only small-scale demonstration projects are available for assessing high-temperature applications of solar energy. Even though electrical energy can be produced with reasonable efficiencies through direct-conversion photocells (Section 7 of this chapter), both the monetary and the energy cost of presently available cells limits their use to specialized applications.

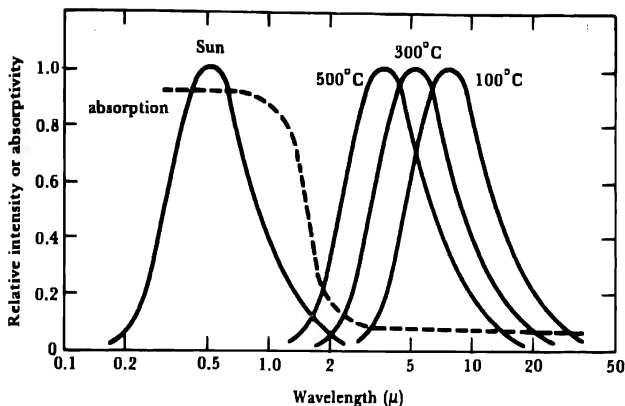
Electricity produced via thermal conversion of solar energy by means of a conventional Rankine cycle is technically achievable. The potential technical problems appear minimal especially when placed in the perspective of those associated with other energy conversion systems, namely, the liquid-metal-cooled fast breeder fission reactor and the yet to be achieved fusion reactor. The economic competitiveness of solar-produced electricity will depend upon the complexity of the eventual system utilized and hence can be ascertained only through the construction of a series of small- and moderate-sized experimental power plants.

The abundance of incident solar energy, particularly in large desert regions with higher than average incident energy for a given latitude and with few interruptions due to cloud cover, lends to its appeal. For example, the Southwest area of the United States (Arizona, New Mexico, and Nevada), and the North African coastal region are ideal for solar energy power stations. These regions have a seasonal minimum average daily horizontal incident radiation of 250 to 300 langleys that is, 2.9 to 3.5 kWh/m<sup>2</sup>. An incident energy of 3 kWh/m<sup>2</sup> corresponds to an average power of 125 W/m<sup>2</sup> and therefore a 10% conversion efficiency results in an average electrical output of 12.5 W/m<sup>2</sup>. An area of  $8 \times 10^7$  m<sup>2</sup> or 80 km<sup>2</sup> would therefore be necessary for a 1-GW (electrical) power plant. While this area, which corresponds to a square with sides of 8.9 km, seems large compared to that needed by conventional power plants, it is minute looking at presently available and unproductive desert areas. An average power of 267 GW (total average U.S. electrical energy consumption rate—1979) requires an area of only  $2.1 \times 10^4$  km<sup>2</sup>: approximately 7% of the area of Arizona. Considering the area strip-mined over a 50-year period for coal-fired power plants such as those of the Four Corners area of the Southwest, solar energy collection (based upon a 10% efficiency) is not excessively land-intensive. As in the case with all uses of solar energy, solar electrical power plants will be extremely capital intensive. They do, however, offer a long-term "solution," supplying energy nearly unlimited by the availability of resources.

The conversion efficiency of a thermodynamic cycle depends upon the collector temperature achieved. A conventional flat-plate collector with an upper temperature on the order of 100°C would result in a Carnot efficiency of only 21% for a low-temperature discharge of 20°C. The achievable efficiency of a Rankine cycle engine would very likely be less than 10% for these conditions, resulting in an overall conversion efficiency of only 3% for a 30% collector efficiency. Temperature losses associated with heat storage would further reduce the overall efficiency. Concentrating collectors are therefore necessary. A parabolic cylinder similar to that of Figure 8.23 or possibly a spherical parabolic collector could be used. For a concentration ratio of  $X$ , the collector efficiency has been shown to be the following.

$$\eta = \tau^n - P_L/XP_i$$

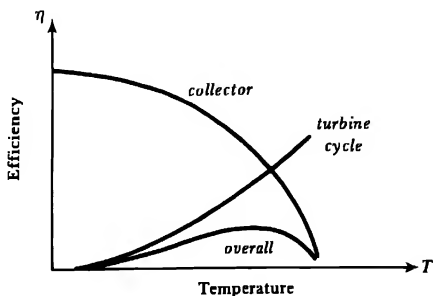
The loss,  $P_L$ , for the high-temperature heat transfer target can be reduced by the use of a selective surface, that is, a surface that is a strong absorber of visible incident short-wavelength radiation but which has a low emissivity (and hence absorption) for long-wavelength infrared radiation. The wavelength distribution involved as well as an ideal absorptivity curve are indicated in Figure 8.26. Selective absorption



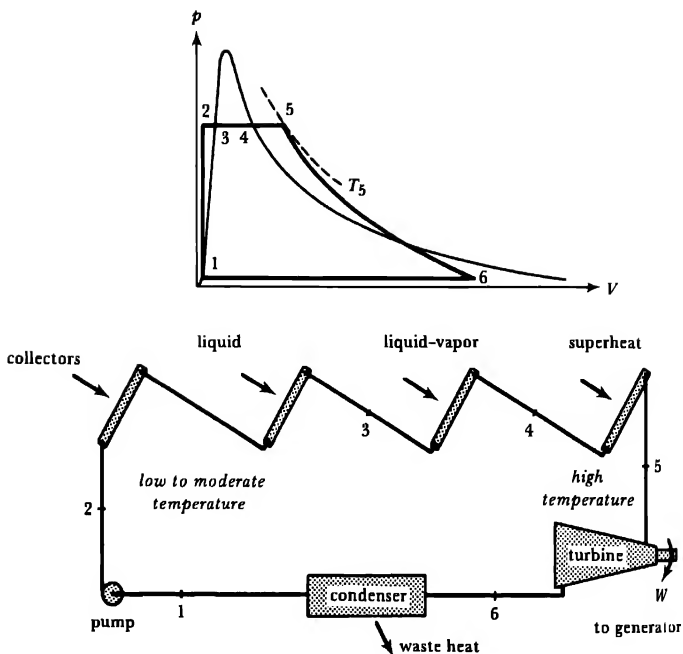
**FIGURE 8.26** *Wavelength Distribution of Black-Body Radiation and Ideal Absorption Characteristic.*

may be accomplished by materials which have an intrinsic change in their optical characteristics, for example, silicon, or by means of the interference characteristics of multilayered thin films. Additional development is necessary to achieve coatings with desired long-term stability at elevated collection temperatures. In order to reduce conductive and convective losses, the heat-transfer portion of the collector will very likely be vacuum enclosed.

While high collection temperatures are important, an optimum temperature that maximizes the overall efficiency is desired (Figure 8.27). For a Rankine cycle engine, however, not all the collector units need to operate at the maximum engine



**FIGURE 8.27** *Temperature Dependence of Overall Efficiency.*

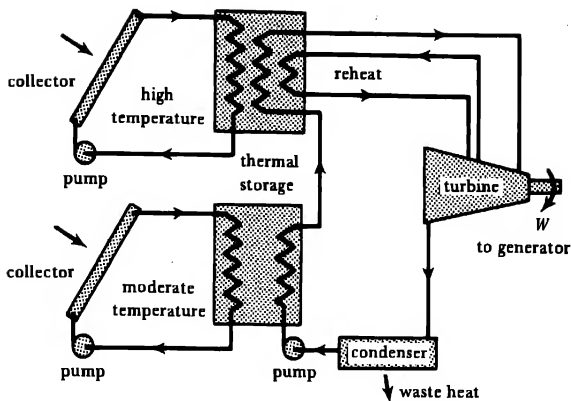


**FIGURE 8.28** *An Idealized Solar Collector Boiler.*

temperature,  $T_5$  (see Figure 8.28). The heat input for the portion of the cycle between 2 and 4 can be derived from lower-temperature and hence higher-efficiency collectors. A solar collector "boiler" optimized for the cycle could very likely result in a higher overall efficiency than a single high-temperature collector utilizing feedwater heaters.

A continuous electrical output or an output responding to consumer demand necessitates a thermal storage system. Molten salt mixtures could be used for both storage and a collector heat-transfer fluid. The high pressures necessary if water is used at high temperatures are thus avoided. A conceivable design for a power plant with two heat reservoirs is shown in Figure 8.29. While a high temperature corresponding to that used in conventional power plants (500 to 600°C) would be desirable, lower temperatures (300 to 400°C) could probably be more readily obtained, at least in initial experimental power plants.

Desalinization is a possible usage of the waste heat for power plants located adjacent to large bodies of salt water (the Mediterranean or the Gulf of California, for example). The Meincls are probably the best known advocates of large-scale



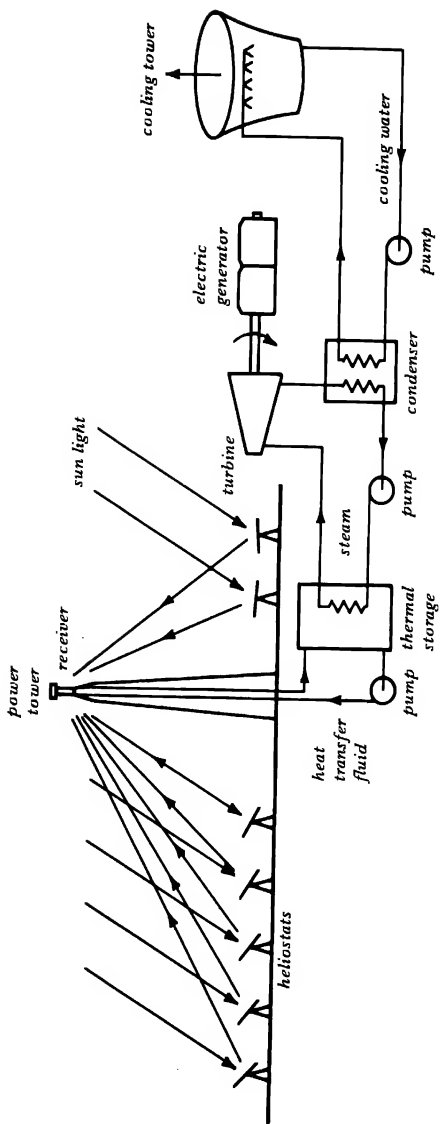
**FIGURE 8.29 A Solar Energy Power Plant with Thermal Storage.**

solar energy power plants, "solar farms," producing both electricity and fresh water [41].

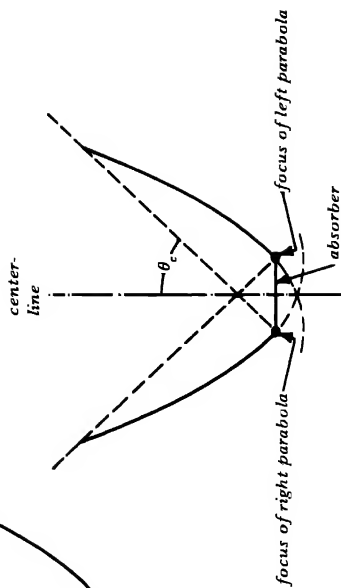
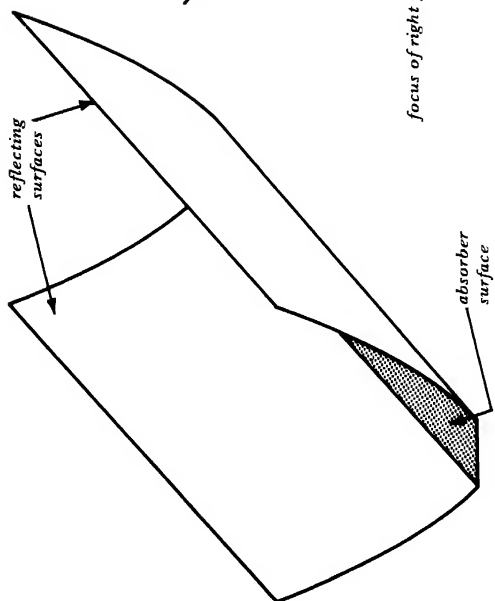
Another system, suitable for a moderate-sized electric power generating facility, utilizes a central collector (receiver) mounted on top of a high tower [42–47]. Radiation from the sun is reflected by a field of mirrors onto the receiver to achieve an extremely high effective concentration ratio (Figure 8.30). A *power-tower* pilot facility at Barstow, California, utilizes 1818 mirrors, of 40 m<sup>2</sup> each, to reflect the sun's radiation to a receiver on a 70-m high tower. Even though losses from the exposed receiver collector surface, per unit of area, are very high for the design temperature (500°C), the ratio of these losses to the power density of the concentrated incident radiation is small.

The orientation of each mirror, for a particular position of the sun, depends upon its location relative to the central receiver. Each mirror system, a *heliostat*, requires an individual control system to keep the sun's reflected radiation concentrated on the central receiver. Thermal storage is not required for a test facility; the central receiver can serve as the boiler for the steam-turbine system. Thermal storage, however, will be required for a commercial facility to mitigate interruptions caused by clouds and to provide for an evening and/or nighttime output. If a molten salt is used for latent heat storage, it could also be used as the heat-transfer fluid for the receiver. Operation of the Barstow and other pilot systems in Italy and Spain is expected during the first half of the 1980s, but larger units will have to be constructed and evaluated before large-scale commercial power generating facilities can be built.

Large quantities of process heat, including process steam, with a temperature of 200°C or less, are used by industries. Intermediate-temperature (100 to 350°C) solar energy collectors could be used to displace fossil fuels currently being used for



**FIGURE 8.30** A Central Tower Solar Electric Plant (Reprinted with Permission from Jerrold H. Krenz, *Energy: From Opulence to Sufficiency*, Washington: Hemisphere/Praeger, 1980, p. 200).



cross-sectional view

compound parabolic  
concentrator trough

**FIGURE 8.31 A Compound Parabolic Concentrator.**



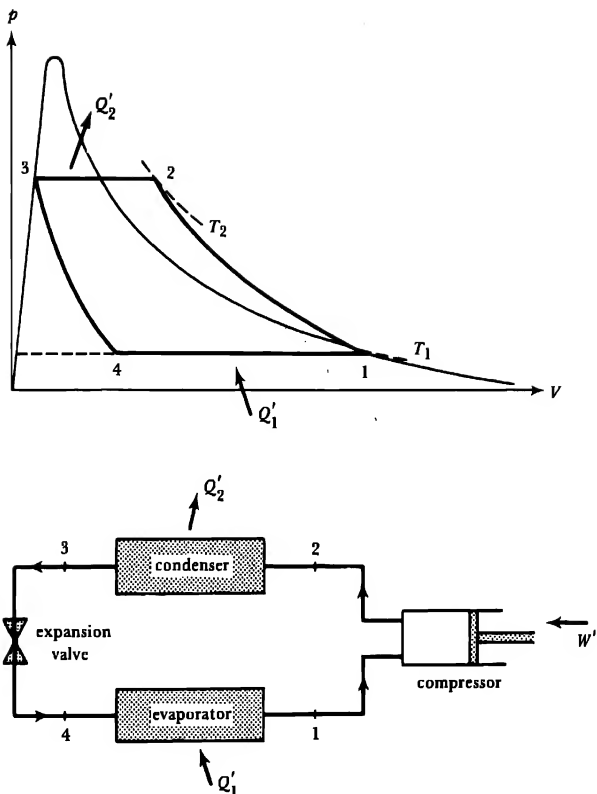
these needs [48]. Solar energy systems used for industrial applications would tend to be used throughout the year in contrast to solar space-heating systems which are seasonally used. Hybrid systems, using fossil-fueled furnaces for a back-up energy supply, are presently feasible, and with the development of efficient energy storage, systems utilizing only solar inputs would be feasible.

Although intermediate temperatures are too high for flat-plate collectors, collectors with concentrating ratios considerably less than those for efficient electric power generation can be used. These collectors, often referred to as *concentrators*, when mass produced, are expected to be considerably less expensive than the collector systems being considered for electric power generation. A trough-type compound parabolic concentrator, one type of collector being considered for intermediate-temperature applications, is illustrated in Figure 8.31 [49]. Each side of the concentrator is parabolic in shape. Incident radiation within an angle  $\theta_c$  of the symmetry plane will, after reflecting off the parabolic surfaces, fall on the absorber plate at the bottom of the concentrator. The concentration ratio for this collector may be shown to be  $1/\sin \theta_c$ . Various modifications of this configuration which result in more desirable absorber surfaces, such as a cylindrical tube, have been developed. For a trough-type concentrator (a cylindrical configuration is also possible), the sun can be followed by a daily rotation about an axis which runs along the length of the collector. With low concentration ratios, a simple clock-driven mechanism, as opposed to a more complex sun-tracking servomechanism needed for heliostats, is adequate.

## 6. COOLING

Even though absorptive-cycle air conditioning units that could be powered by heat energy derived from flat-plate collectors have been developed, conventional air conditioners of the type extensively used in the United States utilize a vapor compression cycle in which an electric motor is normally used to drive the compressor. Widescale usage of air conditioning has caused excessive electrical power demands resulting in the now familiar summer "brown outs" on the east coast of the United States. Furthermore, in many tropical and semitropical areas in which cooling is needed, the cost of electrical energy has been prohibitive. Moderate cooling of buildings in these areas could result in an increased productivity and significantly reduce the spoilage of food. While the discussion in this section will be concerned primarily with cooling or air conditioning, that is, achieving temperatures of 20 to 30°C, the same type systems could be used for refrigerators, 0 to 5°C, or for freezers, -20°C; the lower the temperature, however, the more complex the system.

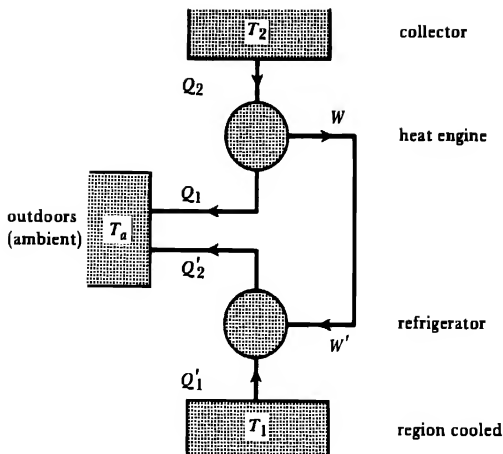
The conventional vapor compression cycle relies on a phase change of the working fluid and is, in many respects, similar to the Rankine power cycle operating in a reversed direction. The elements of an idealized cycle are indicated in Figure 8.32. Heat is removed from the region to be cooled by the evaporator (low temperature isotherm 4 to 1) and given off at a higher temperature by the condenser (2 to 3). Starting at point 1, the low-temperature vapor leaving the evaporator is



**FIGURE 8.32** *Vapor Compression Refrigeration Cycle.*

adiabatically compressed to the pressure of the condenser. The removal of heat,  $Q'_2$ , from the evaporator causes the working fluid to condense or liquify. The liquid (point 3) is allowed to expand adiabatically to the evaporator pressure. This is the function of the throttling or expansion valve (3 to 4) which frequently consists of a small restrictive capillary tube. Since there is no transfer of heat from the fluid or work done on the fluid, its enthalpy remains constant. The mixture of vapor and liquid leaving the throttling valve (point 4) evaporates and absorbs heat from the region to be cooled.

The coefficient of performance for a Carnot cycle,  $Q'_1/W'$ , has already been obtained (Chapter 4, Section 4).

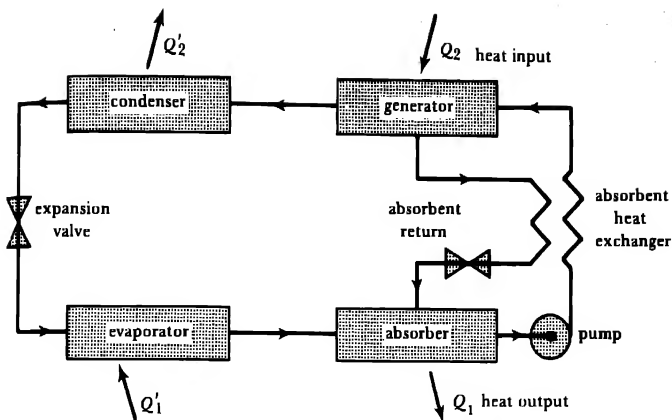


**FIGURE 8.33 Solar Energy Cooling System.**

$$\frac{Q'_1}{W'} = \frac{T_1}{T_2 - T_1} \quad \text{Carnot cycle}$$

A coefficient somewhat less than this maximum value would be expected for a vapor compression cycle owing to the nonreversible expansion within the throttling valve. Tables of thermodynamic properties similar to those for steam are required for explicit calculations. Early refrigeration systems used ammonia or sulfur dioxide as refrigerants. At present, the most commonly used refrigerants are the halogenated hydrocarbons, or freons. For example, freon-12, which is widely used in refrigerators, is  $\text{CCl}_2\text{F}_2$ . These refrigerants are nontoxic and result in pressures that, while not excessive, are greater than atmospheric pressure.

Cooling could be achieved by means of a solar heat engine that is used to mechanically drive the compressor of a conventional refrigeration system as indicated in Figure 8.33. For this system, the high-temperature heat input,  $Q_2$ , would be derived from the solar collectors while the ambient temperature sink,  $T_a$ , would receive both the waste heat of the engine and the heat from the region being cooled. An alternative cycle which avoids the inherently inefficient mechanical energy transfer depends upon the absorption of the refrigerant by a suitable solution. In the absorption cycle, the vapor compressor is replaced by an absorber and a generator (Figure 8.34). A system in which ammonia is the refrigerant and water the absorber is frequently used in conjunction with boiler-generated steam for a high-temperature source. A system which operates at lower temperatures, temperatures achievable with flat-plate collectors, can use water for a refrigerant and a lithium bromide water



**FIGURE 8.34** *An Absorption Refrigeration Cycle.*

solution for an absorber. While temperatures suitable for air conditioning can be achieved with this cycle, temperatures for refrigerators and freezers (below the freezing point of water) are unachievable.

The vaporized refrigerant leaving the evaporator is absorbed by the weak absorber solution and is accompanied by a release of heat. For the basic configuration shown in Figure 8.34, the absorbent solution with a high concentration of refrigerant is pumped to an upper pressure corresponding to that of the condenser. Since pumping involves only a liquid, very little mechanical work is necessary for this process. The heat input to the generator boils off the refrigerant, leaving a weak absorber solution. The vaporized refrigerant that enters the condenser then completes the refrigeration cycle as in the vapor-compression cycle, whereas the weak absorbent is returned to the absorber. A heat exchanger is usually included in the absorption cycle. While a mechanically driven pump is used for very large cooling units, a heat driven vapor life percolator (similar to that of a coffee pot) can usually be used in smaller units, thus eliminating all moving parts. An alternate scheme (the Servel Electrolux refrigerator) uses a third gas, hydrogen, to equalize the total pressure throughout its ammonia-water cycle. A gas flame is usually used for the high-temperature heat source. Numerous descriptions of absorptive refrigeration systems are available [50, 51].

For an absorption cycle, the coefficient of performance is the ratio of the heat removed,  $Q'_1$ , to that supplied to the generator,  $Q_2$ .

$$\text{Coefficient of performance} = Q'_1/Q_2$$

A calculation of the expected performance requires the use of both tabulated refrigerant properties and refrigerant-absorbent solubility data. The resultant

performance will typically be considerably less than that of a comparable vapor-compression cycle. The input for the absorption cycle, however, is relatively low-temperature heat energy rather than more valuable mechanical energy. If the inefficiency of the heat engine which produces the mechanical energy is considered, the performance of an absorptive cycle can be comparable to that of the overall vapor compression system. For a solar system, the prime advantage of the absorption cycle is that moderate-temperature heat energy may be used to directly power the system.

The performance of a solar cooling system may be estimated by first calculating the maximum coefficient of performance that may be expected. For this it is assumed that the refrigerator and heat engine of Figure 8.33 are both ideal, that is, they operate on a Carnot cycle. Based upon the nomenclature of that figure, the work output of the heat engine is the following.

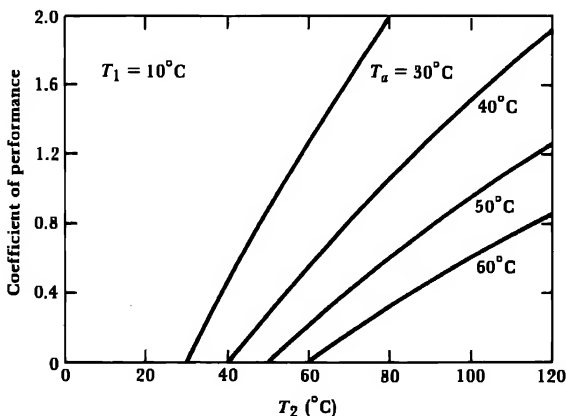
$$W = \eta_c Q_2 = \frac{T_2 - T_a}{T_2} Q_2$$

The cooling,  $Q'_1$  for a Carnot refrigerator is also readily obtained.

$$Q'_1 = \frac{T_1}{T_a - T_1} W'$$

Since the quantities of work are equal, an expression involving only heat quantities may be obtained through the elimination of  $W$  and  $W'$ .

$$Q'_1 = \left( \frac{T_1}{T_a - T_1} \right) \left( \frac{T_2 - T_a}{T_2} \right) Q_2$$



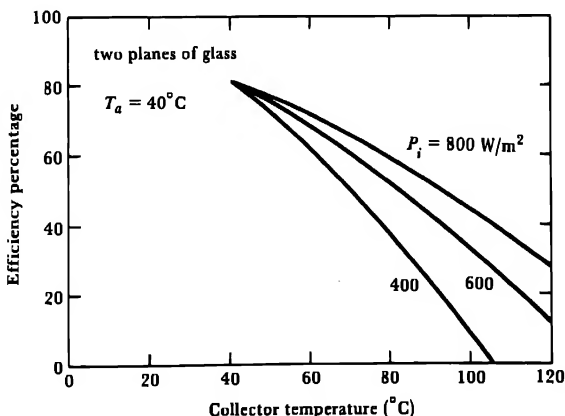
**FIGURE 8.35** Coefficient of Performance for an Ideal System.

$$\frac{Q'_1}{Q_2} = \left( \frac{T_1}{T_a - T_1} \right) \left( \frac{T_2 - T_a}{T_2} \right)$$

The last expression is the maximum coefficient of performance, which cannot be exceeded by an absorption cycle or any other cycle. A plot of this quantity for temperatures typical of a solar cooling system is given in Figure 8.35.

Air conditioning requires a substantially high collector temperature,  $T_2$ , than that needed for heating. Collector heat losses, however, depend upon the ambient temperature. For an ambient temperature of  $40^\circ\text{C}$ , collector temperatures of  $100^\circ\text{C}$  are readily achieved, as indicated in Figure 8.36. For maximum cooling, the tilt of the collector should be optimized for summer collection, but a compromise solution is necessary if the system is to be used for both heating and cooling.

An estimate of the cooling that can be achieved with solar energy may be obtained from the data of Figures 8.35 and 8.36. For a condenser temperature of  $45^\circ\text{C}$ , that is, five degrees above an ambient temperature of  $40^\circ\text{C}$ , the coefficient of performance of an ideal system would be 1.2 ( $T_2 = 100^\circ\text{C}$ ). An actual unit with its inherent losses would have a coefficient on the order of half this value, or .6. An incident collector power density of 600 to  $800 \text{ W/m}^2$  results in an output of 200 to  $360 \text{ W/m}^2$ . Approximately 100 to 180 W of cooling for a coefficient performance of .6 could thus be obtained for each square meter of collector surface. A frequently used measure of cooling is the ton\* which corresponds to 12,000 Btu/h or 3.5 kW.



**FIGURE 8.36 Efficiency of a Flat-Plate Collector: High Ambient Temperature.**

\* One ton of cooling corresponds to the average rate that heat must be extracted in producing one ton of ice in a period of one day.

Based upon the upper incident power density of  $800 \text{ W/m}^2$ , a collector area of approximately  $20 \text{ m}^2$  would be required ( $600 \text{ W/m}^2$  requires  $35 \text{ m}^2$ ). As for heating, the cooling required depends upon the constructional details of a building. For the moderately high ambient temperature of  $40^\circ\text{C}$ , a cooling of 1 ton would probably be sufficient for a well-constructed and insulated residence with a floor area of  $100 \text{ m}^2$ . Higher ambient temperatures, unfortunately, reduce the air conditioner's coefficient of performance while concurrently increasing the heat infiltration to the structure. For these conditions, achieving a desired indoor temperature of say  $25^\circ\text{C}$  would require excessively large collection areas. Nevertheless, solar systems can provide a moderate degree of cooling, that is, they can achieve indoor temperatures 15 to 20 degrees below ambient temperatures.

An alternative cooling scheme that utilizes thermal storage may be suitable for some climates. If sufficient cooling occurs at night, heat could be removed from a storage system which could, in turn, be used to remove heat from the residence during the day. This would be particularly attractive for arid regions in which nighttime evaporative cooling (low ambient wet-bulb temperatures) is suitable for extracting heat from the storage system. The cost of a cooling system of this type used in conjunction with the storage system of an existing solar heating system would be minimal.

## 7. PHOTOVOLTAIC SOLAR CELLS

Photovoltaic solar cells, as opposed to conventional collectors which convert the sunlight to thermal energy, directly utilize the energetic photons of the incident radiation. Conversion efficiencies for thermal systems are limited by the collector temperatures achieved. Even though the conversion efficiency of photocells is limited by other considerations, high efficiencies are possible for the conversion of incident energy directly to electrical energy.

In 1980 photovoltaic cells with a total peak electricity-producing capacity of about 3.6 MW were manufactured. The efficiency of these cells was 10 to 12%, and their average cost was over \$10/W of peak capacity. To accelerate the production and usage of photovoltaic cells, the Solar Photovoltaic Energy Systems Research Development and Demonstration Act of 1978 (PL 95-590) was passed. It is intended that about \$1.5 billion be spent over a ten-year period to achieve the principal goals of the Act: establishing a photovoltaic industry with a yearly production capability of 2 GW (peak capacity) by 1988 and reducing the cost of photovoltaic cells to \$1/W. Even though two years after passage of the Act the initial goals seem overly optimistic, significant progress in increasing production levels and reducing costs has occurred [52].

The energy of a photon of radiation,  $E_{\text{photon}}$ , is proportional to the frequency of the radiation,  $\nu$ .

$$E_{\text{photon}} = h\nu$$

where  $h = 6.625 \times 10^{-34} \text{ J-s}$  (Planck's constant)

Utilizing the relationship between frequency and wavelength, an expression for energy in terms of wavelength is obtained.

$$\nu = c/\lambda$$

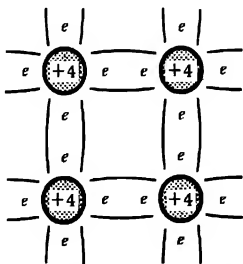
where  $c = 3 \times 10^8$  m/s (velocity of light)

$$E_{\text{photon}} = hc/\lambda$$

For visible light with a wavelength of  $.5\mu$  ( $.5 \times 10^{-6}$  m), each photon has an energy of  $4 \times 10^{-19}$  J. Division by the electronic charge ( $1.6 \times 10^{-19}$  C) yields an energy of 2.5 eV. A photon that interacts with an electron can impart this energy to the electron. An electron energy of this value is comparable to potential barriers within semiconductors. Photovoltaic devices depend upon photon-electron energy exchanges which subsequently permit electrons to overcome existing barrier potentials.

A detailed treatment of photocells is provided in several texts on direct-energy conversion already referred to in the discussion of MHD generators and fuel cells as well as in other texts and articles [53-67]. While several types of semiconductors are available and have been used, the most common one, silicon, is extensively used for transistors and integrated circuits as well as for most of the presently available photocells. Silicon (atomic number 14) has an outer electron shell of four valence electrons. It is neither a good conductor nor a good insulator. In its crystalline state, a silicon atom forms covalent bonds by sharing valence electrons with four neighboring atoms. As indicated in Figure 8.37, the nucleus and the inner electronic shells have a net positive charge consisting of four electronic charges. In an actual crystal, the atoms are arranged in a three-dimensional tetrahedral pattern, each atom being equally distant from its four neighbors.

The energy necessary for an electron to escape from the valence bond of a silicon crystal is 1.12 eV. An electron which leaves a valence bond is referred to as a *free electron*, since it is available for conduction. On the average, each valence electron has a thermal energy of  $\frac{3}{2}kT$ , or approximately .038 eV at room temperature. Owing to the Maxwellian velocity distribution of electrons, however, a few electrons will have sufficient energy to break their valence bonds. Each free electron leaves behind a vacancy or hole in the valence bond. A valence electron of an adjacent



**FIGURE 8.37** Two-Dimensional Representation of a Silicon Crystal.

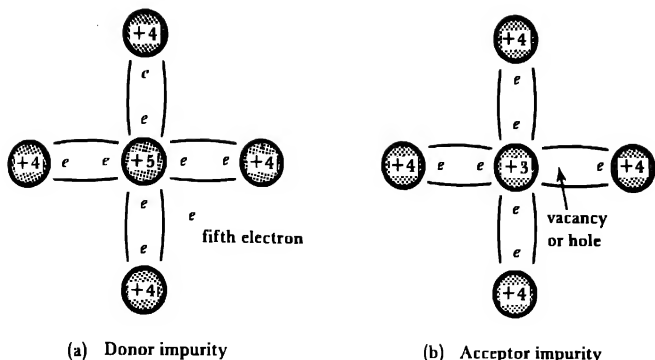


bond may readily move into this vacancy and, in the process, leave behind a new vacancy. In the presence of an electric field, both the free electrons and the valence electrons experience a force and tend to drift in the direction of the force. The net effect for the valence electrons is a drift of the vacancy or hole in a direction opposite to that of the valence electrons. The drift of a hole is therefore in the same direction as that of a positive charge. Rather than consider the motion of the valence electrons, it is more convenient to deal with the motion of the hole, that is, the "missing" electron. An electric field gives rise to a drift of holes in the direction of the field and a drift of free electrons in the opposite direction. Since free electrons and holes are formed in pairs, a pure or intrinsic semiconductor material has one hole for each free electron.

The current of a semiconductor depends upon the mobility of the carriers (the free electrons and holes) and the quantity of carriers. While electron-hole pairs are continuously generated within the crystal, the concurrent recombination of free electrons and holes results in an equilibrium value for their density. Recombination occurs when a free electron literally "falls" into a hole and in the process gives up its potential energy. Since the generation rate of carriers depends upon thermal energy, their density is temperature dependent. For silicon, the equilibrium density of free electrons and holes,  $n_i$ , is  $1.5 \times 10^{16}/\text{m}^3$  at  $20^\circ\text{C}$ . This is an exceedingly small density compared to that of the valence electrons: one electron-hole pair for each approximately  $10^{13}$  valence electrons. Incident light with photon energies greater than the escape energy for valence electrons (1.12 eV for silicon) can generate additional electron-hole pairs which in turn increases the conductivity of the semiconductor. This effect, photoconductivity, is used in many light-sensing devices. An energy conversion cell, however, requires a more complex semiconductor configuration.

Extrinsic semiconductors contain selected impurities that are introduced by the process of doping. In doped semiconductors, conduction can be predominantly the result of either free electrons or holes, rather than due to equal quantities of carriers. Consider the case in which an atom of a silicon crystal has been replaced with an atom with five valence electrons such as antimony, arsenic, or phosphorus (Figure 8.38a). Four of the five valence electrons complete covalent bonds with adjacent silicon atoms. The fifth or excess electron will, at room temperature, have sufficient energy to become a free electron. The free electron in this case, however, does not leave behind a hole. An impurity atom of this type is referred to as a *donor atom*. If sufficient donor atoms are present in the crystal (a density much larger than  $n_i$ ), the density of free electrons will greatly exceed that of the holes, and as a result, conduction will be primarily due to free electrons. A semiconductor with donor impurities is referred to as an *n-type semiconductor* (*n* standing for negative).

A semiconductor in which the current is due primarily to holes can be formed by the addition of impurities with three valence electrons, such as boron, aluminum, or indium (Figure 8.38b). This impurity atom has insufficient valence electrons to complete four valence bonds. The incomplete bond results in a hole that can contribute to the semiconductor current. Impurity atoms with three valence electrons accept an additional valence electron and are hence referred to as *acceptor atoms*. A

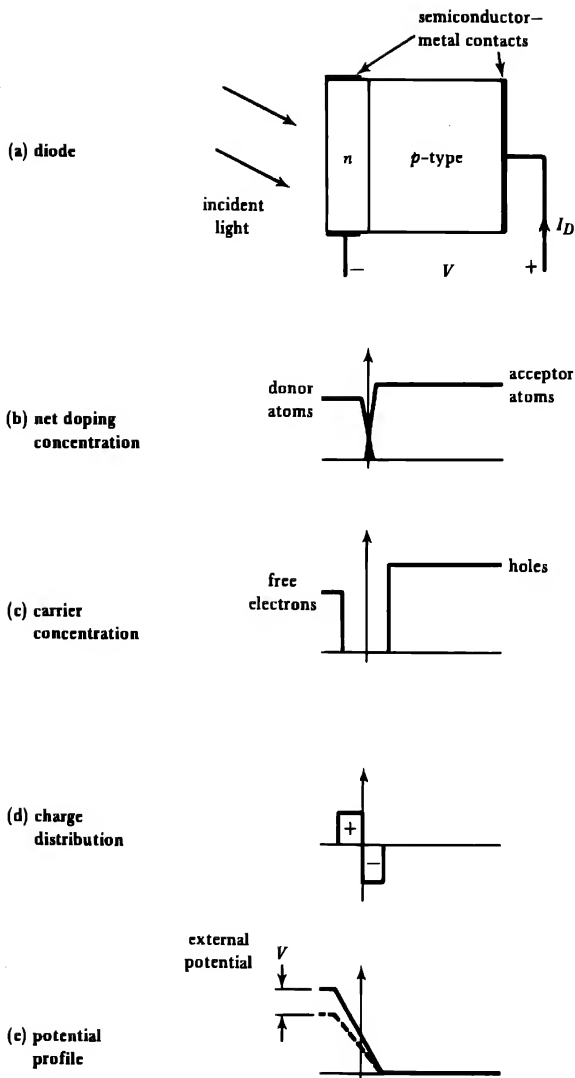


**FIGURE 8.38** *Doping of a Semiconductor.*

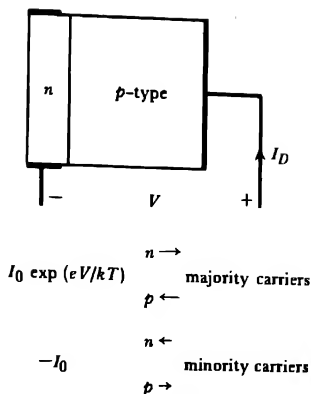
sufficient density of acceptor atoms (a density much larger than  $n_i$ ) results in a semiconductor in which holes are the dominant carriers. Doping with acceptor atoms results in a  $p$ -type (positive) semiconductor.

A solar cell depends upon the behavior of a semiconductor diode to convert photon energy to electrical energy. A diode is the result of an abrupt transition from  $p$ -type to  $n$ -type semiconductor material. For the holes to readily migrate from one region to the other influenced only by the resultant electric fields, the diode must be formed from a single crystal. Figure 8.39a illustrates a diode junction and Figure 8.39b indicates the doping concentrations on each side of the junction (the densities of donor and acceptor atoms need not be equal). While light energy would normally be incident on the left face of the diode shown, the behavior of a diode in the absence of light will first be considered. Recombination occurs in the region of the junction, as indicated in Figure 8.39c. Free electrons from the  $n$ -type material tend to diffuse into the  $p$ -type region where they readily recombine because of the very large hole concentration. Also, holes from the right side of the junction tend to diffuse into the  $n$ -type material and rapidly recombine with the numerous free electrons. These movements result in a net charge density in the region of the junction. Free electrons moving from the left side of the junction leave a net positive charge behind; the migrating holes leave a net negative charge behind on the right side of the junction.

The charge distribution gives rise to an electric field and hence a potential difference across the junction (the zero for the potential profile indicated in Figure 8.39e is arbitrary). The potential (or electric field) retards the diffusion of carriers across the junction and results in an eventual equilibrium condition. In addition to the free electrons of the  $n$ -type semiconductor and the holes of the  $p$ -type (the majority carriers), the few holes of the  $n$ -type and free electrons of the  $p$ -type (the minority carriers) must also be considered. Minority carriers tend to cross the junction readily since the potential profile is such that it enhances their migration.



**FIGURE 8.39** A Semiconductor Junction Diode.



**FIGURE 8.40** Current Components for a Junction Diode.

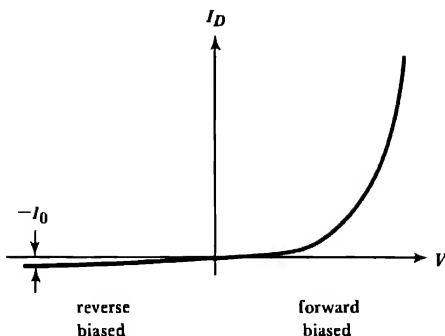
Therefore, the potential for zero current allows only the diffusion across the junction of sufficient majority carriers to cancel the current of minority carriers.

The potential difference across the junction is not directly available to an external circuit. For zero current (no light), the contact potential differences of the semiconductor-metal junctions cancel the diode junction potential and result in a zero terminal voltage. An externally applied potential difference modifies the junction potential. A small positive potential decreases the potential barrier and hence allows a greater quantity of majority carriers to cross the junction. A negative potential increases the potential barrier and decreases the current due to majority carriers but leaves the current due to the minority carriers essentially unaffected. The diode current,  $I_D$ , is the sum of the current due to majority carriers and that due to minority carriers (Figure 8.40). For large negative potentials, only minority carriers cross the junction. The minority current depends primarily upon the generation rate of minority carriers and is hence determined by the temperature and the semiconductor material. For a given temperature, however, it is a constant,  $-I_0$ , which is independent of the junction potential. The current due to the majority carriers depends upon the potential profile. Only free electrons and holes with energies in excess of the barrier potential will cross the junction. Since the carriers tend to have a Maxwellian velocity distribution, a current with an exponential dependence on the potential is obtained. The net result (a rigorous derivation is beyond the scope of this treatment) is given by the following expression.\*

$$I_D = I_0 \exp\left(\frac{eV}{kT}\right) - I_0$$

majority carriers      minority carriers

\* Often the behavior of a junction diode corresponds to an expression with an exponential argument of the form  $eV/AkT$  in which  $A$  is a dimensionless constant with a value greater than one but less than two.



**FIGURE 8.41** Current-Voltage Characteristic of a Junction Diode.

The constant  $I_0$  multiplying the exponential term of the current majority for the carriers is necessary to result in a zero current for zero voltage. Figure 8.41 is a plot of the current-voltage characteristic of a junction diode in which the exponential voltage-dependence results in a very rapid increase in current with voltage. The current  $-I_0$  for reverse bias is frequently negligible compared to the forward current. Appreciable forward current for a silicon diode requires a voltage of .5 to .7 V.

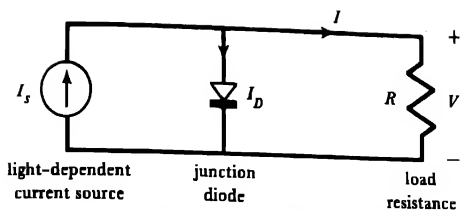
Incident radiation with photon energies equal to or in excess of the energy required to liberate valence electrons has the effect of generating additional electron-hole pairs in the semiconductor. Ideally, the  $n$ -type region of the diode of Figure 8.39a is sufficiently thin that the incident light is absorbed in a region immediately adjacent to the junction. Equal quantities of majority and minority carriers (free electrons and holes respectively for the  $n$ -type material) will be generated. Since large quantities of majority carriers are present even in the absence of light, the excess majority carriers will have little effect. Without light, however, only a few minority carriers are present, and, therefore, the current due to the photon-generated minority carriers can be significant. Since minority carriers readily cross the junction, a minority carrier current proportional to the generation rate of minority carriers results.

An elementary circuit model of a solar cell consists of a light-dependent current source,  $I_s$ , which is in parallel with a junction diode (Figure 8.42). An expression for the load current,  $I$ , is readily obtained.

$$\begin{aligned} I &= I_s - I_D \\ &= I_s - I_0 \left[ \exp \left( \frac{eV}{kT} \right) - 1 \right] \end{aligned}$$

A second current-voltage relationship is determined by the value of the load resistance.

$$I = V/R$$



**FIGURE 8.42** An Idealized Circuit Model for a Photocell.

Knowing the output voltage and current, the output power,  $P$ , is obtained.

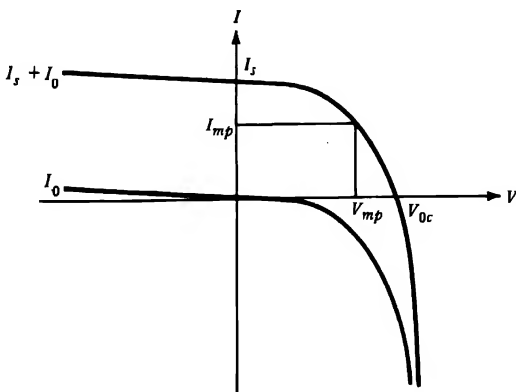
$$P = VI$$

For a given light intensity, the output power depends upon the value of the load resistance. Often of interest is the maximum obtainable output power,  $P_{\max}$ , the corresponding voltage and current being  $V_{mp}$  and  $I_{mp}$ , respectively.

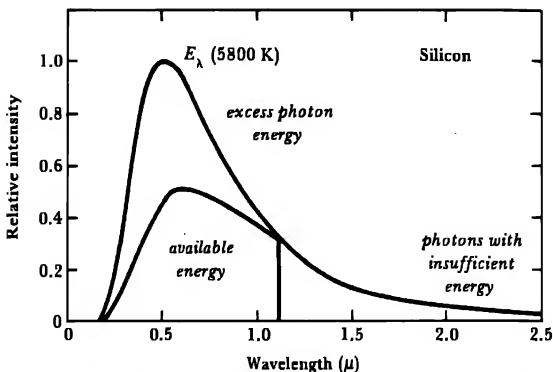
$$P_{\max} = V_{mp}I_{mp}$$

The maximum power as indicated in Figure 8.43, represents the area of the rectangle formed by  $V_{mp}$  and  $I_{mp}$ , which is clearly less than the product of the open-circuit voltage,  $V_{oc}$ , and the short-circuit current,  $I_s$ .

$$P_{\max} < V_{oc}I_s$$



**FIGURE 8.43** Current-Voltage Characteristic of Solar Cell.



**FIGURE 8.44** *Energy Utilization for a Silicon Photocell as a Function of Wavelength.*

An expression for  $V_{oc}$  may be obtained by setting the load current,  $I$ , equal to zero.

$$0 = I_s - I_0 \left[ \exp\left(\frac{eV_{oc}}{kT}\right) - 1 \right]$$

$$\exp\left(\frac{eV_{oc}}{kT}\right) = \frac{I_s}{I_0} + 1$$

$$V_{oc} = \frac{kT}{e} \ln\left(\frac{I_s}{I_0} + 1\right)$$

Optimizing the performance of solar cells is the subject of the now classic paper by Loferski [68]. Since only photons with energy in excess of that required to generate free electron-hole pairs ( $E_g$ ) contribute to the photo current, the spectral distribution of the incident radiation is important. Figure 8.44 shows the wavelength distribution for a 5800 K black-body radiator, a distribution approximately that of the sun's radiation. Since a photon energy of 1.2 eV is required to generate free electron-hole pairs in silicon (the gap energy), photons with wavelengths greater than  $1.1\mu$  have insufficient energy to generate carriers. The area under the wavelength distribution curve to the right of  $1.1\mu$  is therefore proportional to the energy not utilized: approximately 23% of the total area under the curve. While all photons with wavelengths less than  $1.1\mu$  produce carriers, not all the energy of the individual photons is used. Since each photon can produce only one free electron-hole pair, energies in excess of the gap energy serve only to thermally heat the semiconductor.

If  $E_\lambda$  is the spectral energy density,  $E_\lambda d\lambda$  is the energy in the range  $d\lambda$ . The corresponding number of photons in this differential range of wavelengths,  $dn$ , may be obtained by dividing  $E_\lambda d\lambda$  by the energy of one photon, namely,  $h\nu$ .

$$dn = \frac{E_\lambda d\lambda}{h\nu} = \frac{\lambda E_\lambda d\lambda}{hc}$$

The useful energy of each photon is  $E_g$ , the energy necessary for an electron to break its valence bond.

$$E_g dn = \frac{E_g \lambda E_\lambda d\lambda}{hc}$$

The gap energy may be related to the wavelength of a photon with that energy.

$$E_g = \frac{hc}{\lambda_g}$$

$$E_g dn = \left( \frac{\lambda}{\lambda_g} \right) E_\lambda d\lambda$$

This energy distribution, valid for  $\lambda \leq \lambda_g$ , is indicated in Figure 8.44. The net result shows that for silicon only approximately 44% of the incident radiation energy is utilized. While semiconductors with larger gap energies tend to more fully make use of the energy of those photons which interact, a greater proportion of the incident photons have insufficient energy to interact. Conversely, lower gap energies allow more photons to interact but less fully utilize the energy of the reacting photons. Based upon this consideration alone, the optimum gap energy is on the order of 1.5 eV, somewhat greater than that of silicon.

For typical solar cells, the output voltage is considerably less than the gap potential. The value of the open-circuit voltage,  $V_{oc}$ , depends upon the ratio of the photo current,  $I_s$ , and the minority carrier current for no light,  $I_0$ .

$$V_{oc} = \frac{kT}{e} \ln \left( \frac{I_s}{I_0} + 1 \right)$$

The photo current depends upon the quantity of incident photons utilized. For sunlight, the average photon energy is approximately 1.48 eV. Therefore, an incident power density of 1000 W/m<sup>2</sup> corresponds to a photon flux of 1000/(1.48 × 1.6 × 10<sup>-19</sup>) or 4.2 × 10<sup>21</sup> photons/s·m<sup>2</sup>. If only 77% of these are utilized, a current equivalent to that of 3.3 × 10<sup>21</sup> electrons/s·m<sup>2</sup> of 520 A/m<sup>2</sup> results. The photocurrent of a cell with a junction area of one square centimeter,  $I_s$ , will be 52 mA and the corresponding minority carrier current,  $I_0$ , will be approximately 5.9 × 10<sup>-9</sup> mA [68]. This yields an open-circuit voltage,  $V_{oc}$ , of .57 V for  $kT/e = .025$  V. Since the voltage for maximum power,  $V_{mp}$ , will be less than  $V_{oc}$ , the output voltage will be no greater than approximately one-half of the gap energy expressed in electron volts (1.12 eV for silicon). A conversion efficiency for a silicon solar cell will therefore be no greater than 22%.

Other factors tend to further reduce the conversion efficiency. Reflection of incident energy results in a small loss. Recombination of photon-generated minority carriers and resistive losses also decrease the output of a solar cell. Finally, the output voltage and current for maximum power,  $V_{mp}$  and  $I_{mp}$ , are less than  $V_{oc}$  and  $I_s$ , respectively. This effect is frequently referred to as the *curve factor*. A theoretical consideration of all these effects predicts that the conversion efficiency of a silicon cell will be no greater than 20%. Conversion efficiencies of 18% have been achieved.



The same analysis, however, predicts a conversion efficiency of 24% for a material with a gap energy of 1.5 to 1.6 eV. Semiconductor compounds formed from atoms with three and five valence electrons (III-V compounds), such as gallium arsenide, have gap energies greater than those of silicon. The highest experimentally observed efficiency for a conventional single-junction cell, 22%, was achieved with a gallium arsenide cell.

Research and development efforts have been directed toward both improving the efficiency of photovoltaic cells and reducing their cost. Multiple photovoltaic cells, called *tandem cells*, can have a photon efficiency greater than that predicted for a single cell. Photons with a wavelength near to (but less than) the wavelength corresponding to the gap energy,  $\lambda_g$ , are the most energy efficient in generating electron-hole pairs (Figure 8.44). To take advantage of this condition, the incident radiation could be split into discrete wavelength bands (for example, by a prism) of radiation, each band being directed to an individual photovoltaic cell. The gap energy of each cell is such as to maximize its conversion efficiency. A group of cells with appropriately chosen gap energies could, in principle, utilize most of the energy of the incident photons. Alternatively, thin photovoltaic cells could be stacked in a sandwich-like array. Photons with wavelengths greater than that corresponding to the gap energy of a particular cell do not interact, that is, the cell is transparent to these wavelengths. Therefore, if the outer cell has a high energy gap, long-wavelength radiation will pass through it and be absorbed by a cell with a lower energy gap. Cells stacked according to their gap energies (highest on the outside, lowest at the interior) could thus be utilized. However, the manufacture of tandem cells will be more complex than that of single-junction cells.

Another approach to improving the efficiency of photovoltaic cells is to concentrate the incident solar radiation by means of reflecting surfaces or lenses. The light-dependent current source of a diode,  $I_s$ , depends upon the rate at which electron-hole pairs are generated and hence depends upon the incident power density. The open-circuit voltage also depends upon the diode current,  $I_s$  (it has a logarithmic dependence). To the extent that both the load current and voltage for maximum power,  $I_{mp}$  and  $V_{mp}$ , respectively, increase with the diode current and open-circuit voltage, an output power that increases faster than the incident power density is predicted. But, as the incident power density is increased, the power absorbed by the semiconductor also increases. An increase in temperature, caused by the increase in absorbed power, tends to have the opposite effect on efficiency. For highly concentrated radiation, a cooling of the photovoltaic cell is required.

Even if, owing to thermal effects, the efficiency of a photovoltaic cell is not increased with concentrated incident energy, an economic benefit may still result. For a given output power, a cell with a much smaller cross-sectional area is required than that of the cells used in a flat panel (nonconcentrated radiation) producing the same power. Per unit of output power, the cost of photovoltaic cells using concentrated radiation is expected to be less than that of flat-panel collectors. Furthermore, high-efficiency tandem cells which may be too expensive for flat panels could be used to increase the efficiency of concentrating systems. Unlike flat-panel photovoltaic collectors, concentrating systems require, as do concentrating thermal systems, a

mechanical tracking system to follow the sun. This increases the cost and complexity of systems using concentrating photovoltaic cells.

To reduce the cost of photovoltaic cells, alternative manufacturing techniques are being devised. Commercially produced photovoltaic cells are presently (1980) fabricated from silicon ingots grown from purified molten silicon. These ingots, about 10 cm in diameter and 1 m in length, are sawed into thin wafers from which photovoltaic cells are fabricated (about 70% of the ingot is lost in the sawing process). The junction is formed by a diffusion doping process. Thin metallic contacts are then deposited on the exposed surface and an antireflecting coating is applied. To reduce fabrication costs, processes are being developed for drawing ribbons or sheets directly from the molten silicon, thus eliminating sawing. Concurrently, less expensive doping processes, such as ion implantation, and the use of transparent metal films for contacts are being perfected. Automation of these steps, as well as that of encapsulating the cell, will be required to achieve a cost of \$1/W.

As has been mentioned, experimental gallium arsenide cells have demonstrated higher energy-conversion efficiencies than those of silicon. Other semiconductor materials such as cadmium telluride, cadmium sulfide, and copper sulfide, along with various junction configurations, are being studied. While efficiencies higher than that of gallium arsenide cells are not anticipated, the use of these materials may result in lower costs. Commercial photovoltaic cells are formed from a single semiconductor crystal. Smaller crystals, forming what are known as *polycrystalline materials*, are much less costly to produce. Hence, efforts are being directed toward improving the relatively low energy-conversion efficiencies that characterize polycrystalline photovoltaic cells.

Besides using photovoltaic cells for earth-based collectors, it has been proposed that a power-producing satellite with photovoltaic collectors be placed in a geostationary orbit [69-73]. The electrical power produced by its photovoltaic cells would be transmitted to an earth-based receiving antenna by means of a microwave beam. Being outside the earth's atmosphere, the average radiation incident on the satellite collectors would be equal to the solar constant, 1353 W/m<sup>2</sup>. Except when the satellite passes through the earth's shadow, near midnight, the electrical power would be continuous. A proposed system would utilize a photovoltaic collector array 5 × 10 km and would produce, at the earth, an electrical power of about 5 GW. The program to place up to 60 such satellites in orbit would be the largest engineering task ever undertaken. Not only would new components, including light-weight, low-cost photovoltaic cells, need to be developed but these components would also have to be first transported into orbit and then assembled. Much of the required space-transport technology would also have to be developed. An evaluation of this system by the National Academy of Sciences was not favorable—it concluded that the system's cost would be prohibitive [74]. The Academy concluded that although power satellites might be useful in the twenty-first century, it would be premature to spend research and development funds on them in the 1980s.

processes. But, energy-producing forests will need to be cultivated and harvested and very likely fertilized and irrigated in order to sustain rapid tree growth. Since no new technology is required, it is anticipated that these energy farms will be tried and evaluated in the 1980s.

Other schemes to produce energy from biomass convert either crop wastes or specially grown crops to high-quality fuels such as methanol, ethanol, and methane. Biological and/or chemical conversion processes have been utilized. Certain microorganisms digest biomass materials and produce both methane and carbon dioxide, a process known as *anaerobic digestion*. The gaseous mixture of methane and carbon dioxide, a "low-Btu" gas owing to the noncombustible carbon dioxide, can be used either directly or upgraded to a fuel with a higher energy value. Anaerobic digestion is frequently employed for municipal and industrial sewage treatment since not only is a low-Btu gas obtained (generally used to supplement fuel needs of the treatment facility) but the volume of the organic waste is also reduced. Concentrated animal manures, such as produced at feedlots, can also be converted to methane by anaerobic digestion.

Fermentation is another biological process that has been widely used for producing chemicals and fuels. Sugars extracted from sugar beets and sugarcane and starch extracted from grains can be converted to ethanol by the microorganisms responsible for fermentation. Ethanol, derived from grains, is presently being blended with gasoline (10% ethanol) and being marketed in the United States as *gasohol*. It is not clear, however, that producing ethanol from grains is a net energy gain since the growing of grains (as well as of other agricultural products) in the United States is very energy intensive [94]. Energy derived from fossil fuels is used not only for highly mechanized farms but also to produce fertilizers and other required chemicals and in many areas to pump water for irrigation. Only sugars and starches can be converted to ethanol by fermentation. Enzymatic and chemical processes are being developed to convert cellulose (woody) type materials to sugars which in turn can be fermented.

Pyrolysis is an important chemical reaction used to convert wood to more desirable fuel products. When heated in the absence of air, wood decomposes and produces primarily charcoal and methanol. Combustible fuel gases can also be produced. The heat input for pyrolysis is generally obtained through the combustion of the wood feed stock, thus resulting in a significant energy loss. It has been proposed that solar energy be used for pyrolysis. The solar energy input would, in effect, augment the energy of the biomass feed stock [90].

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## PROBLEMS

1. The position of the sun at  $40^\circ$  north latitude and for winter solstice is desired.
  - (a) What is the angular elevation of the sun at noon?
  - (b) Obtain an expression for the angular elevation of the sun that depends upon the hour-related angle  $\phi$ .
  - (c) The compass position of the sun may be specified by the angle between its horizontal projection and due south. Obtain an expression for this angle,  $\beta$ , that is a function of  $\phi$ .
  - (d) At what angle,  $\beta$ , does sunrise occur? Tabulate values of  $\beta$  and elevation for each hour from sunrise to noon.
2. The incident solar energy at the equator is desired. The assumptions of Section 2 (35% reflection, 20% absorption) may be used.
  - (a) Determine the maximum and minimum daily energy density incident on a horizontal collector during a year.
  - (b) Determine the energy density for a collector facing the sun at noon for both these conditions.
  - (c) Determine the optimum orientation for a fixed collector that is to supply energy throughout the year.
3. The length of a day at the summer solstice in the northern hemisphere is to be found. Determine the length of a day for latitudes of  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$ . At what latitudes is the length of a summer day the greatest?
4. To increase the summer energy collection of a flat-plate collector, the collector has been oriented for maximum incident energy at the equinox. What is the incident power density, at noon, for the summer and winter solstices? What are the daily incident energies (kilowatt-hours per square meter) for the solstices? Assume the same conditions as those of Figure 8.9.
5. The power incident on a collector which tracks the sun is desired. The transmission coefficient of Section 2 for a 23-km visibility (Hottel) applies. The collector is located at  $30^\circ$  north latitude.
  - (a) Determine the incident power densities at noon for summer and winter solstices. What is the power density at equinox?
  - (b) Estimate the daily incident energy, per unit area of collector (kilowatt-hours per square meter), for the sun declinations of part (a).
  - (c) What are the incident energies corresponding to a constant radiation intensity of .52S (the approximation used for several of the figures of Section 2)?



6. The curves of Figure 8.9 are to be modified to account for atmospheric effects on the direct component of incident radiation. Assume the transmission coefficient of Section 2 for a 23-km visibility (Hottel) applies. Sketch the new power density functions and estimate the corresponding daily incident energies.
7. Suppose that a water storage system of 1000 gal (8 lb/gal) is used for a solar heating system. Determine the storage capacity in Btu per Fahrenheit degree and in kilowatt-hours per Celsius degree.
8. Consider a solar heater that is being used with a thermal storage system that has a heat capacity of 4 kWh per Celsius degree temperature change. The power output of the collector system is 15 kW and may be assumed constant from 8 A.M. to 4 P.M. A constant heating power of 10 kW is extracted from the system from 8 P.M. to 8 A.M. The noontime temperature of the storage system is 50°C. Assuming the daily collection and heating rates remain unchanged, plot the temperature of the storage system for a 24-hour period. What are its maximum and minimum temperatures?
9. An initial debt of  $Y_0$  is repaid at an exponentially increasing repayment rate.

$$C = C_0 e^{\alpha t}$$

- (a) Show that for a yearly interest rate of  $i$ , the constant  $C_0$  is given by the following expression.

$$C_0 = \frac{(\alpha - i)Y_0}{e^{(\alpha - i)T} - 1}$$

- (b) Show that for  $\alpha > i$ , the debt,  $Y$ , increases before it decreases. At what time is the debt a maximum?
10. An initial sum of \$10,000 is borrowed for a period of 20 years. The yearly interest rate is 8%.
  - (a) What is the repayment rate if the debt is repaid at a constant yearly rate? At what time is the remaining debt equal to one-half its original value?
  - (b) Suppose that repayment is at an exponentially increasing yearly rate of 12%. What is the initial repayment rate for this condition?
  - (c) When is, for the conditions of part (b), the debt a maximum? What is the maximum debt?
  - (d) When is the debt of part (b) equal to one-half its initial value, that is, \$5000?
11. Consider a residence that requires a yearly heating energy of 200 million Btu. A solar heating system and energy conserving measures designed to reduce the space- and water-heating requirements by 80% are to be installed. The life of the system may be assumed to be 20 years and the yearly interest rate to be 10%. Determine the maximum allowable total investment cost if fuel costs are \$4/10<sup>6</sup> Btu. What cost would be acceptable if fuel costs were \$10/10<sup>6</sup> Btu?
12. An investment of \$20,000 is required to insulate a residence and install a solar heating system. The investment reduces the yearly use of heating fuel from  $200 \times 10^6$  Btu to  $30 \times 10^6$  Btu. An income tax credit of \$4000 is allowed for the investment, but yearly property taxes increase by \$100 as a result of the invest-

ment. A 20-year period is to be considered and the effective yearly interest rate is 5%. Yearly maintenance expenses are \$100 and the initial fuel costs are \$4/10<sup>6</sup> Btu.

- (a) Assume fuel costs remain unchanged. Determine the present values of the benefits and costs for the investment. Can the investment be economically justified for the conditions given?
  - (b) Suppose the investment, after 20 years, still has a worth of \$10,000. What is the present value, a benefit, that can be associated with the \$10,000 remaining value?
  - (c) Assume fuel prices increase exponentially each year. Determine, based upon the conditions of parts (a) and (b), the minimum yearly increase for which the present value of costs is equal to the present value of benefits.
  - (d) What are the savings, based upon present values, if fuel prices increase at a yearly rate of 10%?
13. An initial sum of \$10,000 is borrowed for a period of ten years at a yearly interest rate of 6%. It is repaid at a constant yearly rate. Since interest paid on a loan may be deducted from one's income tax liability, the tax saving is a benefit that should be considered when evaluating an investment.
- (a) Determine the repayment rate and an expression for the yearly rate of interest payments. Since the debt changes with time, so too do interest payments.
  - (b) Suppose a tax saving equal to 30% of the interest payments is realized. Assume this benefit occurs at the same time that the interest is paid. What is the present value of the tax savings?
14. A sum of money,  $Y(0)$ , is borrowed at a yearly interest of  $i$ . The interest is compounded at the end of each one-year period when a repayment of  $C$  is made. For this situation, a finite difference equation applies for the  $n$ th year.

$$Y(n) = Y(n-1) - [C - iY(n-1)]$$

- (a) Show, by developing expressions for the first few terms of the debt,  $Y(1)$ ,  $Y(2)$ , etc., that the debt for the  $n$ th year is given by the following expression.

$$Y(n) = -C \sum_0^{n-1} (1+i)^m + (1+i)^n Y(0)$$

- (b) Show that for the debt to be retired in  $T$  years, the yearly repayment must be the following.

$$C = \frac{(1+i)^T Y(0)}{\sum_0^{T-1} (1+i)^m}$$

- (c) Assume an initial debt,  $Y(0)$ , is \$10,000. What is the yearly repayment,  $C$ , for a yearly interest of 10% and a ten-year repayment period?
  - (d) What is the yearly repayment rate if the debt of \$10,000 is repaid (in ten years) on a continuous schedule and the yearly interest rate is 10%?
15. The time necessary for the energy savings that result from a solar energy invest-

ment (or any other energy saving investment) to compensate for the initial investment is often desired. If the investment is  $Y_0$ , and the value of the fuel saved the first year is  $S$ , a quantity known as a simple pay-back period,  $T_s$ , may be obtained.

$$T_s = Y_0/S$$

Both the interest rate and anticipated fuel price changes should be considered in determining a pay-back period.

- (a) Assume that the yearly interest rate is  $i$  and that fuel savings increase at a yearly rate of  $\alpha$ . Determine the present value of the fuel savings for a time interval of  $T$ . What is the time at which the present value of the fuel savings equals the initial investment?
  - (b) The simple pay-back period for an investment is ten years. What is the actual pay-back period, based upon the results of part (a), when fuel prices increase at a yearly rate that is 5% greater than the interest rate. What is the pay-back period if the simple pay-back period is five years?
  - (c) Repeat part (b) for the condition in which the yearly interest rate exceeds the rate at which fuel prices increase by 5%.
  - (d) What is the pay-back period, in terms of  $T_s$ , when  $(\alpha - i) = 0$ ?
16. The analytical result of Problem 15 for a pay-back period will be used. Using the notation of Problem 15, the pay-back period,  $T$ , is a function of  $T_s$  and  $(\alpha - i)$ . Plot a curve of  $T$ , as a function of  $(\alpha - i)$ , for a range of  $\pm .1$  ( $\pm 10\%$ ) for  $(\alpha - i)$  and for a simple pay-back period of five years. A family of curves may be obtained if different simple pay-back periods are considered. To illustrate this, obtain two additional curves for simple pay-back periods of two years and ten years.
17. Tracking collectors are to be used for a solar electric power plant that produces an average electrical output power of 100 MW (24-hour period). Assume that an incident power density of  $800 \text{ W/m}^2$  is available for 8 hours a day, the collection efficiency is 50%, and the thermal to electrical power conversion efficiency is 30%.
- (a) Determine the collector area necessary.
  - (b) Suppose a collector cost no greater than \$1000 per kilowatt of average electrical output power is acceptable. Determine the maximum permissible cost of a square meter of collector.
  - (c) Assume the lowest angle for the sun is  $25^\circ$  from the horizontal. What is the minimum land area necessary for the collection system in order to assume that one collector will not shade another?
18. Suppose that sodium is used for a thermal storage system of an electric power plant with an average output of 100 MW (24-hour average). Assume the thermal to electric conversion efficiency is 30%. The specific heat of sodium is .3 cal/ K-g and a temperature change of  $50\text{C}^\circ$  is acceptable. Determine the mass of sodium that would be necessary to provide for 48 hours of operation of the power plant. Determine the required diameter of a spherical vessel which could be used to hold the sodium (the density of sodium is .93 g/cm<sup>3</sup>).

19. The impact of strip mining coal for a power plant is to be determined. Consider an electrical power plant (40% efficient) with an average output power of 1 GW. Determine the quantity of coal necessary to operate the plant for a 50-year period. Assume the coal is produced by strip mining a vein with an average thickness of 10 m and a density 1.5 times that of water. Determine the land area that would need to be stripped over the 50-year period.
20. A silicon junction diode has a current,  $I_D$ , of 100 mA for a terminal voltage of .6 V. The diode is not illuminated,  $I_s = 0$ . Assuming  $kT/e$  is equal to .025 V, determine  $I_0$ . Illumination produces a current,  $I_s$ , of 30 mA. What is the corresponding open-circuit voltage,  $V_{oc}$ , of the diode?
21. A comparison of the energy produced by a solar power satellite to the energy required to place the satellite in orbit is desired. The gravitational attractive force,  $F$ , between the satellite of mass  $M$ , and the earth of mass  $M_E$ , depends upon their center-to-center separation,  $r$ .

$$F = k \frac{MM_E}{r^2}$$

Since the attractive force at the surface of the earth is  $Mg$ , the proportionality constant  $k$  may readily be evaluated.

- (a) Determine the orbital radius that results in a rotational period of one day (a geostationary orbit).
  - (b) What is the potential and kinetic energy, per unit of mass, of a satellite that is in the geostationary orbit of part (a).
  - (c) Assume that the photovoltaic cells are 12% efficient, that they occupy 75% of the collecting surface, and that power is transmitted to the earth with a 50% efficiency. If the collectors have a mass of .3 kg/m<sup>2</sup>, what is the ratio of the electrical energy produced (at the earth) in a one-year period to that associated with the satellite's orbit?
  - (d) Assume that the launch vehicles are 10% efficient in placing the satellite in orbit. What is the ratio of the output of part (c) to the energy expended by the launch vehicles?
22. The basic carbohydrate molecule, CH<sub>2</sub>O, has an energy of combustion of  $112 \times 10^6$  cal/kg-mole. Determine the energy in Btu per metric ton of CH<sub>2</sub>O. What is its energy of combustion relative to that of coal?

## CHAPTER 9

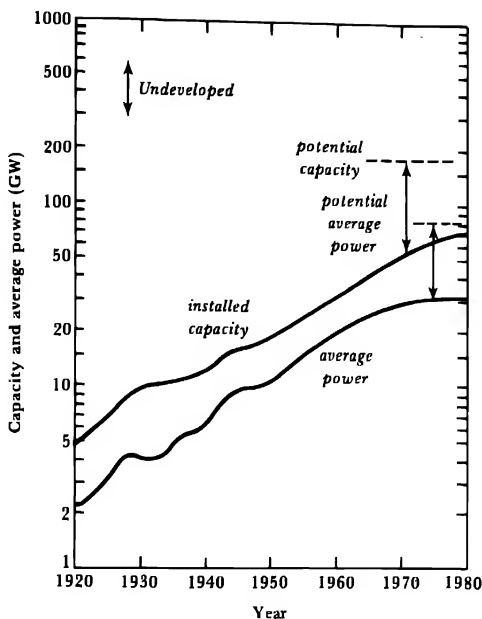
# *Water, Wind, and Geothermal Power*

### 1. HYDROPOWER

Significant quantities of electrical energy are generated by hydroelectric power plants. At the end of 1979, the installed capacity of these plants in the United States was 76 GW, that is, about 12% of the total generating capacity [1]. Hydroelectricity accounted for an energy of  $2.83 \times 10^9$  kWh (average power of 32.3 GW) in 1979 (also 12% of the total electrical energy). The steady growth of hydroelectric generating capacity and average power is indicated in Figure 9.1 [1-3].

On a world basis, hydroelectric capacity accounts for a greater portion of the total installed capacity: 23% (1979) [4]. Approximately 22% of all electrical energy is produced by hydroelectric plants. Due to their topographical features and plentiful rainfall, several countries are highly dependent upon hydroelectricity. For example, it accounts for over 99% of the electrical energy used in Norway, 75% in Switzerland, and 69% in Canada (1979). Hydropower is thus an energy source as important for some nations as fossil fuels are for others. Unlike fossil fuels, water power is a renewable source of energy which does not become depleted through usage.

At present, hydropower is used almost exclusively for the generation of electricity. In the nineteenth century, before the introduction of electrical energy, hydropower was used to directly drive industrial machinery. While precise data is lacking, Landsberg and Schurr [5] estimate that in 1850 wind and water power combined accounted for two-thirds of the mechanical power used within the United States. According to Rosenberg, in the New England states, where manufacturing industries tended to concentrate, 40% of the mechanical power used in 1869 was

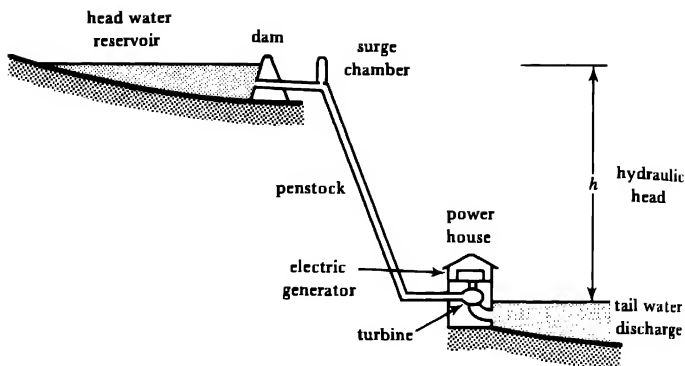


**FIGURE 9.1** *United States Hydroelectric Capacity and Average Power (References 1 and 2).*

due to water [6]. The use of steam power, however, grew much more rapidly than water power during the latter part of the nineteenth century.

Small-scale hydropower installations, typical of those of nineteenth-century New England, utilized only a small fraction of a stream's or river's potential. Modern large-scale hydroelectric power plants are frequently built in conjunction with a reservoir from which water may be withdrawn according to the demand for electrical energy. A reservoir will also tend to mitigate the effect of seasonal variations in stream flow. Figure 9.2 is a schematic diagram of a common hydroelectric installation. Although a concern in designing a hydropower system is that of maximizing the hydraulic head,  $h$ , the resultant configuration is dictated to a high degree by topographical considerations. Many power plants (Grand Coulee Dam and Hoover Dam) are located within the dam itself. Others, such as those at Niagara Falls where a dam is unnecessary, rely on a system of canals and forebays to bring the water to the entrance of the penstocks.

The potential energy of a volume of water with a mass  $M$  and a hydraulic



**FIGURE 9.2 A Typical Hydroelectric Power Plant.**

head  $h$  is given by the following expression. The slight dependence of the acceleration of gravity on latitude may usually be ignored.

$$E = Mgh$$

where  $g = 9.8 \text{ m/s}^2$  acceleration of gravity

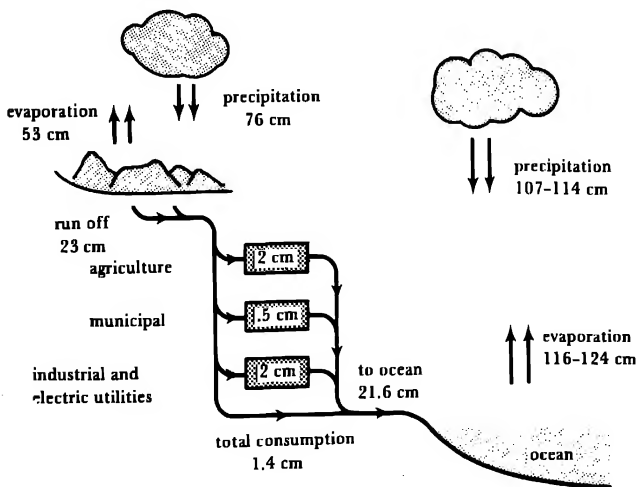
Since one cubic meter of water has a mass of 1000 kg, the potential energy of this volume for a head of 100 m would be  $9.8 \times 10^5 \text{ J}$ . This is equal to an energy of .27 kWh or, alternatively, a volume of  $3.7 \text{ m}^3$  of water ( $h = 100 \text{ m}$ ) has a potential energy of 1 kWh. A flow rate of one cubic meter per second, again for a head of 100 m, results in a power of nearly one megawatt, that is  $9.8 \times 10^5 \text{ W}$ . A one-gigawatt installation would thus require a water flow rate in excess of  $1000 \text{ m}^3/\text{s}$ .

In a hydroelectric power plant, mechanical potential energy,  $Mgh$ , is converted to the mechanical energy associated with the rotation of the shaft coupling the water turbine to the electric generator. There are usually small turbine inefficiencies that may be attributed to the kinetic energy of the discharged water and the hydraulic and eddy losses within the turbine (a slight heating of the water). Large hydraulic turbines have conversion efficiencies in excess of 85%. The thermodynamic inefficiency of converting thermal energy to mechanical (as in conventional thermal power plants) is avoided. Since essentially all hydropower is used to generate electricity, a question arises in comparing energy inputs for hydro and thermal electric power plants. One commonly used procedure is to specify the energy input for a hydroelectric power plant in terms of the thermal energy that would be required by a conventional power plant to produce the same electrical energy. Based upon the present average heat rate for fossil-fueled power plants in the United States (Figure 5.4), approximately 3 units of thermal energy are required for each unit of electrical energy. This accounting procedure is frequently used in the United States. World

data (such as data compiled by the United Nations) usually specifies the energy input for hydropower in terms of the input potential energy. Variations in statistical data may frequently be attributed to this accounting difference. Based upon a thermal equivalence, hydropower accounts for approximately 4% of the total input in the United States and nearly 5% for the world.

While a thermal energy equivalent for hydropower is not unreasonable, it does tend to overstate energy requirements of nations with large hydropower resources. Countries such as Switzerland, which lack fossil-fuel reserves, tend to rely on hydro-produced electrical energy for direct thermal applications (such as space and water heating). Direct heating by fossil fuels, even considering combustion inefficiencies, would require less energy than the thermal equivalent energy corresponding to the electrical energy used. Conversely, specifying hydro energy directly in terms of mechanical energy understates its impact for conventional electrical uses (such as lighting and electric drive). Either accounting method introduces apparent distortions in energy dependence.

Hydro potential for the United States, as indicated in the upper right-hand portion of Figure 9.1, depends upon annual precipitation which in turn determines runoff and topography, the latter influencing the achievable hydraulic head for a potential power plant. Precipitation, evaporation, runoff, and related quantities are usually expressed in a depth-related measure. Precipitation, for example, is specified in terms of the depth of water that would be accumulated in a specified interval,

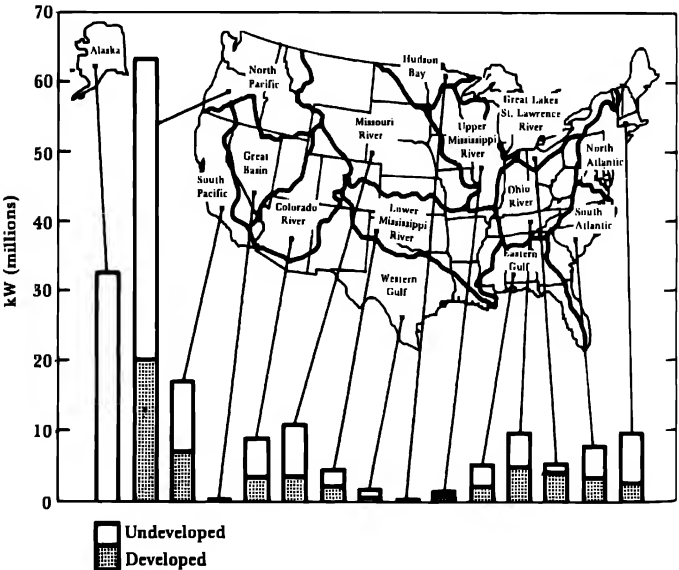


**FIGURE 9.3** Average Annual Water Rates for the Contiguous United States (Reference 7).



assuming no runoff. Average annual values for the United States are given in Figure 9.3 [7]. For land areas, precipitation is greater than evaporation, whereas for large bodies of water, particularly the oceans, evaporation is greater than precipitation. The effect of rivers that flow into the ocean is thus balanced. Based on the estimates of Penman [8], the average precipitation and runoff quantities of Figure 9.3 do not differ greatly from averages for land areas throughout the world.

The average yearly stream flow of 23 cm for the United States is equivalent to a volume of 23 cm<sup>3</sup> per square centimeter of land surface or .23 m<sup>3</sup> per square meter. On an average basis, the runoff for each square kilometer is  $2.3 \times 10^5$  m<sup>3</sup> per year. This is equivalent to an average volume flow rate of  $7.3 \times 10^{-3}$  m<sup>3</sup>/s. To achieve an average power of 1 GW (ignoring turbine and generator losses) for an hydraulic head of 100 m would require a drainage area of  $1.4 \times 10^5$  km<sup>2</sup> (a flow rate of 1020 m<sup>3</sup>/s). Since many regions of the United States have elevations much in excess of 100 m, substantially smaller drainage areas are required for these regions. An hydraulic head of 1000 m, a head that would normally require a series of power plants, would require an area of only  $1.4 \times 10^4$  km<sup>2</sup> for an average power of 1 GW.



Note: The potential of Hawaii, amounting to 54,000 kW, of which 19,000 are developed, is not shown.

**FIGURE 9.4** *United States Developed and Undeveloped Hydroelectric Capacity (Reference 2).*

Figure 9.4, prepared by the Federal Power Commission, is an estimate of hydroelectric potential based upon detailed river basin studies in the United States [2]. These estimates are based upon both technical considerations and average stream flows. Many of the proposed hydroelectric sites used in obtaining the estimates may not necessarily, at least at present, be economically viable. Approximately 30% of the total estimated potential capacity is now developed. If fully developed, hydropower would result in a generating capacity of 178.6 GW and an average power of 80 GW, that is, a power sufficient to supply 30% of the electrical energy consumed in 1979. Water potential, however, is not uniformly distributed throughout the United States. For example, although 76.6% of the potential capacity of the Great Lakes and St. Lawrence drainage is developed, the total installed capacity is only 4 GW (nearly half of which is on the Niagara River). The North Pacific drainage system, however, accounts for an installed capacity of nearly 20 GW, and it has an ultimate potential of 70 GW. Even though the ultimate potential capacity for Alaska is over 30 GW, only a negligible portion is presently developed. Other considerations, not related to power, may reduce the number of potential hydroelectric sites. For example, the Wild and Scenic Rivers Act of 1968 precluded the development of plants with a potential capacity of 1.42 GW. Additional segments of river systems are also being studied for inclusion in the national wild and scenic rivers system. These segments, if they are included, would remove an additional 7 GW of potential capacity. Increases in irrigation and other water usage could also result in further reductions of hydro potential.

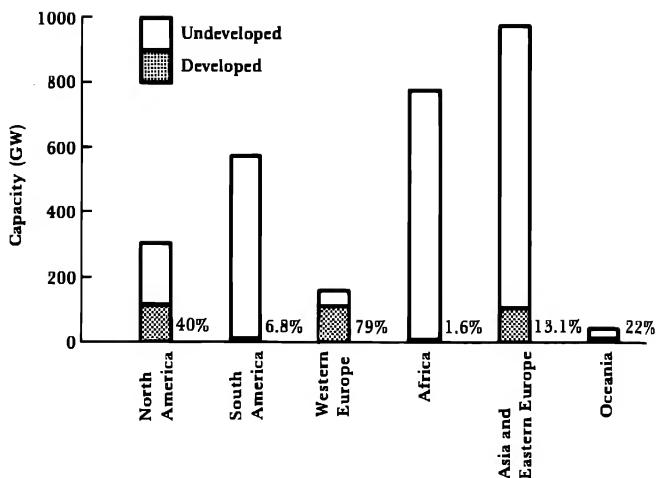
Both the developed and estimated hydropower potential for the world is presented in Table 9.1 and Figure 9.5 [4, 9]. On a world basis, the estimated total potential capacity of 2857 GW is over six times the installed hydroelectric capacity (1979). Hydropower is most fully developed in Western Europe, 79% of its ultimate potential. Very large quantities of hydropower are available in the less developed regions of the world, the potential quantities of energy being many times that presently consumed within these regions. With the advent of extra high-voltage transmission lines capable of transmitting in excess of one gigawatt of electrical power, the develop-

**TABLE 9.1 World Hydropower**

	<i>Estimated potential capacity (Hubbert 1962<sup>a</sup>)</i>	<i>Developed potential (1979<sup>b</sup>)</i>	<i>Average power (1979<sup>b</sup>)</i>
North America	313 GW	125.3 GW	62.9 GW
South America	577	39.4	19.1
Western Europe	158	124.5	51.1
Africa	780	12.8	6.3
Asia and Eastern Europe	984	129.1	53.8
Oceania	45	9.7	4.3
World	2857	440.8	197.5

<sup>a</sup>Reference 9.

<sup>b</sup>Reference 4.



**FIGURE 9.5** *World Developed and Undeveloped Hydroelectric Capacity (References 4 and 9).*

ment of water power at sites far removed from population centers is feasible. Water power could therefore become a very significant energy source for many of the developing nations.

Closely allied to hydropower is pumped storage. Two reservoirs interconnected with a reversible turbine generator unit are required for a pumped storage system. During periods of low electrical demand, the generator/motor operated as an electric motor is used to drive the turbine and pump water from the lower to the upper reservoir. The water in the upper reservoir is then available when required. Overall system efficiencies in excess of those of thermal peaking units such as gas turbines have been achieved. In addition, thermal steam base load facilities that supply the pumping energy are able to operate at or near their peak capacity, and hence at their maximum efficiency even when demand is low. As of January 1, 1979, the developed pumped storage generating capacity in the United States was 10.3 GW. Furthermore, 40 GW of pumped storage facilities were either proposed or under construction.

## 2. TIDAL POWER

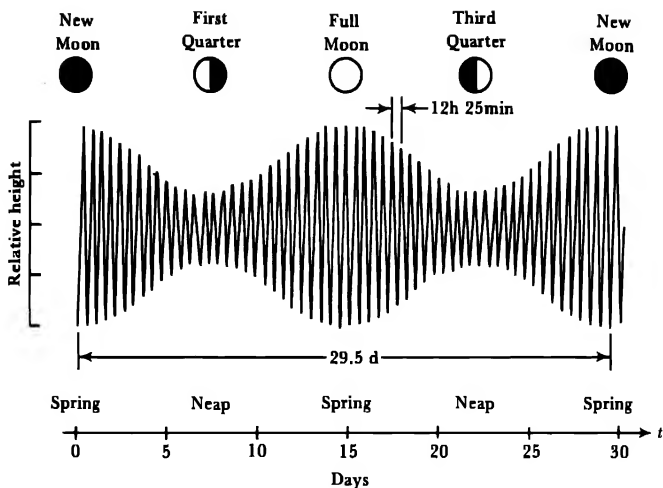
Both hydro and tidal power has been used throughout the history of mankind. Tidal mills were used during the Middle Ages in England and Continental Europe to grind

grains. During the seventeenth and eighteenth centuries, numerous mills were constructed for milling grain and sawing wood in New England, Britain, and Russia. A tidal waterwheel, constructed in 1580 and located under the London Bridge, supplied water to London in the nineteenth century.

It was not, however, until the twentieth century that large-scale tidal projects were considered. The French proposed building a pilot plant at l'Aber Vrach in 1928, while the Americans started (although soon abandoned) construction of a plant at Passamaquoddy Bay on the Canadian-U.S. border in 1935. The British considered building a tidal plant on the Severn River in 1933 and the Russians designed a pilot plant in 1940 for the Kislaya Inlet (on the Barents Sea near the Norwegian-USSR border). Although these projects were technically feasible, they were not economically justifiable at the time. The only large-scale plant that has been constructed is the French tidal project at the Rance estuary on the Brittany coast [11, 12]. At the entrance to the estuary, the maximum tidal range is in excess of 13 m. To effectively utilize this large tidal range, the plant has an installed electric capacity of 240 MW (the yearly averaged power is approximately 62 MW). While the plant was expensive compared to a thermal or conventional hydroelectric installation, over \$300/kW in 1962, the Rance project does demonstrate the technological feasibility of tidal energy.

The rhythmic rise and fall of tides is due to the gravitational forces of the sun and moon, which act together with that of the earth on the oceans. These forces balance the centrifugal force of the water due to the earth's rotation. The tidal flow, however, depends not only upon gravitational forces but also upon the profile of the ocean's bottom and the irregularities of the continental shore lines. In many regions a resonance-like effect results in very large tidal ranges. These regions, ideally suited for tidal power plants, include the Bay of Fundy, the English Channel, the Patagonian coast of Argentina, the Murmansk coast (Barents Sea), and the coast along the Sea of Okhotsk (north of Japan). While tidal ranges as large as 10 m are common for these areas, other coastal regions have much smaller if not negligible tidal ranges.

One of the first scientific explanations of the tides, a theory that is still largely accepted, was put forth by George Darwin (son of the noted biologist, Charles Darwin) [13]. More recent treatments of tidal phenomena are also available [14, 15]. A text by Bernshtein [16] provides a comprehensive discussion of electric tidal power plants, while the proceedings of a tidal power conference held in Nova Scotia lend a recent perspective on the utilization of tidal power [17]. A typical tidal pattern (Figure 9.6) consists of two high and two low tides that occur every 24 hours and 50 minutes. Since this effect is primarily the result of the moon's gravitational attraction, the double period corresponds to the period of the apparent rotation of the moon about the earth. Two high tides occur simultaneously, one on the side of the earth facing the moon and the other on the opposite side. The largest tidal range, the spring tide, occurs when the gravitational forces of the moon and the sun act together, either in conjunction or in opposition, a new moon or full moon, respectively. The minimum tidal range, the neap tide, occurs when the forces of the moon and sun are in quadrature, that is, at the first- or third-quarter phase of the moon. The spring-neap tide cycle corresponds to approximately one-half of a



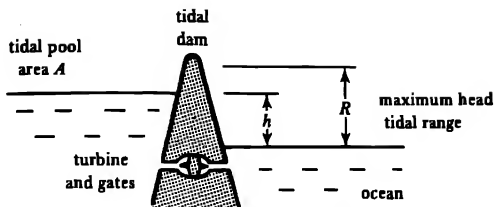
**FIGURE 9.6** A Typical Tidal Pattern.

lunar month (a lunar month is 29.5 days). Since the typical range of neap tides is about one-third that of spring tides, the operation of tidal power plants is significantly influenced by this effect.

Tides are also affected by variations in solar distances which are the result of the ellipticity of the earth's orbit about the sun and the moon's orbit about the earth. The changing declination of both the sun and the moon also affect the tides. As a result, tidal variations are frequently more complex than those indicated in Figure 9.6. Accurate long-term predictions based upon a detailed harmonic analysis of observed tidal patterns have been prepared for most coastal regions (tide tables). While tidal fluctuations are often irregular, they are highly predictable.

The viscous friction associated with the tidal motion of the oceans has been considered in discussing the solar energy balance (Chapter 3). While the total power dissipation by the tides is on the order of  $2.4 \times 10^{12}$  W, Munk and MacDonald estimate that no more than  $10^{12}$  W are dissipated in shallow seas and coastal regions [18]. Though the rate at which energy is dissipated in these more accessible areas does not necessarily set a physical limit to the useful power that can be extracted from the tides, it seems unlikely, considering the dams and barrages that would be needed, that more than a small fraction of  $10^{12}$  W could readily be utilized.\* Since

\*The prophetic narrative of Haldane, *The Last Judgment* [19], in which tidal power supplies all power for the world, a power much in excess of natural tidal friction losses, does not violate any physical laws, though harnessing the tides on this scale does not, at least at present, seem technically feasible.



**FIGURE 9.7 A Tidal Power Dam and Turbine.**

$10^{12}$  W are only 12% of the 1978 rate of energy consumption for the world, tidal power will, at best, provide a rather small portion of future energy requirements.

Proposed schemes for extracting tidal energy depend upon the availability of estuaries and bays that can be economically enclosed by a dam to form a tidal pool. The simplest system uses a single pool that is emptied at low tide and filled at high tide. The flow achieved during both the filling and emptying is used to drive a reversible turbine electric generator. As a consequence, electric power is generated twice during each tidal period of 12 hours and 25 minutes. To obtain an estimate of the maximum energy and hence the average power that can be produced, a tidal pool with an area  $A$  is assumed to have been filled at high tide. If the water is then used to drive the turbine approximately six hours later at low tide, the available hydraulic head is equal to the tidal range,  $R$  (Figure 9.7). Extracting water from the tidal pool, however, decreases its level and hence the hydraulic head. If  $h$  is the head, the differential energy corresponding to a differential volume flow,  $dV$ , may be obtained (losses will be ignored).

$$dE = g \rho h dV$$

where  $\rho$  = density,  $1000 \text{ kg/m}^3$

The differential volume, however, is related to the change in hydraulic head,  $dh$  (a negative quantity).

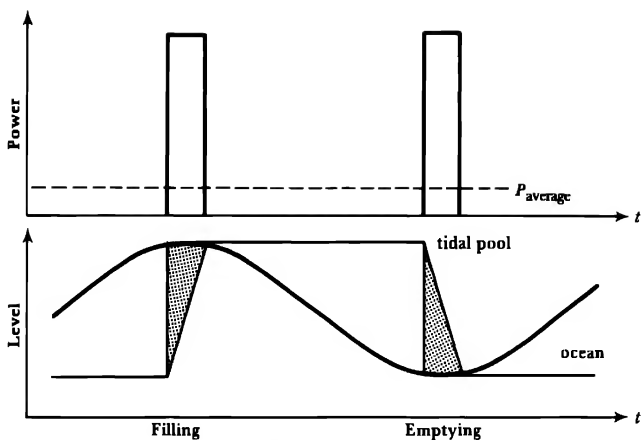
$$dV = -A dh$$

$$dE = -g \rho A h dh$$

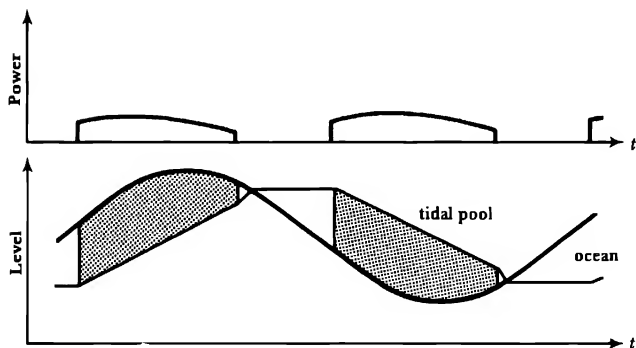
Integrating over the entire range of the hydraulic head, that is, from  $R$  to zero, an expression for the total energy is obtained.

$$\begin{aligned} E &= \int_R^0 dE = -g \rho A \int_R^0 h dh \\ &= \frac{1}{2} g \rho A R^2 \end{aligned}$$

For the above result, the ocean level was assumed to have remained unchanged, that is, the emptying of the pool and the concurrent power extraction were done



(a) Maximum energy output



(b) A more uniform power output

**FIGURE 9.8** Water Levels and Power Output: Single Tidal Pool.

over a relatively short interval of time. A high peak power, as indicated in Figure 9.8a, is therefore obtained. If, after emptying the tidal pool, the water passages are again closed while the tide rises, an equal energy is obtained when filling the tidal pool at high tide. The total theoretically extractable energy for each tidal period is therefore  $g\rho AR^2$ .

$$E_{\text{period}} = g\rho AR^2$$

Based upon this calculation, a tidal range of 5 m could produce an energy of  $2.45 \times 10^{11}$  J/km<sup>2</sup>. Dividing by the tidal period of 12 hours and 25 minutes ( $4.57 \times 10^4$  s) yields an average power of 5.48 MW/km<sup>2</sup>. Introducing the appropriate numerical values, an expression for average power in terms of the tidal range,  $R$ , is obtained.

$$P_{AV} = .219R^2 \text{ MW/km}^2$$

The high peak values of power corresponding to a rapid emptying and filling of the tidal pool not only necessitate large capacity turbines but are also difficult to absorb in an electrical power system. The peaks which occur at times of high and low tides do not necessarily correspond to the times of peak electrical demand (in addition, the peaks occur 50 minutes later each day). Figure 9.8b illustrates the tide and pool levels corresponding to a more uniform power output, albeit an output with significant zero periods. Since a smaller average head is used, the average power is less than the theoretical maximum. This is essentially the scheme utilized by the Rance tidal project, in which the decrease in hydraulic head and other inefficiencies diminish the energy output to no more than one-quarter of the theoretical value. Since the French electric power system has a large hydroelectric component, 40% of the total capacity, the storage capacity of the conventional hydroelectric facilities, is in effect used to compensate for the fluctuating output of the tidal power plant. Since Great Britain lacks significant quantities of hydropower and relies on thermal power plants for electricity, tidal power, despite the equally large tidal ranges available, is less attractive.

Although the extraction of large quantities of tidal energy is theoretically possible, the immensity of the facilities required precludes development of all but a very moderate tidal capacity. For example, the Bay of Fundy, with an area of 13,000 km<sup>2</sup> and an average tidal range of 8 m, could produce an average power of 182 GW. Even if only 25% of this is realized, the output of 46 GW is substantial (approximately twice that consumed by Canada). However, since enclosing the Bay of Fundy is neither technically nor economically feasible, tidal sites are limited to more modest developments. Using only those basins and estuaries for which construction of tidal power plants was deemed technically feasible, Hubbert estimated that the maximum total average power that could be produced would be 64 GW [20]. These sites correspond to regions already mentioned with tidal ranges of 5 to 10 m. Assuming that one-quarter of the theoretically available power could be achieved, the total output would be only 16 GW, a rather modest average power.

While the impact of tidal power on the world power supply will necessarily be rather small (16 GW is less than 2% of the average electricity consumption rate for the world), this is not true for Eastern Canada. The potential sites on the Bay of Fundy account for nearly one-half of the world's theoretical tidal potential, that is, 29 GW [21]. This region, with unusually large tidal ranges, contains the best tidal sites in the world. A proposed plant utilizing the Minas Basin and Cobequid Bay would result in a tidal pool with an area of nearly 800 km<sup>2</sup>. Due to the very large tidal range with a mean value of 10.9 m (16.5 m spring and 7.3 m neap tides), the site has a theoretical potential of 20 GW. Even if only one-quarter of the potential



is realized, the output would be considerably greater than that which can be presently utilized in the region.

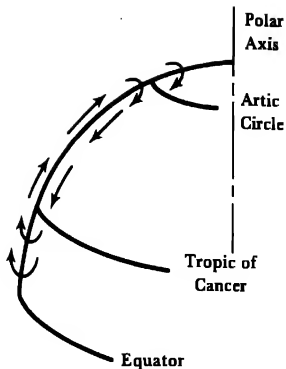
Other sites, on the Bay of Fundy that have received greater attention include the estuaries on the Chignecto Bay, the Annapolis Basin, the Cobscook-Passamaquoddy Bays. The latter site on the international border between Canada and the United States has probably received the greatest attention [22, 23] even though, according to Trenholm [21], the other sites wholly in Canada are technically superior in many respects. The proposed Passamaquoddy Tidal Project would utilize two tidal pools. The high pool would be the Passamaquoddy Bay. It would be filled at high tide through the gates of the dam, separating it from the ocean, while the low pool, Cobscook Bay, would be emptied at low tide. The pools would be separated by a dam containing the electric power plant. Power would be obtained by the flow of water from the high to the low pool through the turbines. Even though less of the potential tidal energy is utilized by this scheme, the resultant output is less dependent upon the tidal period than that obtained with a single tidal pool. An international study initiated in 1956 resulted in a proposal to develop a base load power plant with an output of 300 MW. Subsequently (1961), the project was deemed to be economically unfeasible, based upon the then existing conditions. A new study was prepared in 1963 by the U.S. Department of the Interior, recommending that a facility with peak capacity of 1 GW be constructed. The construction of this facility in conjunction with a hydroelectric development on the Upper Saint John River was considered economically justifiable due to the greater market value of peaking power. Construction would involve a joint effort by Canada and the United States with both countries sharing the benefits. To date (1980) an agreement between the governments has not been achieved. While the tidal potential for the Bay of Fundy is large, the transformation of engineering designs into functioning electric power plants is yet to be realized.

### 3. OCEAN THERMAL ENERGY CONVERSION (OTEC)

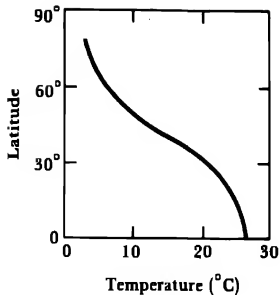
Incident solar energy is absorbed by the surface waters of the oceans as well as by land areas. Since the oceans comprise over 70% of the earth's surface area, the absorbed energy that is stored as a latent heat of the oceans represents a very large potential source of energy.

An estimate of the rate at which thermal energy exchanges occur (power) may be obtained by determining the energy necessary to produce the average yearly evaporation for the oceans of approximately 120 cm (Figure 9.3). For a surface temperature of 20°C, an energy of 580 cal is needed to evaporate one gram of water ( $2.4 \times 10^6$  J/kg). Since an evaporation of 120 cm implies the evaporation of  $1.2 \text{ m}^3$  (1200 kg) of water for each square meter of surface area, a solar energy input of  $2.9 \times 10^9$  J is required. Averaged over a year, this is equivalent to a power density of 92 W/m<sup>2</sup>. In tropical regions with higher surface temperatures and hence higher evaporation rates, energy exchange rates in excess of 100 W/m<sup>2</sup> are required.

As a result of the small variation in the density of water with temperature,



(a) Thermal ocean circulation  
(northern hemisphere)



(b) Ocean surface temperature

**FIGURE 9.9 Ocean Circulation and Surface Temperature.**

the ocean's temperature is not uniform with depth. Pure water has a maximum density at a temperature of  $4^{\circ}\text{C}$ ; at both higher and lower temperatures it is less dense. Even though the density change with temperature is very small (the density at  $25^{\circ}\text{C}$  is only .3% less than at  $4^{\circ}\text{C}$ ), it is sufficient to retard the mixing of the surface layers of water that have been warmed by the absorbed solar flux and the deeper cold waters. The cold water, being more dense than the water at the surface, is prevented from rising by gravitational forces. Hence, the mixing that occurs due to thermal conduction proceeds at an extremely slow rate.

In the tropics, ocean surface temperatures in excess of  $25^{\circ}\text{C}$  occur. Currents, however, tend to eventually carry these waters to the polar regions, where they are cooled to  $4^{\circ}\text{C}$ . The chilled water then tends to settle to the depths of the ocean. This thermal-syphon process, indicated in Figure 9.9, produces a slow mixing of the ocean waters. As a result, the warm surface waters that occur in tropical regions are found immediately above cold deeper waters that have temperatures within a few degrees of  $4^{\circ}\text{C}$ . Depths of less than 1 km are often sufficient to obtain a temperature difference of 15 Celsius degrees. The combination of the warmed surface water and cold deep water provides the very condition needed to operate a heat engine, a combination recognized as early as 1881 by D'Arsonval as a potential energy source.

A temperature differential of 15 Celsius degrees is considerably smaller than that utilized by a high-efficiency modern steam power plant (500 Celsius degrees). The inherent low efficiency of an ocean power plant is compensated for by the large quantities of heat energy available. The efficiency can be no greater than the Carnot efficiency, which depends only upon the absolute temperature of the heat source and the waste heat sink.

$$\eta_c = 1 - T_1/T_2 = (T_2 - T_1)/T_2$$

$$\Delta T = T_2 - T_1$$

$$\eta_c = \Delta T/T_2$$

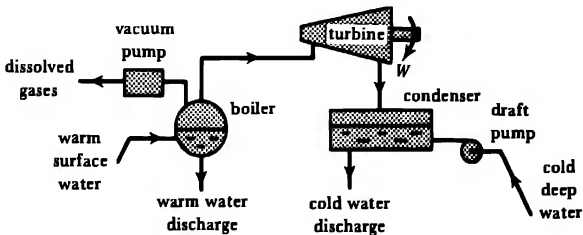
A high temperature,  $T_2$ , of 25°C (298 K) results in the following efficiency expression.

$$\eta_c = .336 \Delta T\%$$

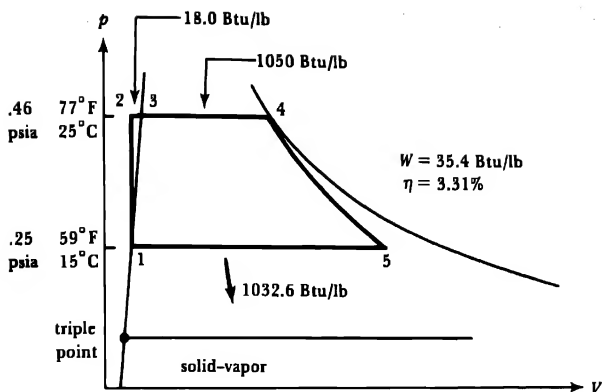
For an efficiency of 3%, a temperature difference of 9 Celsius degrees is required. Due to unavoidable temperature differentials that produce the necessary heat flows, a working temperature differential in excess of 9° is unlikely. If the energy required to move the large volumes of water necessary to operate the power plant is considered, the overall efficiency will probably be no more than 2%.

The earliest attempt to tap this energy resource was made by Georges Claude [24] in 1929. An experimental power plant was built on the Mantanzas Bay, Cuba. It utilized the waters of the warm Gulf Stream for the high-temperature heat source. A submarine cliff adjacent to the Bay, which sinks nearly vertically to depths of 100 to 200 m, was the cold water source. The greatest difficulty associated with the construction of the power plant was encountered in laying the cold water pipe, a pipe 2 m in diameter, 2 km long, and weighing 400 tons. Two pipes were "lost" before success was achieved. With the third pipe, water at a temperature of 13°C for an hourly flow rate of 4000 m<sup>3</sup> was obtained. The temperature of the water at the inlet of the pipe was estimated to be 11°C. In order to minimize the heating of the cold water as it was brought to the surface, a large-diameter pipe was required. As a consequence, the pumping power, 80 kW, was considerably greater than the 22 kW produced by the small turbine that was available. Since only 10% of the cold water was actually used, an output of ten times that obtained, 220 kW, would be expected if all the cold water were effectively used. The pumping power then would be only 36% of the output, a quantity not as excessive as many of Claude's critics predicted.

The system used by Claude, Figure 9.10, employed an open cycle in which



**FIGURE 9.10 Open-Cycle Sea Thermal Plant.**



**FIGURE 9.11** A Low-Temperature Difference Rankine Steam Cycle.

the sea water, due to the reduced pressure, served as the working fluid. Figure 9.11 is a pressure-volume diagram for conditions typical of a sea power plant: a boiler temperature of 77°F (25°C) and a condenser temperature of 59°F (15°C). The efficiency of a closed Rankine cycle for these conditions is 3.31% (assuming pure water), a value only slightly less than the Carnot efficiency. Boiling of the warm surface water occurs at a pressure of only .46 psia, 3% of atmospheric pressure. Since dissolved gases are liberated from the boiling water, a vacuum pump is required to maintain the subatmospheric pressure. The steam is expanded in a low-pressure, low-temperature, high-volume turbine before being condensed by the cold water. An advantage of this cycle is that heat exchangers with their attendant temperature differentials are unnecessary. The disadvantage is the very small pressure drop and the large specific volumes that must be utilized by the turbine. The turbine inlet specific volume,  $v_4$  (695 ft<sup>3</sup>/lb) is 3700 times that of Figure 5.6 (a more conventional steam cycle without superheat). In addition, owing to the inherent low efficiency associated with the small temperature difference, the output work per unit mass of steam is only 5 to 15% the work of a conventional power plant. A low efficiency combined with a large specific volume requires a very large turbine.

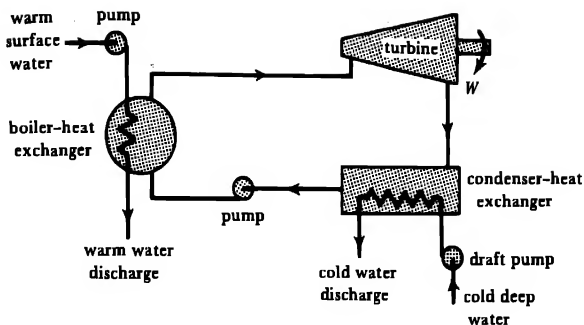
The heat input corresponding to the path 3-4 of Figure 9.11 (1050 Btu/lb), is obtained through the cooling of the warm water. A cooling of 9 Fahrenheit degrees (5 Celsius degrees) yields 9 Btu for each pound cooled. The heat input of 1050 Btu which is required for each pound of steam thus requires the cooling of 117 pounds of warm water. The greater the cooling that occurs, however, the lower the steam temperature and the overall efficiency. A temperature decrease of only two Celsius degrees requires a warm water flow rate 292 times the steam rate, that is, only one pound of each 292 pounds of high-temperature water is converted to steam.

Approximately the same flow rate is required for the condenser water, since increasing the temperature difference of the cooling water also decreases the usable temperature difference for the heat engine.

If the unavoidable inefficiencies of a thermal sea power plant are accounted for, even larger quantities of warm and cold water are required. A two percent conversion efficiency requires a heat energy input fifty times the output energy, that is, 50 kWh of thermal energy are required for each kilowatt-hour of output. Since one kilowatt-hour is equal to  $3.6 \times 10^6$  J, a thermal input of  $1.8 \times 10^8$  J or  $4.3 \times 10^7$  cal is required. For a temperature change of two Celsius degrees,  $2.15 \times 10^7$  g or 21.5 metric tons ( $21.5 \text{ m}^3$ ) of water must be circulated through the power plant. Each kilowatt of output power would, for these conditions, require a flow rate of  $21.5 \text{ m}^3/\text{h}$  ( $6 \times 10^{-3} \text{ m}^3/\text{s}$ ). A moderate sized power plant of 100 MW ( $10^5$  kW) would therefore need warm and cold flow rates of  $600 \text{ m}^3/\text{s}$ . The quantities of water involved per unit of output energy are comparable to those of a hydropower plant (the  $21.5 \text{ m}^3/\text{kWh}$  corresponds to a hydraulic head of 17 m). However, while viscous friction losses of hydropower plants can usually be ignored since they are overcome by the hydraulic head, a sea thermal power plant must use a significant portion of its output to overcome these losses.

As indicated by the number of papers and reports published over the last several years, a renewed interest in solar sea power plants is evident [25-38]. In order to evaporate sea water more efficiently, a controlled flash evaporation process which uses a gravity flow of very thin films of water has been suggested [25, 26]. Flash evaporation eliminates the turbulence associated with boiling and reduces the quantity of dissolved gases that are liberated. Less energy is therefore consumed by vacuum pumps. Various modifications of the system utilized by Claude have been proposed. If the open condenser of Figure 9.10 is replaced by a heat exchanger, the condensed steam can be recovered as potable water. Other proposals include utilizing the deep sea water used for cooling for sea farming, mariculture. The settling of the remains of the living materials which exist primarily in the upper sunlit layer of the ocean results in lower water layers that are rich in nutrients. The nutrient-rich water, when warmed, could be used as the food source for other marine life.

If only energy is to be produced, a floating power plant with a vertical (minimum length) cold water pipe appears attractive. In order to reduce the turbine size and to eliminate the low pressures associated with a steam cycle, closed-cycle heat engines using a working fluid such as ammonia, propane, or a freon have been suggested (Figure 9.12). For a boiler temperature of  $25^\circ\text{C}$ , the vapor pressures of ammonia and propane are nearly ten times the atmospheric pressure and the specific volumes are comparable to those of a conventional steam power plant. While the size, and hence the cost, of a turbine is smaller than that of a comparable low-pressure steam turbine, large heat exchangers are required. In order to reduce heat exchanger costs, exchangers submerged at a depth at which the sea water pressure is equal to the working fluid pressure can be used. A boiler pressure ten times the atmospheric pressure requires a depth of approximately 100 m. The condenser, having an internal pressure less than that of the boiler, would be located above the boiler for such a power plant. In order to minimize the temperature differentials of the heat



**FIGURE 9.12 A Closed-Cycle Sea Thermal Power Plant.**

exchangers, large heat transfer surfaces are required. Several temperature differences are introduced by each heat exchanger, a difference associated with the heating or cooling water, that of the exchanger itself, and a difference in conducting or removing heat from the working fluid of the engine itself. These six additional temperature differences (three for the boiler and three for the condenser) decrease the already small available temperature differential for the heat engine. Well designed, not excessively large and costly heat exchangers with minimum temperature losses will be required.

Electrical energy produced by offshore sea power plants could be transported to shore by submarine cables. Alternatively, high-energy consuming processes such as aluminum reduction could be carried out on a floating platform adjacent to the power plant. Since in the case of aluminum, ore is frequently transported by sea, a floating refinery introduces no additional transportation requirements.

Since the flow rates of warm ocean currents are very large, estimates of the potential total power that could be generated by ocean temperature gradient power plants are also large. A flow rate of  $3 \times 10^7 \text{ m}^3/\text{s}$  (a rate comparable to that of the Gulf Stream) could, assuming a two percent conversion efficiency and a cooling of two Celsius degrees, result in an output power of  $5 \times 10^{12} \text{ W}$ . This is nearly twice the total U.S. energy consumption (1979). Even if only a small portion of the above output is achieved, it could still be a significant part of the U.S. electrical energy consumption rate of  $2.7 \times 10^{11} \text{ W}$  (1979). On a world basis, the potential is obviously immense. It is also significant that many of the less developed nations that lack reserves of fossil fuels are in tropical regions adjacent to ocean waters suitable for temperature gradient power plants.

Undesirable environmental effects could set an upper limit on the power that might be extracted from the sea. Not only would power plants tend to cool the surface waters through their heat extraction but also by their discharge of cold water withdrawn from the ocean depths. Large-scale utilization of these power plants

would tend to accelerate the mixing of the ocean waters. A reduced ocean surface temperature could affect the solar energy balance and introduce significant climatic effects. Concurrent changes that might occur in both the evaporation rate and the carbon dioxide exchange rate could be very important. Information on which to base an estimate of these effects is presently lacking.

A substantial research and development effort to determine the feasibility of obtaining energy from the ocean in this manner was initiated in 1974 [39]. The Department of Energy's program includes evaluating the performance of heat exchangers, developing techniques to minimize the fouling of water passages by biological organisms (biofouling), and estimating environmental consequences. Conceptual designs for various pilot plants with electrical output powers of 10 to 40 MW have been completed and a navy tanker has been suitably modified for testing OTEC components at sea. The testing facility, the S.S. *Ocean Energy*, has a closed-cycle heat engine that uses ammonia as a working fluid and has an electric generating capacity of 1 MW. A retractable, 600 m long bundle of three 1.5 m diameter pipes will be used for the cold water inlet. In addition to this facility, numerous other studies are being carried out to aid in developing a pilot OTEC generating facility during the 1980s.

#### 4. WIND POWER

Of the total incident solar flux absorbed by the earth, approximately 20% or  $2 \times 10^{16}$  W is absorbed by the earth's atmosphere. The absorbed energy results in a heating of the atmosphere, a storage of thermal energy, as well as a convective motion of the atmosphere, a conversion to kinetic energy. Only a small portion of the absorbed energy, however, is transformed into kinetic energy. Hubbert estimates that the atmospheric and ocean convective currents combined account for a power of  $3.7 \times 10^{14}$  W [40]. According to Willett, as quoted by Putnam [41], the winds account for a power of  $2 \times 10^{13}$  W (a quantity  $2\frac{1}{2}$  times the 1978 energy consumption rate for the world). If one percent of the  $2 \times 10^{13}$  W estimate of Willett could be utilized, an output of  $2 \times 10^{11}$  W or approximately 2.5% of the world's energy consumption rate would be obtained. If this power were used for the generation of electricity, as it very likely would be, its thermal equivalent could be 7% to 8% of the total. While the potential extractable wind power is small, it is not insignificant. Technical problems which arise due to the variability of wind intensity tend to partially limit the attractiveness of wind systems.

Windmills are the result of an old, but well-developed, technology. Horizontal mills were used in Persia as early as the tenth century. European mills of the more conventional vertical sail-type were used to grind grain as early as the twelfth century. The large European windmills that were developed during the Middle Ages were used for both grinding grain and pumping water. Outputs as great as 50 hp were achieved. The most familiar example of windmills used for pumping of water were those of the drainage system employed for the reclamation of portions of the Netherlands. An excellent description of the European windmills that were widely

used until the introduction of the steam engine during the first half of the nineteenth century is given by Reynolds [42]. An even older use of wind power, the sailing ship, was also replaced by the steam engine during the nineteenth century.

Windmills were used in rural America to power irrigation pumps and to drive small electric generators used to charge the batteries that provided current for electric lighting. The American mills were considerably smaller than the European ones: typically only 12 to 15 ft in diameter with outputs (even under ideal conditions) of no more than one horsepower. A description of these windmills and the results of numerous power measurements are presented in a two-paper series prepared by Murphy in 1901 [43]. His measurements indicated that efficiencies of 20 to 30% of that theoretically predicted were achieved by many of the mills tested.

A windmill or wind-turbine converts the kinetic energy of the moving air to an energy associated with a mechanical motion, usually that of a rotating shaft. While the efficiency of a wind-turbine is not limited by a thermodynamic conversion process, other limitations do exist. The kinetic energy of a unit volume of air (one cubic meter) depends upon its mass, that is, its density. If  $M$  is the mass of a kilogram-mole of air, the mass density is simply this divided by the volume of a kilogram-mole.

$$\rho = M/V \quad \text{kg/m}^3$$

Since the atomic weight of air is approximately 29, the mass of one kilogram-mole,  $M$ , is 29 kg. The volume,  $V$ , may be obtained from the ideal gas law.

$$V = RT/p$$

A temperature of 20°C and atmospheric pressure ( $p = 1.01 \times 10^5 \text{ n/m}^2$ ) results in a volume of 24.1 m<sup>3</sup> and a corresponding density for air of 1.2 kg/m<sup>3</sup>. An increase in elevation results in a decrease in density due to the decrease in pressure.

The kinetic energy of one cubic meter of air (in which  $V$  will now be used for velocity) may readily be obtained.\*

$$E = \frac{1}{2}\rho V^2 \quad \text{J/m}^3$$

In one second, a volume element of air moves a distance of  $V$  m. The total volume crossing a plane one square meter in area and oriented normal to the velocity vector is therefore  $V$  cubic meters. Multiplying the kinetic energy,  $E$ , by  $V$  yields the rate at which energy is transferred, that is, power.

$$P = EV = \frac{1}{2}\rho V^3 \quad \text{W/m}^2$$

Therefore, if the total kinetic energy of the wind could be extracted by bringing the moving air to rest, a power of  $\frac{1}{2}\rho V^3$  would be obtained for each square meter that is intercepted. Substantial powers even for moderate wind velocities are predicted, for example, a 10 m/s velocity (22.3 mph) results in a power of 600 W/m<sup>2</sup>. Since power is proportional to the third power of the velocity, a 26% velocity increase

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\* For the remainder of this section,  $V$  will be used exclusively for velocity.



results in a doubling of the power. Conversely, the power drops very rapidly with a decrease in velocity. A 26% velocity decrease results in a power that is one-half the initial value, while a 50% velocity decrease results in an output that is only 12.5% of its initial value (5 m/s corresponds to a power of 75 W/m<sup>2</sup>).

Obviously, not all the kinetic energy of an airstream can be extracted on a continuous basis, since completely stopping the moving airstream implies an accumulation of air at that point. The best that can be achieved is to partially decelerate the airstream and thus leave the emerging air with a velocity component that is sufficient for it to move from the vicinity of the turbine. This may be analyzed by looking at the wind turbine of Figure 9.13a, in which the flow lines are enclosed by a smooth structure. Typical velocity and pressure profiles are presented in Figures 9.13b and 9.13c, respectively. Assuming that the "spent" air is to be exhausted to the atmosphere, the exhaust pressure,  $p_c$ , must be at least equal to that at the entrance,  $p$ . The velocity is assumed to gradually decrease, owing to the increase in cross section of the turbine structure. Since work is done by the air only when moving through the active portion of the turbine,  $a$  to  $b$ , the change in enthalpy of the airstream through the end sections is zero (a zero heat flow is also assumed). Using  $M/\rho$  for the volume, the following is obtained for the entrance section.

$$p(M/\rho) + \frac{1}{2}MV^2 = p_a(M/\rho_a) + \frac{1}{2}MV_a^2 \quad \text{J/kg-mole}$$

Assuming the density remains constant,  $\rho_a = \rho$ , the following results (Bernoulli's equation).

$$p + \frac{1}{2}\rho V^2 = p_a + \frac{1}{2}\rho V_a^2 \quad \text{J/m}^3$$

A similar equation may be obtained for the exit section,  $b$  to  $c$ .

$$p_c + \frac{1}{2}\rho V_c^2 = p_b + \frac{1}{2}\rho V_b^2$$

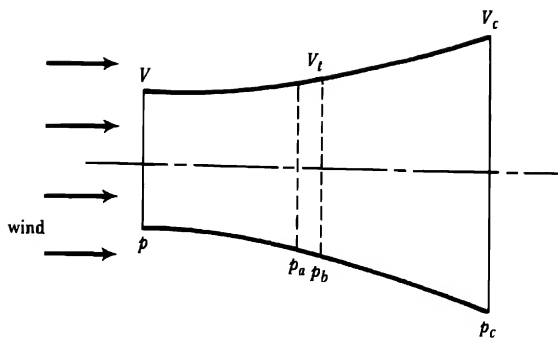
Assuming the exit and entrance pressures are equal ( $p_c = p$ ) and the velocity is the same on both sides of the turbine ( $V_a = V_b = V_t$ ), the following is obtained (by subtraction).

$$\frac{1}{2}\rho(V^2 - V_c^2) = p_a - p_b$$

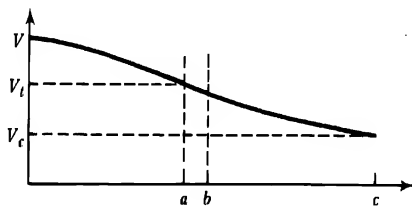
The total axial force on the turbine is the pressure difference multiplied by the turbine area,  $A$ .

$$F = A(p_a - p_b) = \frac{1}{2}\rho A(V^2 - V_c^2)$$

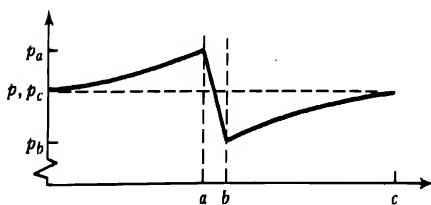
An alternative expression for the axial force may be obtained by considering the momentum change of the air as it traverses the turbine. The mass flow rate



(a) Wind-turbine



(b) Velocity



(c) Pressure

**FIGURE 9.13 An Enclosed Wind-Turbine.**

(kilograms per second), which is constant throughout the turbine enclosure, may be expressed in terms of the velocity of the turbine,  $V_t$ .

$$Q = \rho A V_t \quad \text{kg/s}$$

The momentum transfer per unit time is the mass flow rate multiplied by the velocity. The change in this quantity is the axial force.

$$\begin{aligned} F &= QV - QV_c \\ &= \rho A V_t (V - V_c) \end{aligned}$$

Equating the above expression with that previously obtained results in the following expression for the turbine velocity.

$$\begin{aligned} \rho A V_t (V - V_c) &= \frac{1}{2} \rho A (V^2 - V_c^2) \\ &= \frac{1}{2} \rho A (V - V_c)(V + V_c) \\ V_t &= \frac{1}{2} (V + V_c) \end{aligned}$$

The turbine velocity, for the conditions that have been assumed, is therefore the average of the entrance and exit velocities.

The energy extracted by the turbine is equal to the difference in the kinetic energies of the entering and leaving airstreams. The mass flow rate,  $Q$  (kilograms per second), time  $\frac{1}{2} V^2$  or  $\frac{1}{2} V_c^2$  yields the energy transfer rate, that is, power.

$$\begin{aligned} P &= \frac{1}{2} Q V^2 - \frac{1}{2} Q V_c^2 = \frac{1}{2} \rho A V_t (V^2 - V_c^2) \\ &= \frac{1}{4} \rho A (V + V_c)(V^2 - V_c^2) \quad \text{W} \end{aligned}$$

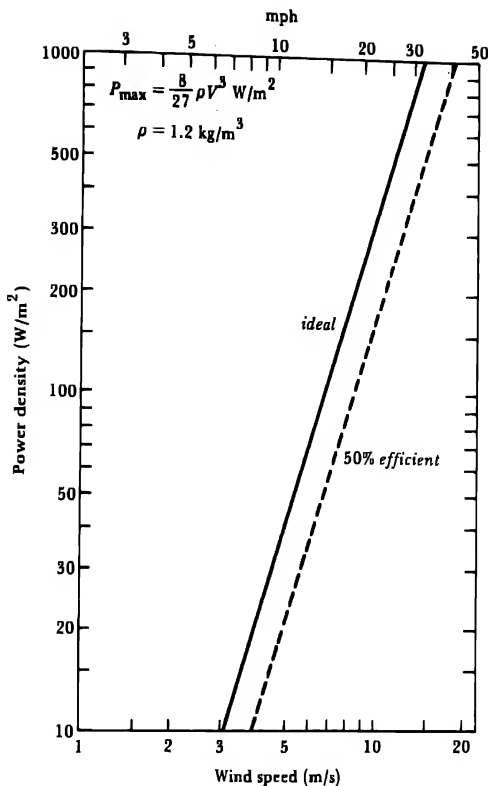
For a specified turbine area,  $A$ , and entrance wind velocity,  $V$ , the output power depends upon the exit velocity,  $V_c$ . A maximum in power (obtained by differentiating  $P$  with respect to  $V_c$  and setting the result equal to zero) occurs for an exit velocity equal to one-third of the entrance velocity.

$$V_c = \frac{1}{3} V$$

Using this result for  $V_c$  in the above relation for power, an expression for the maximum power is obtained.

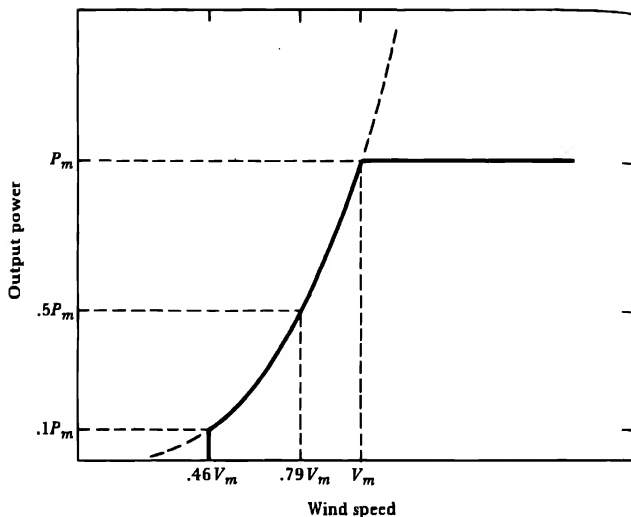
$$P_{\max} = \frac{8}{27} \rho A V^3$$

Since the kinetic energy flux of a wind stream crossing an area of  $A$  is  $\frac{1}{2} \rho A V^3$ , the extractable power is  $\frac{16}{27}$  or 59.3% of this quantity.



**FIGURE 9.14 Maximum Power Density Versus Wind Speed.**

The power output of a nonconfined turbine or conventional windmill would not be expected to be greater than the theoretical maximum of an enclosed turbine. Well-designed windmills achieve outputs which are 50 to 75% of the theoretical maximum. Figure 9.14 is a plot of maximum power density as a function of wind speed. A 10 m/s wind results in a power of 356 W/m<sup>2</sup>. The dependence of power upon the third power of velocity tends to limit the useful energy that can be extracted from the wind. If a wind system is designed for a full output power,  $P_m$ , for a speed of  $V_m$ , speeds in excess of  $V_m$  will not be fully utilized. Also, for wind speeds less than  $V_m$ , the power drops very rapidly, as indicated in Figure 9.15. For a wind speed of .46  $V_m$ , the power is 10% of its full value. Since frictional losses become



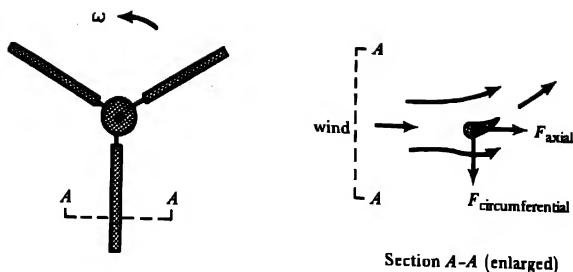
**FIGURE 9.15** *Power Output as a Function of Velocity for a Typical Wind Turbine.*

significant at low power levels, a maximum efficiency occurs only for a small range of wind speeds. The constant output for speeds greater than  $V_m$  implies a reduced efficiency, an effect necessitated by equipment limitations, such as the size of the generator, if the windmill is used to produce electrical power.

The blades of a windmill experience an axial force, the value of which depends upon velocity. Assuming  $V_c = \frac{1}{3}V$ , the condition for maximum power, the following is obtained.

$$F = \frac{4}{9}\rho AV^2$$

Since for a zero exit velocity the force is  $\frac{1}{2}\rho AV^2$ , the force corresponding to maximum power is  $\frac{8}{9}$  or 89% of this value. Windmills must therefore be capable of withstanding significant forces, a condition that makes large windmills expensive. The blades of a windmill experience both an axial and a circumferential component of force, the latter providing the torque (Figure 9.16). The circumferential component of force is analogous to the lift of an airfoil; in fact, a well-designed blade will have a cross section similar to that of an airfoil. This force or "lift" is a result of the differential pressure on the sides of the blade. Speeding up the air reduces the pressure (as on the bottom of the blade of Figure 9.16) and slowing down the air increases the pressure. Varying the pitch of the blade, that is, rotating it about its axis, modifies the forces.



**FIGURE 9.16** *A Propeller-Type Wind Turbine.*

By suitably controlling the pitch, a constant rotational speed can be achieved.

Large European windmills typically had four sails or blades that consisted of a wooden frame covered with canvas. The speed was controlled by the amount of canvas used for the sails. All the canvas was removed when the winds became excessive. Typical American windmills had 15 or more flat or slightly curved sails, the area of the sails being a substantial (over one-half) portion of the area swept by the blades. Designs used and proposed for very large wind turbines to produce electrical power have had two or three propeller-like blades. The largest early unit constructed, the Smith-Putnam Wind turbine (1941-1945) is described by Putnam [41]. A two-bladed turbine, 175 ft in diameter, was used to drive a 1.25 MW synchronous electric generator. The 30 rpm constant speed wind turbine was geared up to drive the 600 rpm generator. Full output power required a 30 mph wind with the "turn on" wind speed being 20 mph. A total of approximately 1000 hours of operation was achieved before the loss of one of the blades. The structural failure was the result of a design flaw and a joint previously repaired by a weld of the skin of the blade. The blade was not replaced and subsequent interest in wind-generated electric power waned. The experiment did, however, demonstrate the feasibility of utilizing wind energy on large scale. According to the then existing economic conditions, Putnam concluded that wind-generated electricity would be competitive with that produced by fossil fuels. While the dollar values of his estimates are no longer valid (for example, an \$800/month salary for the chief engineer), recent fuel price increases suggest that at present wind power could be economically competitive. Concurrently, pollution problems, visual pollution excepted, are avoided.

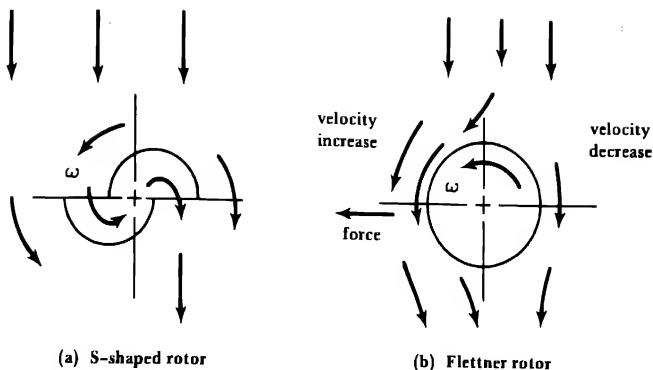
A major disadvantage of wind power is the large number of large wind turbines required for a reasonable output power. For example, a 75% efficient turbine in a 10 m/s wind results in an output of 267 W/m<sup>2</sup>. A 60 m diameter windmill results in an output of approximately .75 MW (only 1000 hp). An output comparable to that of a large steam-turbine power plant, 1 GW, would, for these conditions, require 1325 windmills. Furthermore, maximum output is obtained only when wind speeds are 10 m/s or greater. If this occurs 25% of the time, an average power of 1 GW would require over 5000 60 m diameter wind turbines. While a storage scheme

based upon the hydrolysis of water and hydrogen-oxygen fuel cells is conceivable, presently available fuel cells are inadequate (see Chapter, Section 6). Therefore, a wind-power system would serve primarily as a back-up source for a conventional thermal or hydropower plant. Economic feasibility would hence depend upon the cost of the fuel saved.

While a detailed knowledge of wind conditions, a knowledge not generally available, is necessary to estimate the output of a particular turbine at a given location, preliminary estimates of anticipated energy outputs have been prepared for the United States by the National Science Foundation [44]. A maximum yearly energy of  $1.29 \times 10^{12}$  kWh, an average power of  $1.4 \times 10^{11}$  W, is anticipated for the continental United States. Since the present electrical energy consumption rate is on the order of  $2.7 \times 10^{11}$  W, wind power could, if the estimate is realized, supply over half of the electrical power (this assumes consumption remains unchanged). The number of wind turbines required, however, is enormous, 780,000 based upon the assumptions of the previous paragraph. If the estimate of NSF is achieved, a quantity considerably greater than the  $2 \times 10^{11}$  W estimated in the first paragraph of this section could probably be obtained on a world basis.

A frequently proposed application of wind power is to provide electricity for individual homes [45]. Based upon the data of Chapter 5 (Figure 5.3), the per capita residential consumption rate for electrical energy is approximately 350 W. A typical family of four uses an average power of 1.4 kW (33.6 kWh/day or 1000 kWh/month). Assuming a wind speed of 10 m/s, a 75% efficiency, and a wind factor of one-fourth, a modest-sized windmill (5 m in diameter) would be required. Energy storage, however, requires lead-acid batteries. If four days' storage capacity is deemed necessary, approximately 134 kWh of storage battery capacity is called for (134 batteries with a 1 kWh rating). The number of times that a lead-acid battery can be charged and discharged is limited, one thousand complete cycles being the typical battery-life. Electricity stored by a 1 kWh battery costing \$50 would thus cost 5¢/kWh, based upon a battery-life of 1000 cycles. In addition, the investment in the batteries alone would be \$6700, a substantial sum. As a consequence, battery storage systems are limited to providing rather modest residential electrical requirements such as lighting. Alternative energy sources would be required for the large appliances such as the dishwasher, refrigerator, freezer, range, and the washer and dryer that are found in most homes and are usually electrically powered. Based upon the 5¢/kWh cost of storage alone, electricity costs even for a small system would tend to be in excess of current utility rates.

Two other devices, the S-shaped rotor and the Flettner rotor, have also been successfully used for extracting energy from wind [46-48]. The S-rotor, Figure 9.17a, has a vertical axis and is therefore insensitive to the direction of the wind. Damage due to abrupt wind direction changes is therefore minimized. A disadvantage of the S-shaped rotor is its size, its effective wind area being no greater than that corresponding to the cross section of the rotor. The Flettner rotor relies upon the action of the wind upon a rotating curved surface. As indicated in Figure 9.17b, the air velocity tends to increase on the left side of the rotor and decrease on the right side. A differential pressure, due to the velocity difference, results in a force



**FIGURE 9.17** *The S-Shaped and Flettner Rotors.*

on the rotor which is perpendicular to the wind direction. Since the pressure difference arises from the air moving around the rotor, the effective area of the rotor is considerably greater than its cross-sectional area. Two rotors have been used in place of sails on the *Baden-Baden*, which successfully crossed the Atlantic in 1925. Flettner rotors have been also successfully used for the blades of a conventional windmill, the main advantage of the rotor being that the resultant torque can be controlled by varying the rotational speed of the rotor. A disadvantage of the Flettner rotor is that an external power source, typically an electric motor, is necessary to sustain the rotation. Another vertical-axis-type wind turbine is the Darrieus rotor which has the appearance of an eggbeater.

Numerous reference materials are available which provide a more extensive discussion of wind turbines and of the utilization of wind-produced energy [49–63]. To accelerate the development of wind-produced energy systems, the Wind Energy Systems Act of 1980 (PL 96-345) was passed. Its goal of 800 MW of electrical generating capacity by 1988 was deemed to be readily realizable with a yearly funding of \$100 million [64].

## 5. GEOTHERMAL POWER

Since interior regions of the earth have temperatures higher than that at the surface, an outward flux of heat is observed. The earth's interior is a vast thermal reservoir which can, if favorable geological conditions exist, be utilized as a source of energy. As discussed in Chapter 3, the average outward thermal power density at the earth's surface is  $.063 \text{ W/m}^2$  (1% greater if volcanic effects are included). This power density is equal to  $1.5 \text{ } \mu\text{cal/cm}^2\text{-s}$ , a value frequently cited by geologists [65]. While the average heat flow rate is too small to be directly utilized, many areas have higher



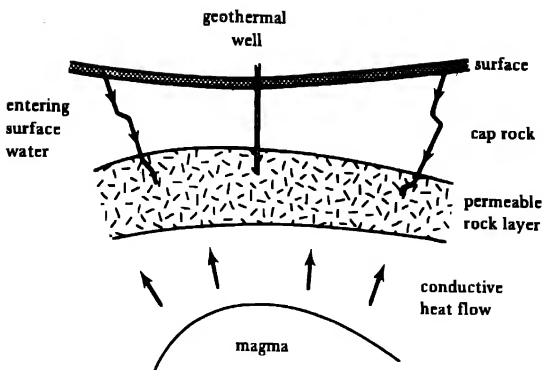
than average rates. Furthermore, many geological formations result in thermal reservoirs located within several kilometers of the earth's surface. By drilling into these high-temperature reservoirs, useful quantities of heat energy are frequently obtained.

The innermost region of the earth, approximately 2900 km from the surface, consists of a liquid core. The core's temperature is estimated to be several thousand Celsius degrees. Surrounding the core is a solid mantle which, in turn, is surrounded by a crustal layer. The crust, relative to the earth's radius, is exceedingly thin: only 30 to 40 km thick under the continents and 5 to 10 km thick under the oceans. It is only the crustal region which has been directly explored; the deepest continental drilling is on the order of only 10 km. Our understanding of the mantle and the core is based primarily upon the observed surface patterns of seismic waves produced by earthquakes. The crustal region contains the fossil-fuel reserves which are, at least at present, the world's primary energy source.

The observed heat flow at the earth's surface is much larger than what would occur due to a simple cooling of the earth's core. The heat flow is now believed to be primarily the result of the radioactive decay of isotopes with long half-lives which are distributed throughout the crust and the upper mantle. Within the crust, the observed heat-producing isotopes are uranium-238, thorium-232, and potassium-40. Normal chemical reactions release insufficient energy to account for the heat flow which has been sustained throughout the geological history of the earth (insufficient for a billion years). The average surface heat flow of  $.063 \text{ W/m}^2$  can be produced by an exceedingly low-volume rate of energy release. If a depth of 100 km is assumed for the heat-producing process, the average volume-power density required is only  $.63 \mu\text{W/m}^3$ . A density of radioactive isotopes measured in grams per cubic meter (which corresponds to a concentration ratio of only a few parts per million, a ratio typically found in surface granites) produces this rate of thermal energy generation.

The surface heat flow, for most regions, is due to thermal conduction associated with a temperature gradient. Gradients of 8 to 50  $^\circ\text{C}/\text{km}$  are common; the average gradient is on the order of 25  $^\circ\text{C}/\text{km}$  [66]. Of interest for geothermal power are regions with uncommonly high temperature gradients which result from lower than average conductivities or higher than average heat flow rates. These regions tend to be located along cracks in the earth's crust called *rifts*. According to recent geological theories, rifts are the result of the rather rapid lateral motion (rapid as measured on a geological time scale) of the tectonic plates which contain the continents. An excellent description of continental drift is contained in a collection of *Scientific American* papers [67]. Along the continental rifts, thermal reservoirs with abnormally high temperatures are frequently found within several kilometers of the surface. References on geothermal power include a highly recommended collection of papers edited by Kruger and Otte [68], an introductory paper by Barnea [69], and many others [70-82].

Figure 9.18 is a cross section of a typical hydrothermal convective system. Surface waters which have settled downward through cracks in the crustal rocks are heated in a permeable layer above a hot magma deposit. Hydrostatic pressure



**FIGURE 9.18** *A Hydrothermal Convective Geothermal System.*

prevents the water from boiling. Since the heated water is less dense than the entering cold water, it tends to rise. A large hydrothermal reservoir consisting of hot water and rocks is formed above the magma. If the water is vented at the surface through natural fissures, hot springs or geysers result. Alternatively, a layer of impermeable cap rock may prevent the heated water from reaching the surface. Geothermal power, for the latter case, is obtained by drilling through the cap rock to reach the pressurized hot water. If the water is sufficiently hot, steam will be produced at the bore hole. The systems that presently use geothermal power are of this type. Before drilling into the thermal reservoir, the surface heat flow from it was small because of the impermeable cap rock. Therefore, if the system was in thermal equilibrium, the heat flow into the reservoir was also small. The natural thermal recharge of the reservoir, compared to the rate at which heat is likely to be extracted through the drill holes, is probably insignificant. This implies a limitation on the energy which can be extracted from a geothermal reservoir or field.

Presently, geothermal power plants utilize steam produced underground in wells that are one to two kilometers in depth. The earliest geothermal electric production was that of a 1904 experimental facility in Larderello, Italy. A permanent 250 kW electric generator was installed at the site in 1913. In Iceland, hot water for heating has been produced since 1928. The largest geothermal installation is that of The Geysers in California which has an electric generating capacity of 1.0 GW. Other large installations are those of Larderello, Italy; Wairakei, New Zealand; and Cerro Prieto, Mexico. Smaller installations are found in Japan, Iceland, and the Soviet Union.

The temperature and pressure of the steam produced by a geothermal well depend upon the production rate: the greater the mass flow rate of the steam, the lower its temperature and pressure. While a low flow rate results in the highest temperature and pressure, the available thermal power which depends upon both

flow rate and the energy content of the steam (its enthalpy) will also be small. The optimum flow rate will produce the greatest output power for a given installation. At The Geysers, dry steam is produced at a temperature of approximately 177°C and at a pressure of 7.8 atm (114 psia). Liquids and suspended solids are removed from the steam at the wellhead by a centrifugal separator.

The geothermal steam, on emerging from the separator, could be expanded in a turbine and exhausted directly to the atmosphere. While this is the simplest and least costly type of installation, its efficiency in converting thermal energy to electrical energy is low. Leaving the turbine would be a mixture of vapor and liquid, the pressure of which must be above that of the atmosphere. Its temperature must therefore be above 100°C. The Carnot efficiency of a heat engine operating between temperatures of 177°C and 100°C is only 17%. A considerable improvement can be achieved if, for example, the exit temperature is reduced to 50°C. The decrease in exhaust temperature by 50 C° increases the differential temperature by 65% and results in a Carnot efficiency of 28%. A turbine exit temperature of 50°C corresponds to a pressure of .12 atm (1.8 psia or 93 mm Hg) for a liquid and vapor mixture. A condenser with an internal pressure less than atmospheric pressure is required. The initial generating units at The Geysers used a barometric condenser of the type indicated schematically in Figure 9.19. Atmospheric pressure acting on the water in the hot well supports a column of water  $h$  meters high. The difference between atmospheric and condenser pressure is the pressure required to support the water column,  $\rho gh$ ,  $\text{n/m}^2$ .

$$\text{Pressure differential} = \rho gh \quad \text{n/m}^2$$

where  $\rho = 10^3 \text{ kg/m}^3$  density of water  
 $g = 9.8 \text{ m/s}^2$  acceleration of gravity

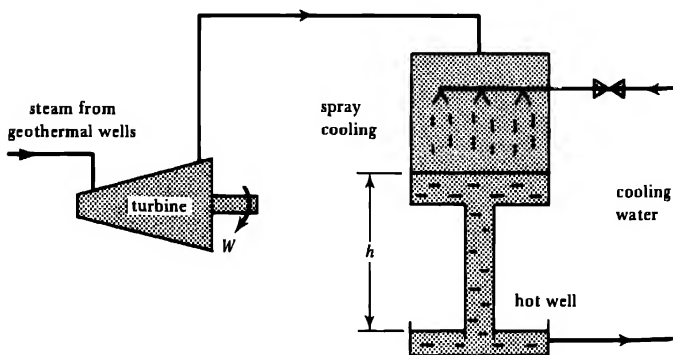


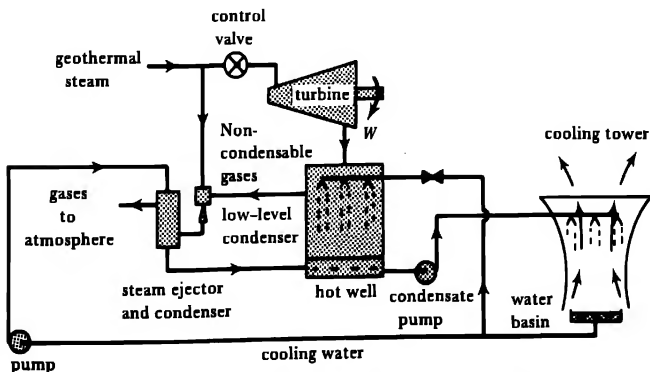
FIGURE 9.19 A Turbine and a Barometric Condenser.

A condenser pressure corresponding to a 50°C turbine exhaust temperature requires a column of water with a height of 9 m. Steam is condensed through direct contact with the cooling water.

An alternative condenser which does not require an external hot well is a low-level type in which a condensate pump is used to extract the heated cooling water from the condenser. Wet cooling towers in which a substantial portion of the condensed steam is evaporated to the atmosphere (80%) are used at The Geysers. Since the thermal efficiency of a geothermal power plant is considerably less than that of a conventional fossil-fueled plant, much larger cooling towers are required for the same electric power output. In addition to suspended solids, geothermal steam also contains gases such as carbon dioxide, hydrogen sulfide, methane, ammonia, nitrogen, and hydrogen. Concentrations depend upon the particular geothermal field. At The Geysers, these gases comprise approximately 1% of the steam by weight. These noncondensable gases which expand with the steam in the turbine must be removed from the condenser in order to maintain the low condenser pressure. Either electric-motor-driven mechanical vacuum pumps or steam-driven gas ejectors such as indicated in Figure 9.20 can be used. Two stages of gas ejectors are required at The Geysers to maintain the desired condenser pressure.

Several steam wells are required for each generator. At The Geysers, seven wells for each 55 MW electric-generating unit are typical. Through usage, the production rate of the wells declines and hence new wells must be continually drilled to maintain the steam rate. The useful life of a well has been found to depend upon well spacing. The production decline implies a depletion of the thermal energy of a reservoir or its available water or both. Several years of useful steam production are usually obtained for each well.

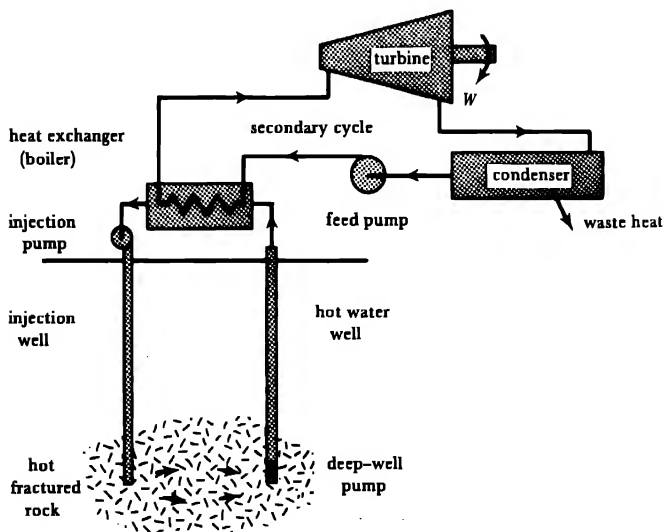
Geothermal wells producing dry steam (such as the steam produced at The



**FIGURE 9.20** A Geothermal Power Plant with a Low-Level Condenser.

Geysers) are ideal for electric power generation. The number and extent of dry steam fields, however, is limited. More common are hot water wells which produce water above  $125^{\circ}\text{C}$  and can, at the wellhead, be flashed to steam. Lower quality geothermal fields which produce hot water below  $125^{\circ}\text{C}$  are frequently used for health spas. Electric power can be produced from the thermal energy of hot water wells by using a secondary closed thermodynamic cycle with a working fluid such as freon or isobutane. The low thermodynamic efficiency that results for the low-temperature source of heat requires a power plant that has large heat exchangers and uses vast quantities of cooling water. The low-quality heat associated with hot water fields can be used for space heating and for low-temperature industrial processes [83, 84]. Hot geothermal water is also used for greenhouse heat in Iceland and to power absorption air conditioners (see Chapter 7, Section 6) at a hotel in New Zealand. In arid regions, hot geothermal water can be used as a source of heat for sub-atmospheric pressure evaporators which produce potable water from the brackish well water.

Dry geothermal fields in which no water is present are another potential source of geothermal energy [85]. A heat transfer fluid, most likely water, will need to be injected into the field. After drilling, fracturing of the field in the vicinity of the well will be required to improve thermal contact with the fluid. Both hydraulic fracturing similar to that used for low-producing oil wells and nuclear explosives have



**FIGURE 9.21 A Dry Geothermal Field and Power Plant.**

been suggested for activating dry geothermal fields.\* A series of pressurized injection wells and extraction wells can then be used to tap the thermal energy. As with the hot water geothermal fields, a secondary closed thermodynamic system will be required for producing electric power (Figure 9.21). The viability of extracting energy from a dry field depends upon the degree to which a field can be fractured. Preliminary results by researchers at the Los Alamos National Laboratory are encouraging.

Wide variations in the estimates of the ultimate power capacity of geothermal plants may be found. Estimates of the in-place stored geothermal energy reserves, however, are straightforward to calculate. White presents a detailed energy estimate [86]. While hydrothermal reservoirs associated with hot springs as well as reservoirs capped with impermeable rocks (hydrothermal systems of a composite type) are included in White's estimates, it is the latter which account for the greatest stored energy. Both the Salton Sea region of California and the Larderello fields of Italy are composite types. Based upon the lateral extent of the fields and the temperatures by which the fields exceed the ambient temperature, White arrives at two estimates of stored energy. For a depth of 3 km, a depth corresponding to the more accessible region of the fields, the total stored energy of known fields is approximately  $8 \times 10^{21}$  J. The energy stored in fields of the hot springs type is estimated to be only 10% of the above quantities. White assumes that 1% of the thermal energy can be recovered, that is,  $8 \times 10^{19}$  J or  $2.5 \times 10^{12}$  Wy for the 3 km estimate. If this energy is extracted at a uniform rate over a period of 50 years, a thermal power of 50 GW is obtained. A thermal-electric conversion efficiency of 20% results in an electric power of only 10 GW. For the estimate based upon a 10 km depth, an electric power of 50 GW is predicted. Since the United States contains only 5 to 10% of the geothermal fields, the corresponding available electric power would be no more than 1 to 5 GW. A recovery of 10% of the stored energy, however, results in a tenfold increase, that is, 10 to 50 GW of electric power for the United States. Based upon a more recent estimate, the U.S. geothermal resource base may be considerably larger than that given by White. As a result of a detailed study of igneous systems (volcanic fields), Smith and Shaw concluded that the geothermal energy base was about  $10^{23}$  J [87]. If 1% of the energy is recovered and a conversion efficiency of 20% is realized, an electric power of 127 GW could be obtained for 50 years. If the above estimates are to be realized, techniques to recover geothermal energy from dry fields will need to be developed.

Several undesirable environmental effects can be caused by power plants that obtain steam from hydrothermal fields. Noncondensable gases which are usually vented to the atmosphere include hydrogen sulfide, a very foul smelling gas. Since the thermal efficiency of a geothermal power plant is much less than that of a fossil-fueled power plant with a comparable output, two to three times more water is evaporated by its cooling tower. This results in abnormally high levels of moisture in the atmosphere. That portion of the condensed geothermal steam which is not

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\* Public opposition to the use of underground nuclear explosives may preclude their use. In Colorado, a citizen-initiated referendum which requires voter approval of future underground nuclear experiments was passed in 1974.

evaporated is high in minerals and other impurities. At The Geysers, this waste water effluent is reinjected into the field through nonproducing wells. Reinjection of water may reduce the rate of surface subsidence. The removal of large quantities of steam (or hot water) from geothermal fields located near fault zones may cause additional seismic activity. Another nuisance concurrent with the development and use of geothermal power is noise, as both drilling and the venting of steam from wells result in high noise levels. The adverse environmental effects of geothermal power plants, as contrasted to fossil-fueled plants, tend to be concentrated at the power plant. Pollution from coal mines or oil wells is avoided as well as that introduced by transporting the fuel. While characterizing it as "clean" may overstate the quality of geothermal energy, geothermally produced electricity may result in lower levels of pollution than the pollution produced from many alternative sources of energy.

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### PROBLEMS

1. Determine, for one cubic meter of water, the elevation change which corresponds to a potential energy of one kilowatt-hour.
2. A pumped water storage reservoir has a surface area of  $1 \text{ km}^2$  and a depth of 10 m. The average height above the lower reservoir may be assumed to be constant and equal to 100 m. Determine the energy storage capacity in joules and kilowatt-hours for this reservoir.
3. Assume that the average runoff for the United States is that of Figure 9.3, 23 cm. For the continental United States, the total developable average hydropower is estimated to be approximately 76 GW. Determine, assuming a continental area of 3 million square miles, the average hydraulic head necessary to produce this power.
4. Suppose two tidal pools are used for a tidal power plant. The upper pool, area  $A_1$ , is filled at high tide and the lower pool, area  $A_2$ , is emptied at low tide. The tidal range is  $R$ .
  - (a) Determine the energy that can be obtained by emptying the upper pool into the lower pool while the gates connecting the two pools to the ocean are closed.
  - (b) A comparison with a single tidal pool area  $A$  is desired. Assuming that  $A = A_1 + A_2$ , the total area enclosed by the two tidal pool scheme, determine the ratio of the energy obtained for each tidal period. *Note:* For the two tidal pool system, energy can be extracted only once for each tidal period. What is this ratio for  $A_1 = A_2 = \frac{1}{2}A$ ?
  - (c) What is the advantage of the two pool system?
5. Consider the steam cycle of Figure 9.11 for an ocean temperature gradient power plant.
  - (a) What is the mass of steam required for an output energy of 1 kWh? What is the quantity of input warm water necessary to produce this output if a temperature drop of two Celsius degrees is utilized?

- (b) Determine the mass flow rate of warm water necessary for an output power of 100 MW.
- (c) Determine the cold water flow rate for a 100 MW power plant, again assuming a two-degree temperature difference.
- (d) If the water velocity of the warm and cold water is 5 m/s, determine the diameter of the piping required for the 100 MW power plant.
6. What would be the Carnot efficiency for an upper temperature of  $25^{\circ}\text{C}$  and a lower temperature of  $15^{\circ}\text{C}$ ? Why is the efficiency of the Rankine cycle of Figure 9.11 very nearly equal to this value, whereas this is not the case for a conventional high-temperature Rankine cycle steam engine?
7. Since an ideal wind turbine has an output power proportional to the cube of the wind velocity, the use of average wind velocities can result in significant errors in predicting energy outputs. Consider the time-dependent wind velocities of Figure P9.7; all have the same average speed ( $V_0$ ) for the period of time,  $T$ . Determine the ratio of energy produced by each of the wind profiles to that produced by a steady wind with a velocity of  $V_0$ . What is the effective wind velocity for each of these cases?

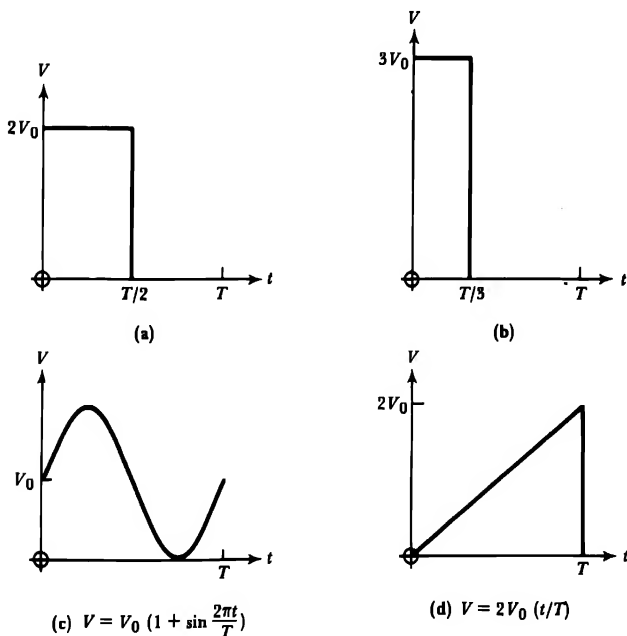


FIGURE P9.7

8. Windmills were, until recently, used for pumping water from wells for irrigation. Determine the pumping rate (cubic meter per second) for the following windmill-pump system which has an overall efficiency of 25%.

Diameter = 5 m

Wind speed = 8 m/s

Well depth = 10 m

What is the water supplied in one day by the system for a constant wind speed of 8 m/s?

9. Assume the surface layers of the earth contain a uniform concentration of uranium-238,  $1 \text{ g/m}^3$ , for a depth of 100 km. The decay of uranium to lead results in a thermal energy of approximately 50 MeV for each uranium disintegration and the rate at which thermal energy is produced is determined primarily by the half-life of uranium-238,  $4.51 \times 10^9 \text{ y}$ .
- (a) Calculate the outward heat flow rate at the earth's surface for the decay of the uranium-238. Assume that the inward heat flow is negligible.
  - (b) Assume that a similar density of uranium-235 (half-life of  $1.59 \times 10^5 \text{ y}$ ) exists. What is the corresponding heat flow rate?
  - (c) Naturally occurring uranium is .71% uranium-235 by mass, the remainder being essentially uranium-238. Calculate the surface heat flow rate for a uniform density ( $1 \text{ g/m}^3$ ) of naturally occurring uranium.
10. An upper estimate for electric power produced from geothermal resources for the United States is desired. Assume that 5% of the land area (total area of three million square miles) contains hot rocks at sufficiently high temperatures for producing power.
- (a) Determine the quantity of heat that could be extracted from the rocks within 3 km of the earth's surface by reducing their temperature 20 Celsius degrees. The heat capacity of rocks is approximately .2 cal/g and their average density is  $2.6 \text{ g/cm}^3$ . Express the result in joules and watt-years.
  - (b) Determine the electrical capacity that could be installed which would operate for 100 years. Assume a thermal to electrical conversion efficiency of 12%.

## CHAPTER 10

# *Energy Usage and Societal Well-Being*

### **1. ENERGY: A PREREQUISITE FOR INCREASING ORDER AND KNOWLEDGE**

Developed societies are highly dependent upon continuous inputs of energy (12 kW per capita for the United States) to provide the goods and services associated with a high standard of living. While energy is usually thought of in purely pragmatic terms, that is, as an input that provides the heat and work utilized by society, the process of development itself may be shown to be theoretically dependent upon inputs of energy. Basically, energy is required to achieve an increase in order and/or knowledge ("thinking," for example, requires a metabolic input by the individual). The theoretical basis for this frequently overlooked concept is based upon the kinetic theory of gases. Hence, a brief discussion of kinetic theory is necessary before considering societal implications. Along with the theoretical developments which now form the basis for classical thermodynamics, a kinetic theory for gases was also developed. Classical thermodynamics is wholly concerned with macroscopic quantities that can be directly measured; a theory based upon the kinetic behavior of the molecules of a gas attempts to predict the observable macroscopic quantities through a consideration of the molecular interactions. Between the constituents of a gas there is a continuous interchange of momentum and energy that can be analyzed through the laws of mechanics. The simplest model is based upon collisions between perfectly elastic spherical particles, a behavior similar to that of billiard balls bouncing off each other. For such a model, the total kinetic energy and momentum of the colliding particles are conserved.

The laws of mechanics, however, are insufficient to predict the behavior of a

gas, since they provide no preferred direction for time. Reversing the sign of the time in these equations simply causes motion to run backward, that is, the system being analyzed would tend to return to its initial state. The concept of increasing entropy, however, implies a direction for time, "time's arrow." If total entropy increased when the system moved in the forward direction, a time reversal would result in a decrease in entropy, an obviously invalid conclusion. The apparent contradiction between kinetic theory and classical thermodynamics arises from the assumption that it is possible to account for the behavior of all molecules. This is an impossible task because of the quantity of molecules involved even in a very small volume of gas. The closest approach to describing microscopic behavior is by means of distribution functions which are based upon statistically averaged quantities. These functions provide greater detail of the microscopic behavior of a gas than the classical thermodynamic variables. In addition, the time-dependent behavior of distribution functions was shown by Boltzmann (*H*-theorem) to be compatible with a concept of nonincreasing entropy [1]. A kinetic theory of gases based upon the statistical behavior of molecules is discussed in numerous texts, often in conjunction with a treatment of classical thermodynamics [2- 7].

While a detailed discussion based upon the statistical behavior of a gas is beyond the present consideration, the results of such an analysis are important. Basically, the molecules of a gas tend to progress to a state corresponding to the maximum disorder: an increase in entropy corresponds to an increase in disorder. An example for which the result is intuitively obvious is that of a gas in which all molecules are assumed to start out with the same velocity vector. After numerous collisions, the direction and magnitudes of the velocity vectors of the individual molecules will become random. Since random motion corresponds to heat and ordered motion to work, it is not surprising that a thermodynamic inefficiency results when heat is converted to work.

Concurrently, the spatial distribution of gas molecules tends to be uniform throughout the container. As an example, consider the analogy of a box divided into two equal portions (left and right sides) which is to be randomly filled with balls. The probability of placing a particular ball in the left portion is  $\frac{1}{2}$ . For two balls being placed in the left portion, the probability is  $(\frac{1}{2})(\frac{1}{2})$  or  $\frac{1}{4}$ ; for three it is  $(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})$  or  $\frac{1}{8}$ , whereas for  $N$  the probability is  $\frac{1}{2^N}$ .

$$\begin{array}{l} \text{Probability of placing} \\ N \text{ balls in left portion} \end{array} = (1/2)^N$$

If the number of balls is 100, the resultant probability is  $7.9 \times 10^{-31}$ . The chance of this occurring is approximately once in  $10^{30}$  attempts. Assuming that a random selection occurs every second, only  $3 \times 10^{17}$  selections would have been accomplished in 10 billion years, a time interval which is on the order of the age of the solar system! A kilogram-mole of gas contains Avogadro's number of molecules. For  $N = N_0$  ( $6.02 \times 10^{26}$ ), the probability of the gas at a given instant being wholly in one of two equal parts of a container is  $\frac{1}{2^{N_0}}$ , a totally insignificant probability. The greatest likelihood, even for the case of 100 balls, is for the balls to be nearly equally divided between the two portions. As the quantity of constituents is increased

as in a gas, the probability of a distribution which deviates by even a minute fraction from spatial uniformity is completely negligible.

A statistically derived definition of entropy depends upon the quantity of distinct states that a system can assume.

$$S = k \ln M$$

where  $M$  = number of distinct states of system  
 $k = 1.38 \times 10^{-23}$  J/K (Boltzmann constant)

The number of states of a system, or using the term introduced by Planck, the number of complexions [8], is a measure of the disorder of the system. If only one state exists, as appears to be the case for a crystal at a temperature of absolute zero, the system is completely ordered and its entropy is zero. As the number of possible states increases, the likelihood of the system's being a particular state decreases and, concurrently, its entropy increases.

Microstates are the result of quantizing energy and positional coordinates, a concept based upon quantum mechanics. Based upon the uncertainty relationship of Heisenberg (1927), the product of the uncertainty of position and momentum for a particle is approximately equal to Planck's constant.

$$\Delta x \Delta p_x \simeq h$$

where  $h = 6.625 \times 10^{-34}$  J-s (Planck's constant)

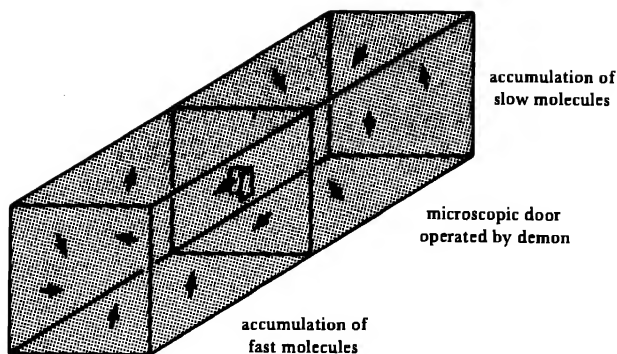
The  $x$  component of momentum,  $p_x$ , for velocities typically encountered in gases, is equal to  $mv_x$ . Since motion occurs in all three coordinates, the following is obtained.

$$\Delta x \Delta p_x \Delta y \Delta p_y \Delta z \Delta p_z \simeq h^3$$

A molecule may be thought of as being in a new state if its position and momentum coordinates differ from those of a previous state in accord with the above relationship. However, a joint consideration of all molecules is necessary. A new position of the momentum coordinate of a single molecule is sufficient to result in a new microstate for the gas. The number of possible states is therefore enormous. The range of spatial coordinates is determined by the dimension of the container and the momentum coordinates are constrained by the energy of the gas. In order to calculate the number of possible microstates, the Schrödinger wave equation must be solved for all molecules.

A paradox proposed by Maxwell in 1871 served to demonstrate the relationship between energy and order. A receptacle (Figure 10.1) is assumed to contain a gas at the same pressure and temperature on both sides of a partition. The partition has a frictionless microscopic door which is operated by a very clever demon—"Maxwell's demon." The demon observes molecules on both sides of the door. When a fast molecule approaches from the right and no molecules are approaching from the left, he opens the door to allow the fast molecule through. He does the same for slow molecules approaching from the left side and, hence, tends to segregate the molecules according to their speed. The average speed of the molecules on the





**FIGURE 10.1** *Maxwell's Paradox.*

left side tends to increase, while the average speed of those on the right tends to decrease. By appropriately selecting molecules the demon achieves a temperature differential between the two sides which could be used to operate a heat engine. The entropy of the system has decreased, since the system is more ordered after the velocity sorting than before. Furthermore, since a frictionless door was assumed, no energy was required according to Maxwell's reasoning to achieve the decrease in entropy.

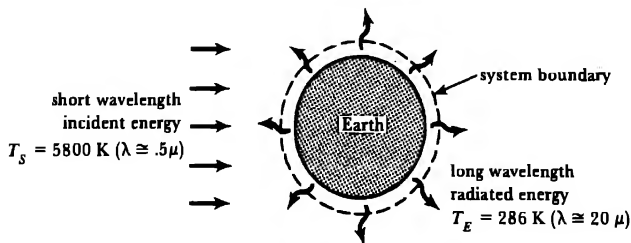
Brillouin is one of those accredited with, as he expresses it, exorcising the demon [9]. A mechanism to ascertain the velocity and position of the molecules was required. In order for us humans to see, light with wavelengths in the visible spectrum is necessary. What we observe is radiation reflected from an object (unless the object is producing its own radiation). The demon of the paradox also needed a source of electromagnetic radiation, not necessarily with visible wavelengths, to see the molecules. Without the illuminating radiation, the information needed for the decision-making process associated with opening the door could not have been obtained. Since a source of energy was necessary to produce the radiation, the sorting required an expenditure of energy. The decrease in entropy of the sorted gas molecules was therefore accompanied by an external entropy increase associated with the generation of the illuminating radiation. (The demon expended negative entropy to do the sorting.) A decrease of total entropy, that of the system plus that of the surroundings, did not occur.

Maxwell's paradox emphasizes the close relationship between the concept of entropy and information theory. The decrease in the entropy of the system required a knowledge of molecular velocities and positions. An energy expenditure was necessary to obtain the knowledge. The expression for information as formalized by Shannon [10] has a very similar form to that of the entropy of a gas. Entropy is a measure of the disorder of a system: the greater the disorder, the greater the quantity of information that is required to specify the state of the system. For example,

a crystal represents a well-ordered system which may be described by specifying only a minimum number of quantities: the type of lattice and the lattice spacing. In order to specify the position and velocity of all molecules of a gas at a particular instant of time, a very large quantity of information is required: three position- and velocity-coordinates for each molecule.

Since the entropy of a system is related to its lack of order, the concept of entropy has often been generalized to apply to processes which are not normally associated with thermodynamics. Biological evolution represents an increase in order for the evolved material (a decrease in entropy) and is frequently referred to by biologists as *complexification*. Cultural evolution and industrialization also represent movements by society from less order to greater order. Even though the entropy change for these evolving processes is difficult if not impossible to quantize, the decrease in entropy associated with the increase in order necessitates an increase in entropy at some other point: a low entropy source of energy was necessary. In addition, life itself requires a continuous input of low entropy energy to compensate for the ultimate entropy increase experienced by all living material, death.

In a closed system, a decrease in entropy of a part of the system can occur at the expense of an increase in the remainder of the system. The earth, however, is not a closed system. The sun provides a continuous input of low entropy energy to the earth, while, owing to its radiation, the earth emits energy with a much higher value of entropy (Figure 10.2). Even though the rates at which energy is absorbed and radiated by the earth are equal (an average power density of  $220 \text{ W/m}^2$ , Chapter 3), the absorption and radiation by the earth result in a degradation of the incident solar energy. If the earth is considered a thermodynamic system, an estimate of the entropy increase of its surroundings may be obtained. Each square meter of the earth's surface has an average energy input rate from the sun of  $220 \text{ W}$ . Assuming the sun's surface temperature,  $T_S$ , for the energy input and the earth's temperature,  $T_E$ , for the energy leaving the earth, the following is obtained for the average rate at which entropy is increasing per unit surface area of the earth.



**FIGURE 10.2** *The Earth as a Thermodynamic System.*

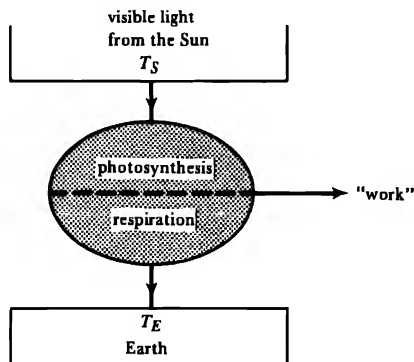
$$\begin{aligned}
 \text{Rate of entropy increase} \\
 \text{of surroundings per unit} &= 220 \left( \frac{1}{T_E} - \frac{1}{T_S} \right) \\
 \text{surface area of the earth} & \\
 &= .75 \text{ J}/(\text{K s m}^2)
 \end{aligned}$$

A concurrent decrease in entropy on the earth is thus possible.

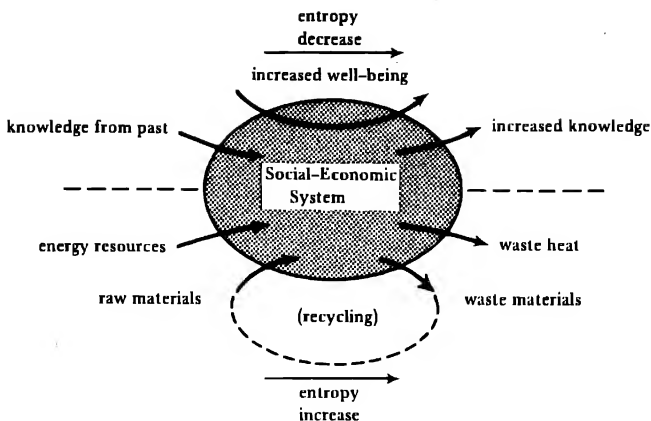
Photosynthesis (Chapter 9, Section 8) is directly dependent upon energetic photons of short-wavelength visible light. Through photon-activated chemical reactions, plants are able to synthesize organic material (for example, carbohydrates) from carbon dioxide and water. Since the energy potential of the organic material is greater than that of its constituents, an entropy decrease is achieved. The organic material also represents a higher degree of order than the carbon dioxide and water from which it was synthesized. The process of respiration, both by the plants and by living organisms which depend either directly or indirectly upon plants, results in low-temperature waste heat. The net effect is a biological heat engine indicated symbolically in Figure 10.3. It is this process upon which life on earth is almost entirely dependent.

Fossil fuels, the principal energy source of modern civilization, are the result of photosynthetic reactions which occurred during previous geological ages. These low entropy fuels have been held within the earth in a state which Brillouin refers to as unstable equilibrium [11]. The natural tendency is for entropy to increase, that is, in the case of fossil fuels, for oxidation or combustion to occur. While geological conditions have prevailed which have retarded this natural tendency, humans have learned to extract these fuels and utilize their low entropy potential.

Highly developed economic systems are dependent upon the earth's low entropy reserves of fossil fuels and minerals. A usage of these reserves, as indicated in the



**FIGURE 10.3** *Biological Heat Engine.*

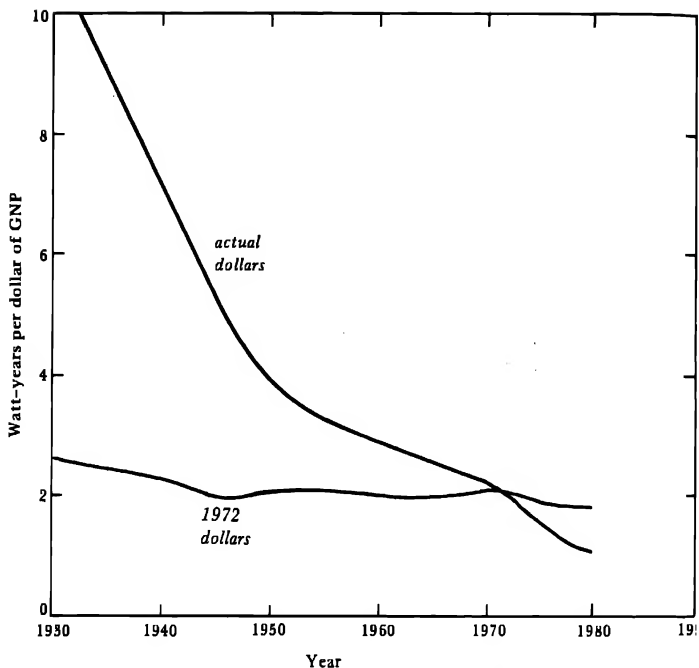


**FIGURE 10.4** *The Social-Economic System.*

lower portion of Figure 10.4, results in an entropy increase. Due to this increase in entropy, a decrease in entropy can be achieved by the social-economic system [12, 13]. An automobile, for example, requires a highly ordered processing of resources and hence represents an entropy decrease for its constituents. Fossil fuels were used to supply the energy required to effect the entropy decrease. While each automobile represents an entropy decrease for its constituents, a decrease in entropy can also be associated with the knowledge system that was required to produce the automobile. In effect, the blueprints from which the automobile was built represent a highly ordered set of information without which the fabrication would not have been possible.

Knowledge is the result of the entire social-economic process, which requires inputs of low entropy reserves. Well-being has also tended to increase even though history has been marked with short-term reversals (for example, wars). Improvements in social systems and increased quantities of material goods represent an increase in order, that is, a decrease in entropy. Again, the earth's reserves were the source of low entropy. Improvement in well-being is often the result of accomplishments that have very subtle effects, for example, a work of art or a musical composition. A new insight, such as a new scientific theory, represents a more ordered image of the world. In addition to increases in order, technological systems frequently produce undesired forms of disorder, that is, pollution. For example, atmospheric emissions represent a "scattering to the winds" of pollutants, an obvious increase in disorder. The release or dumping of many effluents has relied on the process of dispersal to dissipate their negative effects. The earth's atmosphere and waters have, until recently, been treated as if they were infinite in extent.

Recycling of wastes is equivalent to interconnecting the material inputs and



**FIGURE 10.6** *Energy per Dollar of the GNP (References 15-17).*

force necessary to produce goods or services. Figure 10.6 is thus a measure of the energy efficiency of the economy. As of 1979, 1.82 Wy of energy was required to produce each 1972 dollar of goods and services. In terms of 1979 dollars, this amounted to approximately 1.2 Wy per dollar.\* In terms of coal (.936 Wy/kg) nearly 1.2 kg of coal, on the average, would be needed for each 1979 dollar of goods and services, whereas .27 gal of gasoline (125,000 Btu/gal = 4.2 Wy/gal) would be required.

The unchanging quantity of energy per unit of economic activity (1972 dollars) for the 1950 to 1970 period occurred even though the technological improvements introduced during this period should have reduced energy utilization. For example, a marked increase in the average efficiency of generating electricity occurred between 1950 and 1960 (Figure 5.4). Energy costs, however, significantly declined during

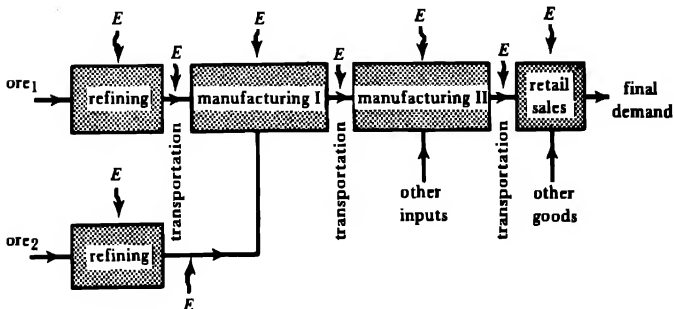
\* Approximately \$1.65 in 1979 dollars was necessary to purchase the same goods and services that \$1.00 would buy in 1972.

this period. Measured in fixed dollars, the average price of electricity in 1970 was approximately one-half that of 1950 [19]. Petroleum prices also declined during the 1957–1967 decade [20]. Government regulation of natural gas resulted in a similar price trend. Since energy became an increasingly better bargain during this period, economic incentives for increasing the efficiency of its usage were lacking.

Conventional economic accounting considers the monetary value added for each processing step. The cost of an output is therefore equal to the cost of all the inputs plus the value added which includes the cost of labor, taxes, and capital equipment. Like these value-added inputs, energy is required for each process. Thus, in an analogous fashion, the energy required could be considered an energy value added. Like a dollar cost, the energy cost would be the sum of all added energy. If an energy accounting existed, goods could be compared on the basis of energy required to produce them. Energy-efficient processes could thus be readily identified.

Industrial energy consumption is most often associated with energy intensive processes like iron and aluminum manufacturing. In general, all industries require substantial quantities of energy for machinery, heating or airconditioning, lighting, and transportation. This is also true for the service sector. While many energy expenditures may be peripheral to the actual good or service being considered, they cannot be ignored. Odum, in addition to dealing with energy flows in biological systems, has considered flows associated with economic systems [21, 22]. Figure 10.7 illustrates an energy flow for a hypothetical process. In addition to direct energy inputs, manufacturing requires indirect energy inputs in the form of capital goods. Transportation services, in addition to direct fuel consumption, require indirect inputs in the forms of trains, trucks, rails, roads, etc. The final energy value of a commodity that reaches the final demand sector must include its share of these indirect inputs.

Individuals and households purchase fuel and electricity that is used in the residence and for transportation. Referring to Figure 1.8 of Chapter 1, residential usage accounts for nearly 20% of the total energy consumed. Since approximately half the transportation sector (25% of total) is for individual usage, direct consumer



**FIGURE 10.7** *Hypothetical Manufacturing Process.*

purchases account for nearly one-third of the total. If governmental fuel purchases are considered, an estimate for all final demand energy purchases of 35% of the total energy consumed is obtained. Hence, nonfinal demand purchases account for approximately 65% of the total energy consumed. Therefore, ignoring the small dollar contribution of direct energy sales to the GNP, the energy required for each 1972 dollar of goods and services was about 1.18 Wy. In terms of 1979 dollars, approximately .73 Wy of energy was required on the average, for each dollar of economic activity.

A summation of direct, as well as indirect, energy requirements (as indicated in Figure 10.7) is necessary to arrive at an overall energy value for a particular item. This suggests the use of an input-output economic analysis as discussed by Leontief [23–26] and others [27–29]. As normally formulated, input-output tables provide a means of arriving at indirect monetary requirements. Input-output analysis divides the economy into sectors, which, in general, sell to consumers as well as to other sectors. Sectors, in turn, purchase from other sectors. Food, textiles, appliances, automobiles, tires, electric utilities, amusements, and real estate are examples of the economic sectors used. Each sector of the economy, for a particular economic activity, has a total output of  $x_i$  dollars. The output of each sector consists of a final demand,  $y_i$ , and its interindustry sales. If  $x_{ij}$  is the dollar sales of industry  $i$  to industry  $j$ , the total interindustry sales by  $i$  must be the sum of these sales.

$$\text{Total interindustry sales by sector } i = \sum_{j=1}^n x_{ij}$$

Adding final demand,  $y_i$ , to the above sales yields the total output of sector  $i$ .

$$x_i = y_i + \sum_{j=1}^n x_{ij}$$

$$y_i = x_i - \sum_{j=1}^n x_{ij}$$

For the summation,  $n$  sectors have been assumed. The sale by sector  $i$  to sector  $j$ ,  $x_{ij}$ , is an input to sector  $j$ , that is, a purchase of sector  $j$ . Table 10.1 illustrates this relationship. The total output of sector  $i$ ,  $x_i$ , is the sum of all the elements of the  $i$ th row, that is, all sales to other sectors as well as that which goes to final demand. Final demand, by convention, consists of all consumer and government purchases as well as capital investments (such as buildings). While imports, exports, and inventory changes are also included in the final demand portion of input-output tables, their effect is usually small and will be ignored in the subsequent discussion. Knowing all interindustry transactions is equivalent to knowing all the terms of the  $x_{ij}$  array, which, for an  $n$ -sector table, requires knowing  $n^2$  elements. Even for a modest sized table, the array represents a rather detailed knowledge of economic transactions.

Since  $x_{ij}$  is a sale by sector  $i$  to sector  $j$ , it corresponds to a purchase by sector  $j$ . For the general case, sector  $j$  purchases from all other sectors.

**TABLE 10.1 An Input-Output Table**

	<i>Sector j</i> (purchase)	<i>Final demand</i>	<i>Total output</i>
<i>Sector i</i> (sale)	$\begin{matrix} \cdot \\ \cdot \\ \cdot \\ \cdot \cdot \cdot \cdot \cdot x_{ij} \cdot \cdot \cdot \cdot \cdot \\ \cdot \\ \cdot \\ \cdot \end{matrix}$	$y_i$	$x_i$ (sum of row)
$n + 1$ row	+ Value added by sector $j$	GNP	
	$x_j$ (sum of column)		

$$\text{Total purchases by sector } j = \sum_{i=1}^n x_{ij}$$

By definition, the output of sector  $j$  is equal to the sum of all its purchases, plus a value-added term.

$$x_j = \sum_{i=1}^n x_{ij} + \text{value added by sector } j$$

The value added by each sector can be included with the array of interindustry sales as the  $n + 1$  row of the table. Therefore, summing a column also results in the total output of a particular sector.

The gross national product is equal to the sum of all final demands, that is, the column sum of the individual final demands.

$$\text{GNP} = \sum_{i=1}^n y_i$$

The gross national product, however, is also related to the value-added terms. The value added for a particular sector is the sum of its wages, taxes, and capital expenditures. These expenditures provide the purchasing power (consumer, capital goods, and government) which produces the final demand of the economy. A summation of all value-added terms must therefore also equal the GNP.

$$\text{GNP} = \sum_{j=1}^n \text{value added by sector } j$$

The structure of an input-output table thus tends to emphasize the relationship of



final demand and value-added quantities, the summation of either being equal to the GNP.

It should be noted that the sum of the final demands, the  $y_i$  terms, is used as an economic indicator (GNP) rather than the sum of the outputs of the various sectors, the  $x_i$  terms. A simple summing of the sector outputs would, for many goods and services, result in a multiple counting. For example, the tire-producing sector sells a substantial portion of its output to the automobile manufacturing sector. The total output of the automobile sector includes the value of the tires purchased by it. Those tires used for new cars would thus be counted twice if the total output of the tire-producing and automobile manufacturing sectors were added. Furthermore, the sum of the output of all sectors is dependent upon the arbitrary divisions used for constructing a table. If, for the above example, the tire and automobile sectors were consolidated into a single tire and automobile sector, a different sum would be obtained even though no real change in economic activity occurred.

Consider the following ratio which is designated as  $a_{ij}$ .

$$a_{ij} = \frac{x_{ij}}{x_j}$$

The numerator,  $x_{ij}$ , is a purchase by sector  $j$  from sector  $i$ . Dividing by the total output of sector  $j$  results in a quantity,  $a_{ij}$ , which is the dollar value of the purchase from sector  $i$  that is required for each dollar of output of sector  $j$ . These quantities, necessarily less than one, represent the intersector or interindustry dependence of the various sectors. If  $a_{ij}$  is zero, no purchase from sector  $i$  took place whereas if it is large, sector  $j$  is highly dependent upon sector  $i$ . These terms depend upon both the technology used to produce a given product (for example, the type of inputs) and the manner in which production is organized.

Each sale or purchase,  $x_{ij}$ , can thus be expressed in terms of  $a_{ij}$  and the output of industry  $j$ , the purchaser.

$$x_{ij} = a_{ij}x_j$$

Using the  $a_{ij}$  terms, the final demand of sector  $i$  may now be expressed in terms of the outputs of all sectors.

$$y_i = x_i - \sum_{j=1}^n x_{ij} = x_i - \sum_{j=1}^n a_{ij}x_j$$

Since  $n$  sectors are involved,  $n$  equations representing the  $n$  final demands are obtained.

$$\begin{aligned} y_1 &= (1 - a_{11})x_1 - a_{12}x_2 && - \cdots - a_{1n}x_n \\ y_2 &= -a_{21}x_1 && + (1 - a_{21})x_2 - \cdots - a_{2n}x_n \\ &\vdots \\ y_n &= -a_{n1}x_1 && - a_{n2}x_2 - \cdots + (1 - a_{nn})x_n \end{aligned}$$

For a specified set of total outputs,  $x_1, x_2, \dots, x_n$ , the above equations yield the final demand of each sector.

The above set of equations relates final demands and total outputs in terms

of the intersector dependence coefficients, the  $a_{ij}$  terms. If, instead of specifying the total outputs ( $x_i$ ), the final demands were specified ( $y_i$ ), the equations could be solved for the corresponding values of the  $x_i$  terms. Since the initial set of equations is linear, a solution for the  $x_i$  terms that is linearly dependent upon the  $y_i$  terms would be obtained.

$$\begin{aligned}x_1 &= A_{11} y_1 + A_{12} y_2 + \cdots + A_{1n} y_n \\x_2 &= A_{21} y_1 + A_{22} y_2 + \cdots + A_{2n} y_n \\&\vdots \\x_n &= A_{n1} y_1 + A_{n2} y_2 + \cdots + A_{nn} y_n\end{aligned}$$

The  $A_{ij}$  coefficients are a "solution" of the original set of equations. They are, in matrix terms, the coefficients of the inverse of the unit matrix minus the  $a_{ij}$  matrix. Consequently, the  $A_{ij}$  terms are known as *inverse coefficients*.

The coefficients  $a_{ij}$ , as well as the derived inverse coefficients  $A_{ij}$ , describe the interrelatedness of the economic sectors. Published tables usually provide both sets of coefficients. While the  $a_{ij}$  terms relate the input of a given sector to its total output, the inverse coefficients relate the output of a sector to all final demands. The term  $A_{ij} y_j$  is the output of sector  $i$  necessary to provide a final demand,  $y_j$ . To be more specific, consider the first of the above equations. The term  $A_{11} y_1$  is the output of sector 1 to produce the final demand,  $y_1$ . For  $A_{11} = 1$ , an output of only  $y_1$  would be required. Since other industries that depend upon the output of sector 1 may either directly or indirectly provide inputs to sector 1,  $A_{11}$  is normally greater than one. The term  $A_{12} y_2$  is the output of sector 1 necessary to supply a final demand  $y_2$ . Since the  $x_i$  terms represent the total output, both direct and indirect demands are included in producing a specified set of final demands. Also, since  $A_{ij} y_j$  is the output of sector  $i$  for a final demand  $y_j$ , each dollar of final demand of the  $j$  sector requires an output of  $A_{ij}$  dollars of sector  $i$ .

Again the automobile manufacturing and tire-producing sectors may be used to demonstrate the significance of the inverse coefficients. If  $y_j$  is the final demand term of the automobile sector and  $x_i$  the output of the tire sector, then  $A_{ij} y_j$  is the output of the tire sector necessary to provide the tires required for the automobile sector's final demand. This term ( $A_{ij} y_j$ ) includes the output of tires purchased by the automobile manufacturer as well as other indirect consumption of tires that occurs. It includes, for example, the tires that are consumed by the transportation sector in transporting both completed automobiles and parts that are used to produce an automobile. In essence, not only do the five tires that are sold as part of a completed automobile need to be provided by the tire-producing sector, but also an additional quantity of tires (probably less than one) as a share in indirect activities that result from the production of an automobile.

### 3. ENERGY PER DOLLAR VALUE OF GOODS AND SERVICES

Inverse coefficients of an input-output table provide a measure (usually in monetary terms) of indirect requirements necessary to produce a given final demand. An

extension of this analysis to estimate indirect energy requirements is not unreasonable [30-34]. Suppose sector  $i$  is an energy-producing sector (for example, coal mining). Its total dollar output is given by a sum of terms involving the inverse coefficients and the final demands.

$$x_i = A_{i1} y_1 + A_{i2} y_2 + \cdots + A_{ij} y_j + \cdots$$

Each term represents the output of sector  $i$  that is required to provide for a particular final demand. The term  $A_{ij} y_j$  is the output of sector  $i$  (for example, the dollar value of coal) necessary to provide all direct and indirect inputs from sector  $i$  to produce the final demand  $y_j$ .

If the energy output of sector  $i$  is  $E_i$ , the ratio  $E_i/x_i$  is the average energy per dollar sale of that sector,  $\kappa_i$ .

$$\kappa_i = E_i/x_i$$

The inverse relationship for  $x_i$  may be converted to an energy relationship through multiplication by  $E_i/x_i$  or  $\kappa_i$ .

$$E_i = \kappa_i A_{i1} y_1 + \kappa_i A_{i2} y_2 + \cdots + \kappa_i A_{ij} y_j + \cdots$$

Each term of the above equation is the portion of the total energy produced by sector  $i$  that is required for the respective final demand. The  $y_j$  coefficient,  $\kappa_i A_{ij}$ , is therefore the energy of sector  $i$  that is required for each dollar of final demand of sector  $j$ . The total energy for a given final demand can be obtained by considering all energy-producing sectors. Energy values obtained in this fashion are based upon an average value of energy per dollar sale of sector  $i$  ( $\kappa_i$ ). Variations in energy prices which might occur due to quantity sales are thus ignored.\*

Several input-output tables have been prepared for the U.S. economy [35-41]. The most recent are those for 1972: a condensed set of tables with 85 sectors [42, 43] and a more detailed set with 496 sectors [44].† These tables, in addition to providing the dollar values of all transactions, include values of the direct coefficients, the  $a_{ij}$ 's, as well as the inverse, or indirect coefficients, the  $A_{ij}$ 's. Final demand is segmented into personal consumption, new residential construction, highway construction, and investment in producer durable equipment (capital goods). In addition, federal defense, other federal expenditures, and state purchases are included. For accounting purposes, inventory changes, imports, and exports are included in the final demand sector.

The 85-sector input-output tables for 1972 will be used to illustrate the method by which energy values for various goods and services may be obtained. This set of tables has four energy-related sectors. Two sectors, coal mining and that labeled crude petroleum and natural gas, may be classified as primary energy sectors. The other two energy-related sectors may be considered secondary or derived-energy sectors. If it were not for imports of refined petroleum products, the petroleum

\* The direct coefficients, the  $a_{ij}$  terms, introduce a similar error.

† A six- to seven-year period has been required by the U.S. Department of Commerce to prepare these tables.

refining and related industries sector would be totally dependent upon the crude petroleum and natural gas sector for its energy inputs (crude oil). A summing of the indirect energy requirements based upon the energy supplied by both the crude petroleum sector and the refined petroleum sector would, in effect, result in a double accounting of the crude oil used to produce a given item. A similar situation occurs for the electric, gas, water, and sanitary services sector. If it were not for hydro and nuclear power inputs of electric utilities, the energy inputs of this sector would be totally dependent upon outputs of the primary energy sectors of coal mining and of crude petroleum and natural gas (ignoring petroleum imports).

To illustrate indirect energy requirements, consider the food (and kindred products) sector. This large sector (9.9% of the consumer final demand) is rarely thought of in terms of being a substantial energy consumer. Its direct purchases of crude oil and natural gas are negligible (< \$.1 million, refined petroleum purchases are \$281 million), and its purchases of coal of \$35 million are small (.7% of the total coal output). The direct coefficient for coal of the food sector is only .00030, that is, for each dollar output of food, .030¢ of coal is purchased. In terms of energy, this purchase is negligible (approximately .025 Wy). Food, however, requires indirect energy inputs for farm machinery, fertilizers, transportation, packaging, processing, etc. The  $A_{ij}$  inverse coefficients reflect indirect requirements. For food, 1.34¢ of crude oil and natural gas is ultimately needed for each dollar of food. Over ten times the direct purchases of coal, .32¢ instead of .03¢, are also required.

In 1972, the overall energy consumption rate was  $2.39 \times 10^{12}$  W while the rate of economic activity was  $\$1183 \times 10^9$  (per year) [17, 43]. An average energy intensity for the overall economy,  $\kappa$ , may be obtained.

$$\begin{aligned}\kappa &= \frac{\text{energy consumption rate}}{\text{GNP}} \\ &= 2.02 \text{ Wy}/\$ \quad (1972)\end{aligned}$$

Table 10.2 provides the 1972 distribution of energy usage based upon categories corresponding to the energy sectors of the Department of Commerce Input-Output tables being used. An average value (the appropriate  $\kappa_i$ ) may be calculated for the primary energy sectors.

**TABLE 10.2 Summary of 1972 U.S. Energy Consumption<sup>a</sup>**

	<i>Per cent of total consumption</i>
Coal mining	17.4%
Crude petroleum and natural gas	70.7
Petroleum imports	7.0
Hydro and nuclear power	4.9
	100.0%

<sup>a</sup>(Reference 17).

$$\begin{aligned} \kappa_{\text{coal}} &= \frac{.174\kappa \text{ (GNP)}}{\text{total coal sales}} \\ &= 41.4\kappa \end{aligned}$$

$$\begin{aligned} \kappa_{\text{oil-gas}} &= \frac{.707\kappa \text{ (GNP)}}{\text{total crude oil and gas sales}} \\ &= 42.8\kappa \end{aligned}$$

The numerators of the above expressions will be recognized as the total energy provided by each sector. The above coefficients account for all energy usage except that due to petroleum imports and hydro and nuclear power. Imports of crude oil have been included in obtaining the energy coefficient for the crude oil and natural gas sector.

An energy coefficient for the petroleum refining sector, based only upon the energy contribution of imports, may be obtained.

$$\begin{aligned} \kappa_{\text{petroleum refining}} &= \frac{.070\kappa \text{ (GNP)}}{\text{total petroleum refining sales}} \\ &= 2.53\kappa \end{aligned}$$

This energy coefficient, when used with the appropriate inverse coefficient ( $A_{ij}$ ), will account for petroleum imports but not result in a double accounting of crude oil used by domestic refineries. The effect of hydro and nuclear power inputs may be handled in a similar fashion.

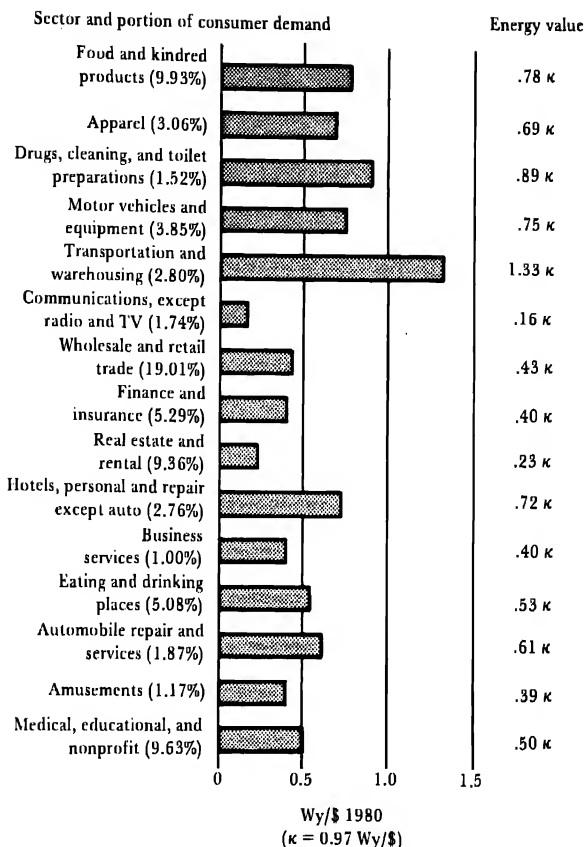
$$\begin{aligned} \kappa_{\text{utilities}} &= \frac{.049\kappa \text{ (GNP)}}{\text{total utility sales}} \\ &= .84\kappa \end{aligned}$$

Returning to the food sector, the inverse coefficients for the energy sectors may now be used to obtain a total energy value for this sector (per dollar of food sales).

$$\begin{aligned} .00323(41.5\kappa) &= .134\kappa && \text{coal mining} \\ .01336(42.8\kappa) &= .572\kappa && \text{crude petroleum and natural gas} \\ .02106(2.53\kappa) &= .053\kappa && \text{petroleum refining, imports} \\ .02809(.84\kappa) &= .023\kappa && \text{utilities, hydro and nuclear power} \\ \hline \text{total} &= .782\kappa \end{aligned}$$

Inserting the value of  $\kappa$  for 1972 (2.02 Wy/\$), an energy value of 1.58 Wy/\$ is obtained for the food and kindred products sector. Each 1972 dollar of sales by this sector entailed, on the average, the usage of 1.58 Wy ( $5.0 \times 10^7$  J or 47,400 Btu) of energy. In terms of a petroleum energy equivalent, each dollar of food required a consumption of about one-third gallon of petroleum.

Figure 10.8 presents energy values for the nonenergy sectors in which personal



**FIGURE 10.8 Energy Required per Dollar Value of Goods and Services.**

consumption was greater than 1% of the 1972 consumer final demand of \$738 billion (62.4% of total final demand of \$1183 billion). (Other final demand components are capital investments, 15.6%, governmental purchases, 21.4%, and small corrections for inventory changes, imports, and exports.) While variations between sectors are not negligible, the differences are considerably less than might initially be anticipated. Attempts to change the mix of consumer products for the purpose of reducing energy consumption could have, at best, a marginal effect.

The bar graph of Figure 10.8 gives the energy intensiveness of the various sectors based upon the 1980 energy value,  $\kappa$ , of .97 Wy/\$. The large change in  $\kappa$  from 1972 to 1980 was primarily due to the inflationary change of the purchasing value of the dollar. The 1980 energy values, it should be stressed, are only approximate since they are based upon the economic structure of 1972, not that of 1980.

Both the food sector and the motor vehicle manufacturing sector have comparable energy values, requiring in 1980 an energy expenditure of about .75 to .78 Wy (24 MJ or 23,000 Btu) for each dollar of final sales. Food (and kindred products) accounted for about 6.2% (62.4% of 9.93% of consumer final demand) of final demand in 1972. If the portion of final demand for food were the same in 1980, per capita expenditures for food would have been \$740. Therefore, on a per capita basis, the rate of energy usage for producing food would have been 560 W (560 Wy/y). This is about 5% of the overall rate of energy consumption.

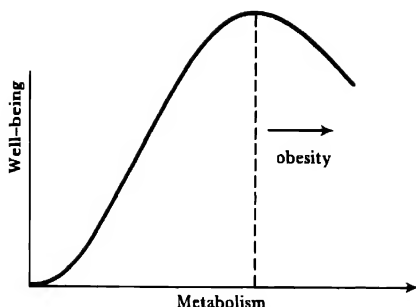
Input-output tables may be utilized to determine the overall energy requirements for various energy-producing or -conserving endeavors. In particular, the energy required to produce a system may, knowing its investment cost, be estimated by using the energy value of the appropriate economic sector. Very expensive systems imply that a large energy expenditure is necessary and while this energy is often ignored, it should be considered when evaluating an energy system. An overly expensive energy-producing system, for example, could require more energy for its manufacture than it provides over its expected lifetime. Data of a more detailed table (496 sectors, for example) is generally required for evaluating most energy-producing or -conserving systems.

#### 4. SOCIETAL IMPLICATIONS

It is often either assumed or implied that an increase in energy consumption represents an improvement in well-being. Complementing the Industrial Revolution which began in the early part of the nineteenth century in the United States was the very rapid increase in energy consumption.

Without underestimating the human "price," industrialization has substantially improved the condition of those of the industrial countries. To the extent that inanimate energy was necessary for industrialization, the concurrent improvement in well-being was dependent upon fuels. However, improvement with the increase of a given input seldom continues indefinitely. The concept of marginal utility or diminishing returns has long been recognized by economists.

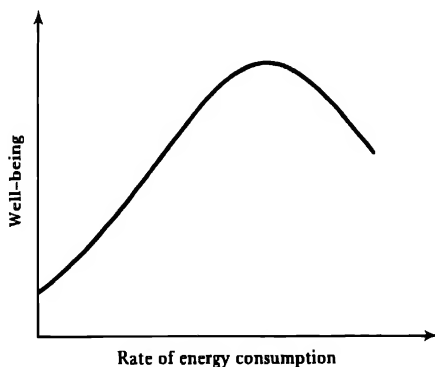
Marginal utility may readily be illustrated for the case of human metabolism. Metabolism (usually measured in kilogram calories per day) is the rate of energy consumption, that is, power, provided by the intake of food. Even without a precise definition of well-being, a curve with a characteristic shape of that of Figure 10.9 is likely. An increase in well-being with an increase in food does not continue indefinitely. While the metabolic input associated with the peak depends upon environment (average temperature for example) and physical exertion, intakes greater than that for the peak result in obesity.



**FIGURE 10.9** *Human Well-Being as a Function of Metabolism.*

A similar situation, on a societal level, may be true relative to nonmetabolic energy consumption. Well-being as a function of energy input is very likely to reach a peak (zero marginal utility) and then decline as indicated in Figure 10.10. A crucial problem is that of determining the location of the peak. For the U.S. society, with its geographical distribution and cultural roots, an optimum consumption rate, dependent to an extent upon a more equitable distribution, undoubtedly exists.

The present average consumption rate, 12 kW per capita, may be such that a further increase does not necessarily reflect a societal improvement. For example, consider residential heating. An increase in fuel consumption results in greater comfort until the desired temperature is achieved. Comfort, however, is not neces-



**FIGURE 10.10** *Probable Relation Between Well-Being and Energy Consumption.*



sarily dependent upon the fuel consumption rate. A non insulated, poorly constructed home requiring a high fuel consumption is, from the standpoint of pure comfort, less desirable than a well-insulated and well-constructed house with a low fuel consumption. Low fuel costs together with high interest and construction costs may, despite the decrease in comfort, encourage the former type of construction.

Individual mobility is obviously important for a developed society. Before the Industrial Revolution, individual mobility was extremely limited. The introduction of the railroad in the eighteenth century and the automobile in the early portion of the twentieth century vastly increased mobility. Widescale usage of air transportation is a further increase. When mobility was minimal, it could readily be argued that all increases represented an improvement in well-being. Today, this is not necessarily true. An increase in automobile fuel consumption may reflect an increase in commuting, which in turn reflects a decline in the quality of urban environments. The additional commuting, for this case, is used to "escape" undesirable urban conditions. An urban improvement might decrease commuting and, hence, reduce fuel consumption. An increase in gasoline consumption may thus be totally unrelated to an increase of pleasure.

Further reflection can reveal many negative social changes that have resulted in a concurrent increase in energy consumption. Increased lighting to reduce crime, higher airconditioning loads associated with crowded working conditions, and a decline of mass transportation services (thus increasing automobile usage) are examples. Furthermore, excessive consumption of material goods (all of which require energy to produce) may reflect efforts to compensate for deteriorating social conditions.

Since variations in energy requirements per dollar value of goods and services are not very great, total energy consumption is not critically dependent upon the mix of goods and services of the economy. A simple reordering of economic activities, for example a growth of the service sectors, would probably have only a minimal effect on total energy consumption. It should be recognized that many proposed methods of conserving energy would concurrently reduce (conserve) economic activity. An improved transportation system would not only conserve energy but would be less costly (economically more efficient).

Predictions of energy consumption for the next few decades have and are being continually revised downward [45-48]. The potential for using energy more effectively may be shown to be very great. With increasing energy prices, it is becoming very expensive to be inefficient. To the extent that energy is used more efficiently, economic growth without a corresponding growth in energy consumption can be achieved. However, arguments for continued economic growth are also being questioned [49-53]. As is the case for increased energy consumption, increased quantities of economic goods and services may not be necessary if they, too, are used more effectively.

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## PROBLEMS

1. *The Economic Report of the President*, published in January or February of each year, is a convenient source of economic data. (More recent data can be obtained from the *Survey of Current Business*, published by the Department of Commerce.) Energy values can be obtained from the *Monthly Review of Energy*, published by the Department of Energy.
  - (a) What was the gross national product (GNP) for the previous year expressed in both 1972 and actual dollars? What was the price index for that year?
  - (b) What was the rate of energy consumption, expressed in watts, for the year used in part (a)?
  - (c) What was the average energy value,  $\kappa$ , expressed in watt-years per dollar, for both actual dollars and 1972 dollars?
  - (d) Determine the value of  $\kappa$ , for 1972 dollars, for the five previous years. Can a trend be associated with the change in  $\kappa$  for this period?
2. The GNP is the sum of the final demands, that is,  $\text{GNP} = \sum_{i=1}^n y_i$ . Why is the quantity independent of the method in which sectors are chosen? Why are governmental expenditures (federal, state, and local) included in this final demand? *Note:* The value added term for each sector includes both wages and taxes paid by that sector.
3. Why must the final demand,  $y_i$ , of a particular sector be less than the total output of the sector? Why, for each dollar of final demand, must the sector have an output of more than one dollar? Consider the petroleum industry.

Why must more than one gallon of petroleum be produced for each gallon of gasoline sold at the local gasoline station? Prepare a list of indirect uses that were necessary to deliver this gallon to the consumer.

4. Consider the following hypothetical input-output table in which  $M$  could represent the manufacturing sector and  $C$  the commercial sector.

	$M$	$C$	<i>Final demand</i>
$M$	700	300	500
$C$	200	300	500
<i>Value added</i>	600	400	1000 GNP

While the monetary units in the above table are arbitrary, they could be thought of in terms of  $\$10^9$ , that is, a GNP of  $\$1000 \times 10^9$ .

- Determine the total output of each sector and the corresponding direct coefficients (the  $a_{ij}$ 's).
  - Express the final demands,  $y_1$  and  $y_2$ , as a function of the total outputs  $x_1$  and  $x_2$  and the  $a_{ij}$ 's. Leave the  $y$ 's and  $x$ 's in the equation rather than replacing them with their numerical values.
  - Solve the above equations to obtain the inverse coefficients (the  $A_{ij}$ 's).
  - Determine the output of each of the two sectors necessary to obtain one unit of final demand for each sector.
5. Consider the following hypothetical input-output table which, in addition to the manufacturing ( $M$ ) and commercial ( $C$ ) sectors, has an energy sector,  $E$ .

	$M$	$C$	$E$	<i>Final demand</i>
$M$	700	300	60	500
$C$	200	300	40	450
$E$	60	40	0	50
<i>Value added</i>	600	350	50	1000 GNP

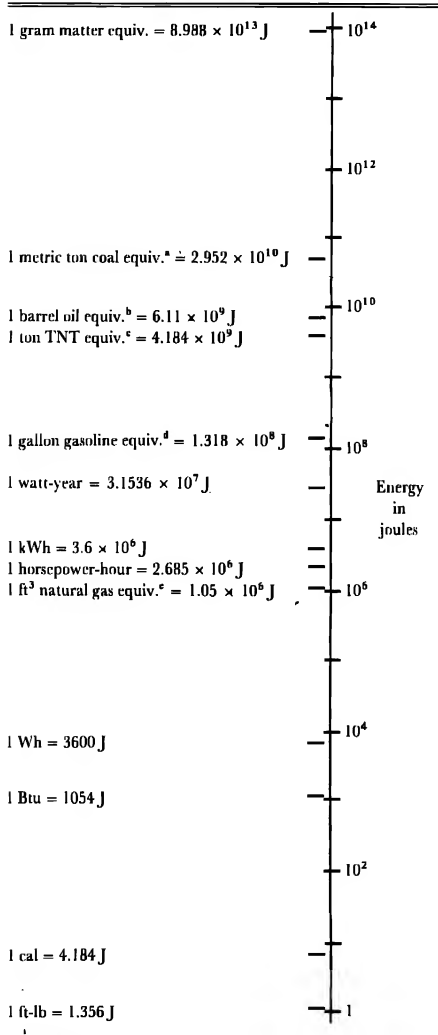
The energy per dollar value of GNP is  $\kappa$ .

- Based upon sales, determine the percentage of energy used by the manufacturing and commercial sectors and supplied to final demand.
  - Determine the direct and indirect coefficients for this table.
  - Assume the GNP of the table is \$1000 billion and the energy value,  $\kappa$ , is 1.5 Wy/\$. Determine the energy requirements per dollar of final demand for the manufacturing and commercial sectors.
6. Consider an automobile that sells for \$8000 (1980 dollars).

- (a) Based upon the data of Figure 10.8, determine the energy required to produce the automobile.
  - (b) Assume that the automobile achieves an average mileage-gasoline rate of 25 mpg and is driven 10,000 miles in one year. Compare the energy required to produce the automobile to that consumed (in gasoline, 125,000 Btu/gal) in one year of driving.
7. Lead-acid batteries are frequently used for electrical energy storage (electric cars or wind generators). Assume a storage battery costs \$50/kWh of storage capacity and has a charge-discharge life of 1000 cycles.
- (a) What is the average battery cost associated with each kilowatt-hour of electricity stored by it?
  - (b) Assume the storage batteries require an energy of approximately .60 Wy/\$ to produce. (Lead-acid batteries are essentially recycled.) What is the energy cost of producing a one kilowatt-hour storage battery? What is the energy cost associated with the storage of one kilowatt-hour of electrical energy? (Since the energy calculated to produce the battery was thermal energy, a direct comparison of this energy to the electrical stored energy would not be appropriate. A comparison to the thermal energy required to produce the electrical energy by conventional means would be more appropriate).
8. Consider a 40% efficient fossil-fueled electric power plant that costs \$1000/kW (electric) to construct. The power plant has an average duty factor of .75, that is, its yearly averaged output energy is 75% of that obtainable if the power plant operated a full power for the entire year.
- (a) Determine, assuming a requirement of .80 Wy/\$, the energy required to construct each kilowatt of generating capacity.
  - (b) What is the rate of thermal power produced by the power plant? What is the thermal energy used (expressed in watt-years) per year?
  - (c) Determine the length of time (in years) for the energy consumed by the power plant to equal that required to produce it. If the power plant has a useful life of 40 years, determine the ratio of the energy to produce the power plant to that used by it.
  - (d) Why was the thermal energy consumption of the power plant used for comparison rather than its electrical energy output?

# Appendix

**TABLE A.1 Equivalent Units of Energy**



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